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Abstract

This paper shows that firms producing homogeneous goods (e.g. Bertrand competitors) can achieve supernormal profits using interfirm bundled discounts, which connect their product with a specific brand of other firm with market power. By committing to a price discount exclusively to buyers of a particular brand of another good, the firms create a sort of artificial switching costs and attain a semi-collusive outcome. In fact, the discount scheme allows the firms with no market power to avoid Bertrand trap by leveraging other firms' market power. Consumers are worse off due to higher prices under bundled discounts.

JEL Classification:

Keywords: Brand-specific discounts, bundling, co-branding, co-promotion

1 Introduction

Nowadays we often find firms offering price discounts conditional on the purchase of a particular brand of other (related or independent) products. This kind of interfirm bundled discounts are becoming one of the most important marketing practices in many industries. Credit card companies provide price discounts or rewards to customers of a specific petrol station, telecommunications or Internet service provider, amusement park, airline, hotel chain, car

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rental company, motor insurance company, and so on. Some grocery stores offer rewards points to consumers who buy from a specific travel agent, internet merchant or auction site.

We observe some notable features of such discount schemes. First, they are brand-specific in the sense that discounts are offered exclusively to those who buy some designated brand of other product. Second, the discount schemes create interlocking relations between the products supplied by different firms in different markets. So it is like interfirm bundling across industries. Finally, firms usually announces discount schemes prior to setting their retail prices.

This paper analyzes a situation where firms selling homogeneous products (Bertrand competitors) offer price discounts to consumers who purchase a specific brand in other differentiated product market. We find that these bundled discounts create a sort of artificial switching costs and relax price competition. The brand specificity of discounts segments consumers who are otherwise homogeneous into two groups, each loyal to a particular product, and the firms use this as a strategic semi-collusive device. This is in fact similar to "puppy dogs ploy" in terms of Fudenberg and Tirole (1985)'s animal spirits taxonomy of business strategies. An interesting result is that brand-specific discounts allow firms with no market power at all to achieve supernormal profits by leveraging other firms' market power. Consumers, however, become worse off due to higher effective prices.

Closely related to our analysis is the article by Gans and King (2006). Analyzing the Hotelling competition with two firms in each of two symmetric differentiated markets, they show that two pairs of firms end up with jointly offering bundled discounts in equilibrium even though no firm benefits from the discount arrangements. So, the bundled discounts appear as a prisoners' dilemma type outcome resulting from discount competition between two coalitions. In fact, in their model effective prices (headline prices minus discounts) with bundled discounts are smaller than equilibrium prices without discounts, although consumer surplus and total welfare fall because consumer choice is restricted by the bundling nature of brand specificity. Matutes and Regibeau (1988) derive a similar result in the context of bilateral (in)compatibility decisions among independent firms in symmetric system markets.[1] In contrast, our model

[1] See also Matutes and Regibeau (1992) for the case where the firms can choose to offer bundles together with compatibility decisions.

explains *profitable* bundled discounts under asymmetric market structures. In particular, we establishes that using unilateral bundled discounts firms with no market power can achieve supernormal profits by leveraging the other firms' market power. In this respect, our paper is similar to the recent work of Katz and Hermalin (2011). Analyzing exclusive contracts in system markets consisting of perfectly complementary goods, they showed that exclusive deals with providers of differentiated applications can raise platforms' margins without reducing applications' margins, so that overall industry profits rise. Our paper in contrast shows that a similar result is via unilateral brand-specific discount schemes in totally independent markets.

This paper is also related to the literature on endogenous switching costs. Banerjee and Summers (1987) show that in a two-period repeat purchase framework Bertrand duopolists can earn monopoly profits in both periods by offering discount coupons to repeated users in period one. Repeated-purchase discounts induce consumer loyalty due to switching costs, which enables the firms to segment the market and charge higher prices in period two. Also, the firms resist cutting prices and taking over the whole market in period one as well, in order to insure a large enough install base of the rival in the later period. Caminal and Matutes (1990) examine loyalty inducing strategies in differentiated product markets, and show that equilibrium profits fall with price commitment but increase with discount commitment. Also related in this line of research is the recent works on behavior-based price discrimination, which analyze the effects of intertemporal discriminatory pricing based on purchase histories.[2] Even though the basic logic is similar, our paper is distinct from these earlier literatures in several aspects. First, we show that the semi-collusive effect of discounts works in a one-shot framework without repeat purchases over time. Second, consumer loyalty in our model is created by interfirm relations across different markets via discount schemes rather than intertemporal consumption relations of the same good. In this respect, our analysis is focused on the role of brand-specific discounts in leveraging market power in one market to another market.

A related work in the bundling literature is Chen (1997). He shows that multiproduct

[2] See Villas-Boas (1999), Fudenberg and Tirole (2000). Fudenberg and Villas-Boas (2007) provide an excellent survey on this topic.

duopolists producing two goods, one in a market with homogeneous consumer valuations and the other in a competitive market with heterogeneous consumer valuations, can use bundling as an effective differentiation mechanism to avoid price wars. Brand-specific discounts in our model play a similar role, allowing firms selling homogeneous good to artificially differentiate their products and thereby escape from the Bertrand trap. In our model, however, the firms produce a single product, and they commit to contractual arrangements in relation to other firms' products rather than physical bundling of their own products.

Our work is also related to some studies in the marketing literature analyzing the effects of coupons in retail markets. Dhar and Raju (1998) investigate how interbrand discount coupons (cross-ruff coupons in their terminology) affect consumers' choice and how to design a coupon to enhance sales and profits.[3] But they do not explicitly consider strategic interactions among competing firms which is the main focus of our paper.

2 Model

We consider markets for two products A and B, each served by two competing firms, A_1 and A_2 and B_1 and B_2 respectively. The firms in each market compete in price. We assume without loss of generality that there are no costs associated with the production of either product. There is a population of consumers of mass one, whose preferences for the two products are independent. Individual consumers have unit demands for each product. All agents have complete information.

The products offered by firms, A_1 and A_2 are identical in the eye of consumers. Let v_A denote the reservation price for a unit of good A, which is common to all consumers, and p_A^i denote the (headline) price charged by firm $A_i, i = 1, 2$. On the other hand, consumers' preferences for the two products produced by firms B_1 and B_2 are horizontally differentiated. We capture this horizontal differentiation using a standard linear city model. So, consumers can be viewed as uniformly distributed along the unit interval $[0, 1]$. A particular consumer's location on this line is denoted by x, with firms B_1 and B_2 each being located at 0 and 1. If a consumer located at x purchases a unit of product B from B_j, she gains net utility

[3] Also see Raju, Dhar, and Morrison (1994) and the papers cited therein.

of $v_B - p_B^j - t\,|x - \widehat{x}_j|$ with $\widehat{x}_j \in \{0,1\}, j = 1,2$, where v_B are the consumer's gross value of product B, p_B^j is the price charged by firm B_j, and $t\,|x - \widehat{x}_j|$ measures the disutility the consumer bears due to the difference between the purchased product and her most preferred product. We assume that v_A and v_B are sufficiently large so that in equilibrium all consumers buy a unit of each product.

We allow individual firms in market A to pre-commit, prior to the announcement of headline prices and consumers' purchasing decisions, to a price discount for consumers who purchase a particular brand of product B.[4] For instance, the firms can send off coupons that promise to give a fixed amount off the headline price to consumers who present evidence of purchases of a specific brand of product B.[5] Attention is restricted to the cases where firm A_i's discount is related to only a single brand of product B, leading to two pairs of exclusive relationship between producers of either product.[6] We assume that the firms successfully coordinate their relationships to avoid the situations where two distinct discount offers are targeted to the same brand of product B. Let $d_i \in [0, \infty)$ denote the amount of discount offered by firm $A_i, i = 1, 2$. We also assume that firms B_1 and B_2 do not provide discounts, and focus on the strategic role of unilateral brand-specific discounts in leveraging market power across two otherwise unrelated markets.[7] In fact, in our model setup it turns out that the firms in market B are ex ante neutral to the presence of brand-specific discounts (see Lemma 1 below).

Then, the net utility a consumer located at x gains when purchasing good A from A_i and

[4] The firms could offer other non-monetary benefits instead of price discounts.

[5] Here we consider the case of additive discounts, deducting an absolute amount of money from the headline price, but the analysis can be easily extended to the case where discounts are in the form of proportional discounts.

[6] More generally, the firms may offer multiple brand-specific discounts d_{ij} that discriminate the amount of discount depending on which brand of product B consumers buy. Given that consumers' purchasing decisions are eventually affected by the effective prices including discounts, and that the firms' objective is to create artificial switching costs as will be shown later, it would be unreasonable to expect the firms to offer nonnegative discounts to both brands of product B.

[7] One may see this as alliance formations where a firm in one market teams up with a single firm in the other market in order to offer a bundled discount to consumers who buy from both firms, as in Gans and King (2006).

B from B_j can be expressed as

$$\left[v_A - \left(p_A^i - D_{ij}\right)\right] + \left[v_B - t\left|x - \widehat{x}_j\right| - p_B^j\right], \; i, j = 1, 2,$$

where $D_{ij} = d_i$ if A_i is connected to B_j via brand-specific discounts and $D_{ij} = 0$ otherwise.

We look for subgame perfect equilibria in the following dynamic game. The timing of moves is as follows. First, the firms in market B set their prices independently and simultaneously. Second, the firms in market A simultaneously commit to individual brand-specific discounts d_i. Third, a sequential pricing game follows between the firms in market A, where a price leader is selected randomly by tossing of a fair coin.[8] Finally, consumers make purchasing decisions given the prices and discounts set in the early stages.

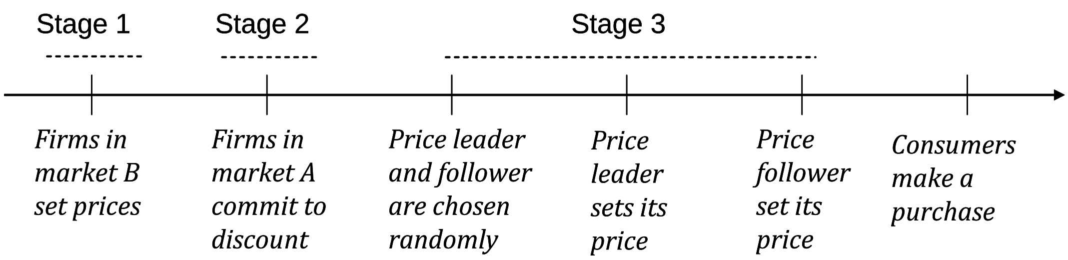

Figure 1: Timeline of the game

3 Analysis

In this section we solve for a subgame perfect equilibrium using backward induction. Before proceeding to the analysis, first note that in the absence of brand-specific discounts the two markets are completely independent, giving rise to standard Bertrand outcome in market A and standard Hotelling equilibrium in market B respectively. Here we assume that marginal cost pricing prevails in the sequential pricing game in market A.[9] This will serve as a benchmark in evaluating the effect of brand-specific discounts later.

[8] We model this stage as sequential moves in order to guarantee the existence of a pure-strategy Nash equilibrium. Note that under simultaneous moves we have to rely on mixed-strategy equilibria for small discounts. See Banejee and Summers (1987) for more detail on this issue.

[9] Note that theoretically there are infinitely many equilibria with the follower undercutting the leader's price greater than or equal to marginal cost.

3.1 First-stage pricing of the firms in market B

Let us first derive some preliminary result regarding the first-stage pricing equilibrium in market B, which will be useful in solving the entire game. The following lemma says that the pricing behavior of the firms in market B is totally neutral to the future discounts and pricing decisions of the firms in market A, given the randomness of pricing sequence in market A in stage 3.

Lemma 1 *Suppose that the firms in market B are risk neutral and the price leader in market A is randomly selected. Then, in equilibrium firms B_1 and B_2 behave as if duopolists in the standard Hotelling model, each firm setting the symmetric price $p_B^1 = p_B^2 = t$ and earning expected profits of $\frac{t}{2}$.*

Proof. Note that the equilibrium market shares and profits of firms B_1 and B_2 depend not only on their own prices but also on the prices and brand-specific discounts set by the firms in market A. Neither of firms B_1 and B_2 knows which firm in market A will be connected to itself via brand-specific discounts and who will be the price leader in market A later, and so a priori they do not know how the presence of brand-specific discounts will affect their profits. However, given all consumers buying a unit of product B in equilibrium, if one firm gains ξ in terms of market share from brand-specific discounts the other firm will lose by the exactly same amount. So, given relevant prices and discounts we can write firm B_1's market share as

$$x^+ = \frac{t - p_B^1 + p_B^2}{2t} + \xi\left(p_A^i, d_i\right)$$

if he benefits from brand-specific discounts, and

$$x^- = \frac{t - p_B^1 + p_B^2}{2t} - \xi\left(p_A^i, d_i\right)$$

if he loses from brand-specific discounts, where $\xi\left(p_A^i, d_i\right)$ denotes the sole effect of brand-specific discounts on market shares. Firm B_2's market share will be then $1 - x^+$ and $1 - x^-$ respectively. Given that a price leader in market A is randomly selected by tossing of a fair coin, the risk-neutral firms $B1$ and $B2$ will choose prices in order to maximize their expected profits:

$$\begin{aligned}
E[\pi_B^1] &= p_B^1 \frac{x^+ + x^-}{2} = p_B^1 \frac{t - p_B^1 + p_B^2}{2t}, \\
E[\pi_B^2] &= p_B^2 \frac{(1-x^+) + (1-x^-)}{2} = p_B^2 \frac{t - p_{B2} + p_B^1}{2t},
\end{aligned}$$

which are nothing but the profit functions obtained in the standard Hotelling model. Therefore, we can expect that they will behave just like Hotelling duopolists. ■

It is as if the firms in market B are unaware of such discounts or even the presence of product A. With their strategic responses to brand-specific discounts being neutralized in equilibrium, we can treat them to behave independently of prices and discounts set by the firms in market A. Of course, as will be shown shortly, brand-specific discounts may inflict a profit loss to an individual firm in market B ex post. However, this loss, if exists, can be compensated via monetary transfer (e.g. side payments) from the corresponding firm in market A. Given this neutrality result, we now proceed to equilibrium analysis using backward induction.

3.2 Third-stage pricing game in market A

A price leader is chosen randomly at the beginning of the stage, and then the two firms select their price sequentially. Let p_l and p_f denote the prices of the leader and the follower respectively. We assume without loss of generality that in the second stage the price leader has committed to a discount d_l to consumers purchasing product B from firm B_1 and the follower has committed to a discount d_f to those purchasing from firm B_2.

3.2.1 Follower's optimal pricing

Given the equilibrium price $p_B^1 = p_B^2 = t$ in the first stage, a consumer located at $x \in [0, 1]$ will face the following effective prices (including transportation costs) for the two products:

$$P = \begin{cases} t + tx + p_l - d_l, & \text{when buying from } B_1 \text{ and } l \\ t + tx + p_f, & \text{when buying from } B_1 \text{ and } f \\ t + t(1-x) + p_l, & \text{when buying from } B_2 \text{ and } l \\ t + t(1-x) + p_f - d_f, & \text{when buying from } B_2 \text{ and } f \end{cases}.$$

Then, the follower's demand function is given as[10]

$$D_f(p_l, p_f; d_l, d_f) = \begin{cases} 0 & \text{if } p_f > p_l + d_f + \min\{t - d_l, 0\} \\ \frac{1}{2} + \frac{(p_l - d_l) - (p_f - d_f)}{2t} & \text{if } p_l - d_l + \max\{d_f - t, 0\} \leqq p_f \leqq p_l + d_f + \min\{t - d_l, 0\} \\ 1 & \text{if } p_f < p_l - d_l + \max\{d_f - t, 0\} \end{cases}.$$

Note that the demand function is discontinuous at $p_l + d_f$ if $d_f < t$ and at $p_l - d_l$ if $d_l < t$.

The follower has two options in responding to the leader's price. One is to share the market with the leader by setting an appropriate price. The other is to corner the market by undercutting the leader's price. Which one is more profitable between the two options depends on the size of two discounts and the level of the leader's price. Intuitively, the follower will corner the market for small discounts as in the standard Bertrand model. For large discounts, however, cornering the market is very costly and therefore the follower will find it optimal to share the market. In what follows, we first calculate the follower's maximal profits obtained under market sharing and cornering, and then determine the optimal choice by comparing those two profits.

First, let us calculate the follower's profit under market sharing. The follower's problem in this case is as follows:

$$\max_{p_f} : (p_f - d_f) \left[\frac{1}{2} + \frac{(p_l - d_l) - (p_f - d_f)}{2t} \right].$$

The first-order condition gives a unique solution $p_f = d_f + \frac{t + p_l - d_l}{2}$. In order for this price to yield market sharing, it is required that $p_l - d_l + \max\{d_f - t, 0\} \leqq p_f^S \leqq p_l + d_f + \min\{t - d_l, 0\}$, which leads to the condition $|t - d_l| \leqq p_l \leqq t + d_l + 2\min\{d_f, t\}$. If $p_l < |t - d_l|$ (i.e. $p_f^S > p_l + d_f + \min\{t - d_l, 0\}$), we have a corner solution with the optimal price $p_f = p_l + d_f + \min\{t - d_l, 0\}$. In this case, the follower's demand is given by $D_f = \frac{1}{2} - \frac{d_l + \min\{t - d_l, 0\}}{2t}$. Finally, if $p_l > t + d_l + 2\min\{d_f, t\}$, the follower optimally corners the market A by setting $p_f^M = p_l - d_l + \max\{d_f - t, 0\} - \varepsilon$ and earns profits of $\pi_f^M = p_l - d_l - \min\{t, d_f\}$. Then, the follower's

[10] Here we assume that consumers, if they are indifferent, buy the good with a discount.

optimal price and profit in the market sharing regime are:

$$p_f^S = \begin{cases} d_f + \frac{t+p_l-d_l}{2}, & \text{if } |t-d_l| \leqq p_l \leqq t+d_l+2\min\{d_f,t\} \\ p_l + d_f + \min\{t-d_l,0\}, & \text{if } p_l < |t-d_l| \end{cases} \tag{1}$$

$$\pi_f^S = \begin{cases} \frac{(t+p_l-d_l)^2}{8t}, & \text{if } |t-d_l| \leqq p_l \leqq t+d_l+2\min\{d_f,t\} \\ \frac{[p_l+\min\{t-d_l,0\}]\max\{t-d_l,0\}}{2t}, & \text{if } p_l < |t-d_l| \end{cases}. \tag{2}$$

The time line of the game indicates that the price leader will avoid a price leading to monopolization of market A by the follower. Since such a strategy will never be supported as a perfect equilibrium of the whole game, we can restrict our attention to subgames where both firms are active in market A. Suppose the follower chooses to share market by setting price p_f^S in (1). Then, the leader's profit is given by

$$\begin{aligned} \pi_l &= (p_l - d_l)\left[\frac{1}{2} - \frac{(p_l-d_l)-(p_f^S-d_f)}{2t}\right] \\ &= \frac{(p_l-d_l)}{4t}\left[3t + d_l - \max\{p_l, |t-d_l|\}\right], \end{aligned}$$

which is continuous in p_l. Note that for $p_l \leq |t-d_l|$ the profit function is monotonically increasing in p_l, which together with the continuity implies that the profit function is maximized at $p_l = p_l^* \geq |t-d_l|$ where p_l^* will be characterized below. Therefore, as long as the follower is expected to share the market, the leader always set p_l greater than or equal to $|t-d_l|$. In what follows, we will focus on this case in characterizing p_l^*, and will show later that (p_l^*, p_f^S) indeed constitutes an equilibrium.

Comparing the optimal profits under the two regimes, π_f^S and π_f^M, leads to the following result regarding the follower's optimal pricing decision.

Lemma 2 *Given $p_l \geq |t-d_l|$, the follower prefers market sharing if*

$$d_f \geq t \tag{3}$$

or

$$d_f \leq t \text{ and } p_l - d_l \leq 3t - \sqrt{8t(t-d_f)}, \tag{4}$$

and market cornering otherwise.

Proof. Note that the follower always corners the market for $p_l > t + d_l + 2\min\{d_f, t\}$. For $|t-d_l| \leqq p_l \leqq t + d_l + 2\min\{d_f, t\}$, the follower's optimal choice depends on the level of the leader's discounted price. The follower's profit under market sharing is larger than or equal to that under monopolization if and only if

$$\Delta\pi \equiv \pi_f^S - \pi_f^M = \frac{(t+p_l-d_l)^2}{8t} - (p_l - d_l - \min\{d_f, t\}) \geq 0$$

Note that $\Delta\pi$ is a convex quadratic function of $z \equiv p_l - d_l$. Suppose first that $d_f \geq t$ (i.e., $\min\{d_f, t\} = t$). Then, we have $\Delta\pi(z) = \frac{(t+z)^2}{8t} - (z-t) \geq 0$ with equality only when $z = 3t$. In this case the follower always chooses to share the market. Suppose next that $d_f \leq t$ (i.e., $\min\{d_f, t\} = d_f$). Then, among the two solutions of equation $\Delta\pi(z) = \frac{(t+z)^2}{8t} - (z - d_f) = 0$, the one that satisfies the condition $p_l \leq t + d_l + 2\min\{d_f, t\}$ is $z^* = 3t - \sqrt{8t(t-d_f)}$. So, the follower prefers market sharing to cornering if $p_l - d_l \leq z^* = 3t - \sqrt{8t(t-d_f)}$ for the case of $|t - d_l| - d_l < z^*$. ■

3.2.2 Leader's optimal pricing

The leader's profit-maximization problem is

$$\max_{p_l} : \frac{(p_l - d_l)}{4t}\left[3t + d_l - \max\{p_l, |t-d_l|\}\right]$$

subject to $\pi_f^S(d_l, d_f) \geq \pi_f^M(d_l, d_f)$. The unconstrained solution of the problem is given by

$$p_l^* = \frac{3}{2}t + d_l.$$

Then, from the previous analysis we have

$$\begin{aligned} p_f^* &= p_f^S = \frac{5}{4}t + d_f, \\ \pi_l^* &= \frac{9}{16}t, \\ \pi_f^* &= \pi_f^S = \frac{25}{32}t. \end{aligned}$$

Note that the condition $|t - d_l| \leq p_l$ is satisfied at the equilibrium p_l^* ($|t-d_l| \leq p_l^* = 3t/2 + d_l$). So, the valid condition for $\pi_f^S(d_l, d_f) \geq \pi_f^M(d_l, d_f)$ is given by (3) and (4). Hence, the follower will decide to share the market if $d_f \geq t$ or

$$\begin{aligned} p_l^* - d_l &= \frac{3t}{2} \leq 3t - \sqrt{8t(t-d_f)} \\ &\Longrightarrow d_f \geq 23t/32. \end{aligned} \tag{5}$$

So, (p_l^*, p_f^*) indeed constitutes an equilibrium of the third-stage pricing game if the discount set by the follower is large enough (more precisely if $d_f \geq 23t/32$), irrespective of the leader's discount. We do not explicitly characterize other possible equilibria of this pricing game for different values of d_l and d_f since the corresponding joint profit cannot be greater than $(\pi_l^* + \pi_f^*)$, as will be shown below.

Note that given the equilibrium prices consumers are divided into two groups at the indifferent type $x = 3/8$.

3.3 Second-stage discount game

Now we consider the second-stage discount game where the two firms in market A choose simultaneously and independently the level of their brand-specific discount. Each firm's problem is to choose its own discount to maximize expected profits $(\pi_l/2 + \pi_f/2)$, or equivalently to maximize ex post joint profits $(\pi_l + \pi_f)$. The following proposition establishes that in equilibrium both firms can make strictly positive profits by committing to sufficiently large brand-specific discounts.

Proposition 3 *In subgame perfect equilibria, firms A_1 and A_2 both commit to a brand-specific discount larger than or equal to $23t/32$. Each firm ex ante earns expected profits of $43t/64$ (the half of the maximum joint profit) and ex post the leader earns profits of $9t/16$ and the follower earns profits of $25t/32$.*

Proof. In order to prove that neither A_1 nor A_2 has incentives to choose $d_i < 23t/32$, it will be sufficient to show that π_l^* and π_f^* at the market sharing equilibrium with $d_f \geq 23t/32$ are truly the maximal profit obtainable by the leader and the follower for all values of d_l and d_f. First, it is obvious that π_l^* is the leader's overall maximal profit since it is the unconstrained solution of the problem and the leader will obtain zero profits under monopolization. Next, we show that the follower's profit with $d_f < 23t/32$ is no larger than π_f^*. Suppose that $d_f < 23t/32$. Then, the constraint $\pi_f^S \geq \pi_f^M$ of the leader's profit maximization problem will be binding, and different equilibrium prices (let us denote them (p_l^{**}, p_f^{**})) will be obtained. Note that constraint $\pi_f^S \geq \pi_f^M$ sets an upper bound on p_l (and $p_l - d_l$) as defined in inequality (4). From (5) we know that the constraint is binding at $d_f = 23t/32$, and in this case $p_l^* - d_l = \frac{3t}{2}$ (upper bound). The binding constraint ($\pi_f^S = \pi_f^M$) for $d_f < 23t/32$ is given by $p_l^{**} - d_l = 3t - \sqrt{8t(t-d_f)}$. Since the upper bound is increasing in d_f, we have

$$\begin{aligned} p_l^{**} - d_l &= 3t - \sqrt{8t(t-d_f)} < \frac{3t}{2} = p_l^* - d_l \\ &\Longrightarrow p_l^{**} - d_l < p_l^* - d_l. \end{aligned}$$

From (2) we can see that π_f^S is increasing in p_l (or $p_l - d_l$), independent of d_f. This implies that the follower's profit reaches it maximum π_f^* at the equilibrium with (p_l^*, p_f^*) and $d_f \geq 23t/32$. Hence, to sustain the maximum equilibrium profits π_l^* and π_f^*, both A_1 and A_2 will commit to discounts $d_i \geq 23t/32$. ■

3.4 Welfare

We can easily see that consumers become worse off due to brand-specific discounts. First, consumers face higher effective prices in market A compared with the benchmark equilibrium:

$$p_f^* - d_f = \frac{5}{4}t > 0 \text{ and } p_l^* - d_l = \frac{3t}{2} > 0,$$

although the equilibrium prices in market B are the same ($p_B^1 = p_B^2 = t$) regardless of the presence of brand-specific discounts. Second, there are additional utility reductions since some consumers (specifically those with $x \in [3/8, 1/2]$) end up with buying their less preferred product B and so bearing larger transportation costs than in the benchmark case.[11]

Total welfare also decreases under brand-specific discounts, because consumer gross utilities decrease due to the increase in transportation costs, and there are no costs in the present set up. Note that the welfare effect would be more adverse if we allow for elastic demands.

[11] Note that, however, this is an artifact caused by sequential pricing in the third stage.

4 Conclusion

This paper has examined a stylized model in which market power can be leveraged across firms in different markets via brand-specific discounts. The central finding is that firms without market power can achieve supernormal profits by committing to interfirm bundled discounts which connect their product with a specific brand of other firm possessing market power. With such discount schemes in place, products that would otherwise be competitively supplied are sold at prices above marginal cost.

Our analysis, although very preliminary, gives some implications for public policy toward interfirm brand-specific discounts. Specifically, the above result suggests that competition policy needs to pay more attention to such discount practices, specially when the amounts of discounts are large compared with headline prices. For instance, the Korea Fair Trade Commission recently announced that they will introduce credit cards which award reward points for all petrol stations in order to respond potentially anti-competitive bundled discounts between credit card companies and petrol stations.

References

[1] Banerjee, Abhijit and Lawrence H. Summers (1987), On frequent flyer programs and other loyalty-inducing economic arrangements, Discussion paper No. 1337, Harvard Institute of Economic Research.

[2] Caminal, Ramon and Carmen Matutes (1990), Endogenous switching costs in a duopoly model, *International Journal of Industrial Organization*, 8, 353-373

[3] Chen, Yongmin (1997), Equilibrium product bundling, *Journal of Business*, 70, 85-103

[4] Dhar, Sanjay K. and Jagmohan S. Raju (1998), The effects of cross-ruff coupons on sales and profits, *Management Science*, 44(1), 1501-1516

[5] Fudenberg, Drew and Miguel Villas-Boas (2007), Behavior-based price discrimination and customer recognition, Economics and Information Systems, Vol. 1, Elsevier: Oxford

[6] Fudenberg, Drew and Jean Tirole (1984), The fat-cat effect, the puppy-dog ploy, and the lean and hungry look, *American Economic Review,* 74(2), 361-366

[7] Fudenberg, Drew and Jean Tirole (2000), Customer poaching and brand switching, *RAND Journal of Economics*, 31(4), 634-657

[8] Gans, Joshua S. and Stephen P. King (2006), Paying for loyalty: product bundling in oligopoly, *Journal of Industrial Economics*, 43-62

[9] Hermalin, Benjamin E. and Michael L. Katz (2011), Product differentiation through exclusivity: Is there a one-market-power-rent theorem?, working paper, Haas School of Business, University of California Berkeley

[10] Matutes, Carmen, and Pierre Regibeau (1988), Mix and match: product compatibility without network externalities, *Rand Journal of Economics*, 19(2), 219-234

[11] Matutes, Carmen, and Pierre Regibeau (1992), Compatibility and bundling of complementary goods in a duopoly, *Journal of Industrial Economics*, 40(1), 37-54

[12] Raju, Jagmohan S., Sanjay K. Dhar and Donald G. Morrison (1994), The effect of package coupons on brand choice *Marketing Science*, 13(2), 145-164

[13] Villas-Boas, Miguel (2004), Price cycles in markets with customer recognition, *RAND Journal of Economics*, 35(3), 486-501