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JANUARY 2014

2014RWP-63

Economic Research Institute
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January 9, 2014

Abstract

This paper investigates the role of collateral constraints in the transmission of monetary policy shocks in a two-sector sticky price general equilibrium model with nondurable and durable goods. While many researchers have stressed the role of collateral constraints in one-sector sticky price model, it has been relatively less explored in the two-sector sticky price model. This study shows that nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable spending in the two-sector sticky price general equilibrium model.

Keywords: durable goods, collateral constraints, sticky nominal wages
JEL Classification: E21, E23, E31, E44, E52

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1 Introduction

Durable goods play a significant role in discussion of monetary policy. According to the empirical results documented by Erceg and Levin (2006), Barsky et al. (2007), and Monacelli (2009), the durable spending responds most procyclically to monetary policy. An important characteristic of durable goods is that households use the durable goods as collateral. Residential mortgage loan accounts for a great part of total household debt recently. For example, in South Korea, the weight of mortgage loans has increased since 2007. About 61% of loans to household of depository corporations consist of mortgage loans in 2012.[1] Moreover, in the United States, according to 2012 Survey of Consumer Finances, collateralized debt represents 88.6% of total household debt in 2010. The role of borrowing constraints at the household level in an one-sector model has been widely stressed by Zeldes (1989), Jappelli and Pagano (1989), and Campbell and Mankiw (1989). While these papers have stressed the importance of financial frictions for macroeconomic fluctuations, they are based on a partial equilibrium model. In addition, Chah et al. (1995) develop and test a theory of optimal consumption behavior in the presence of collateral constraints. Campbell and Hercowitz (2006) study the role of collateralized household debt contributes to macroeconomic stabilization in a real business cycle model. Unlike those researches, this paper investigates how the collateral constraints play a role in explaining the procyclical response of durable spending to monetary policy in a two-sector sticky price general equilibrium model.

Our paper is not the first paper to investigate the role of collateral constraints in a two-sector sticky price general equilibrium model. Iacoviello (2005), Monacelli (2009), Sterk (2010), and Iacoviello and Neri (2010) also analyze how collateral constraints work in the transmission of monetary policy in the two-sector price model. However, Iacoviello (2005) focuses on the nondurable goods because durable stock is fixed. The main issue of Iacoviello and Neri (2010) is mainly not about the role of collateral constraints.[2] Monacelli (2009) and Sterk (2010) reach a opposed conclusion. Monacelli (2009) claims that the introduction of collateral constraints can accelerate the procyclical response of durable spending to monetary policy shocks in a standard two-sector sticky price model. However, Sterk (2010) demonstrates that Monacelli (2009) neglects general equilibrium

[1] The source of data are Economic Statistics System of the Bank of Korea.

[2] Iacoviello and Neri (2010) shows that the housing market spillovers concentrate on consumption rather than business investment.

effects. The changes in borrowing and durable purchases by the borrowers have consequences for the savers. Thus, collateral constraints rather decelerate the response of durable expenditure to monetary policy shocks in the general equilibrium model.

This paper shows that whether collateral constraints at the household level accelerate or decelerate durable spending to monetary policy shocks depends on how we model labor markets. We find that the degree of nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable spending in a two-sector sticky price general equilibrium model. Monacelli (2009) and Sterk (2010) assume Walrasian labor markets. However, if there exits nominal wage rigidity, wage inflation serves as a core in households' labor supply unlike Walrasian labor markets. This has a great impact on households' labor supply. When wage inflation is changed as a consequence of monetary policy shocks, households decide their labor supply by comparing between current and future wage inflations. Since current wage inflation is changed more greatly by an existence of collateral constraints, the effects of monetary policy shocks on durable spending are accelerated in the model with sticky nominal wages.

Although labor market rigidity makes collateral constraints to accelerate the procyclical response of durable spending in the initial period, the sensitivity of durables to monetary policy shocks is too much larger than that of nondurables, which is at odds with empirical data. Since the sticky nominal wages lead to rapid over-shooting of durable purchases, as Carlstrom and Fuerst (2010) shows, this paper additionally considers adjustment costs in durable investment in the model in order to smooth production over time.

The rest of the paper is organized as follows. Section 2 outlines two-sector sticky price general equilibrium model incorporated with collateral constraints and nominal wage rigidity. Section 3 describes the parameter calibration. Section 4 presents the analytical discussion of the model to monetary policy shocks. Section 5 investigates the numerical discussion of the model to monetary policy shocks. Section 6 considers durable investment adjustment costs in the model. Section 7 concludes.

2 The Model

This study analyzes a two-sector sticky price model with nondurable and durable goods in order to investigating the role of collateral constraints. We add nominal wage rigidity to a standard two-sector sticky price model with collateral constraints (Monacelli (2009)).

2.1 Households

The model introduces two types of households with different time rate preference, meaning that one group of households is more patient than the other. They are titled borrowers and savers respectively because the impatient households borrow from the patient households in equilibrium. The borrowers are confronted with collateral constraints. We suppose that There is a continuum of measure 1 of agents in each of the two groups. The size of the total population is normalized to 1 and the weight of the borrowers is set equal to ω.

2.1.1 Borrowers

Borrowers have a discount factor β. They maximize the following utility program:

$$E_0 \sum_{t=0}^{\infty} \beta^t U^b(X_t^b(i), N_t^b(i)), \tag{1}$$

where $U^b(\cdot)$ is a utility function depending on labor supply $N_t^b(i)$ and a CES consumption basket $X_t^b(i)$ that consists of nondurable goods $C_t^b(i)$ and the stock of durable goods $D_t^b(i)$:

$$U^b(X_t^b(i), N_t^b(i)) = \log X_t^b(i) - \frac{\nu^b N_t^b(i)^{1+\varphi^b}}{1+\varphi^b}, \quad \varphi^b > 0, \tag{2}$$

$$X_t^b(i) \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_t^b(i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D_t^b(i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{3}$$

where ν^b is the parameter that indexes the preference for hours worked of borrowers, φ^b is the inverse elasticity of borrowers' labor supply, $\alpha > 0$ is the share of durable goods in the CES consumption basket, and $\eta > 0$ is the elasticity of substitution between services of nondurable and durable goods. The borrowers face the following budget constraints, collateral constraints in real terms (in units of nondurable), and law of motion for durable goods respectively:

$$C_t^b(i) + q_t I_t^b(i) + R_{t-1}\frac{b_{t-1}(i)}{\pi_{c,t}} + AC_{w,t}^b = b_t(i) + w_t^b(i)N_t^b(i), \tag{4}$$

$$R_t b_t(i) \leq (1-\chi)(1-\delta)E_t(D_t^b(i)q_{t+1}\pi_{c,t+1}), \tag{5}$$

$$I_t^b(i) = D_t^b(i) - (1-\delta)D_{t-1}^b(i), \tag{6}$$

where $q_t \equiv \frac{P_{d,t}}{P_{c,t}}$ is the relative price of the durable goods, $I_t^b(i)$ is durable investment as the flow variable, $b_t(i)$ is end-of-period t real one-period debt, R_{t-1} is the gross nominal interest rate on loan contracts stipulated at time $t-1$, $\pi_{c,t} \equiv \frac{P_{c,t}}{P_{c,t-1}}$ is nondurable goods inflation, $AC_{w,t}^b(i)$ is the real cost of adjusting nominal wages, $w_t(i)^b \equiv \frac{W_t^b(i)}{P_{c,t}}$ is the real wage for the borrower, χ is the fraction of the durable good value that cannot be used as collateral, and δ is the depreciation rate for durable goods. This study is based on an assumption that the collateral constraints always binds, that is to say, in a neighborhood of the steady state, equation (5) is always satisfied with equality. Also, labor is assumed to be perfectly mobile across sectors, implying that the real wage rate is common across sectors.

We follow Kim (2000) by modeling nominal wage rigidity through the cost of adjusting nominal wages. The wage adjustment cost function is assumed to be quadratic and zero at the steady state. The real total adjustment cost for a borrower i is given by

$$AC_{w,t}^b(i) = \frac{\phi_w^b}{2}\left(\frac{W_t^b(i)}{W_{t-1}^b(i)} - 1\right)^2 w_t^b, \tag{7}$$

where ϕ_w^b is the wage adjustment cost parameter.

Let $\lambda_t^b(i)$, $\lambda_t^b(i)\psi_t(i)$ be defined as the multipliers on constraints (4) and (5) respectively. We assume a symmetric equilibrium to rule out potential multiple equilibria in which identical agents behave differently because of different initial conditions. In a symmetric equilibrium, each borrower is endowed with the same initial condition and makes identical decision so that $C_t^b(i) = C_t^b$, $D_t^b(i) = D_t^b$, $I_t^b(i) = I_t^b$, $b_t(i) = b_t$, $\lambda_t^b(i) = \lambda_t^b$, and $\psi_t(i) = \psi_t$. The first-order conditions are as follows:

$$U_{c,t}^b = \lambda_t^b, \tag{8}$$

$$q_t U^b_{c,t} = U^b_{d,t} + \beta(1-\delta)E_t(q_{t+1}U^b_{c,t+1}) + (1-\chi)(1-\delta)E_t(q_t U^b_{c,t}\psi_t \pi_{d,t+1}), \tag{9}$$

$$R_t\psi_t = 1 - \beta E_t\left(\frac{\lambda^b_{t+1}}{\lambda^b_t}\frac{R_t}{\pi_{c,t+1}}\right), \tag{10}$$

where $U^b_{c,t}$ and $U^b_{d,t}$ are the marginal utilities of nondurables and durables respectively.

In equation (8), the borrowers' marginal utility of nondurable consumption is equated to the shadow value of budget constraints (4). Equation (9) requires that the borrowers to equate the marginal utility of nondurable consumption to the shadow value of durables. In the right hand side, the last term is proportional to ψ_t, which measures the tightness of the borrowing constraints. Equation (10) is a modified version of the standard Euler equation. It would reduce to the standard Euler equation if the constraint would not be binding, that is, when $\psi_t = 0$ for all t.

2.1.2 Savers

Savers have a discount factor γ. The key feature is that $\gamma > \beta$. This is because the savers are more patient than the borrowers. It is assumed that the savers are the owners of the monopolistic firms in each sector. They maximize the following utility program:

$$E_0\sum_{t=0}^{\infty}\gamma^t U^s(X^s_t(i), N^s_t(i)), \tag{11}$$

where $U^s(\cdot)$ is a utility function depending on labor supply N^s_t and a CES consumption basket X^s_t, that consists of nondurable goods C^s_t and the stock of durable goods D^s_t:

$$U^s(X^s_t(i), N^s_t(i)) = \log X^s_t(i) - \frac{\nu^s N^s_t(i)^{1+\varphi^s}}{1+\varphi^s}, \quad \varphi^s > 0, \tag{12}$$

$$X^s_t(i) \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C^s_t(i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} D^s_t(i)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{13}$$

where ν^s is the parameter that indexes the preference for hours worked of savers, and φ^s is the inverse elasticity of savers' labor supply.

The savers face the following budget constraints in real terms (in units of nondurable) and law of motion for durable goods respectively:

$$C_t^s(i) + q_t I_t^s(i) + s_t(i) + AC_{w,t}^s(i) = R_{t-1}\frac{s_{t-1}(i)}{\pi_{c,t}} + w_t^s(i)N_t^s(i) + (1-\omega)^{-1}\Pi_t(i), \tag{14}$$

$$I_t^s(i) = D_t^s(i) - (1-\delta)D_{t-1}^s(i), \tag{15}$$

where $I_t^s(i)$ is durable investment as the flow variable, $s_t(i)$ is end-of-period t real one-period savings, $AC_{w,t}^s(i)$ is the real cost of adjusting nominal wages, and $\Pi_t(i)$ are aggregate nominal profits from holding of monopolistic competitive firms.

Similarly, the wage adjustment cost function is assumed to be quadratic and zero at the steady state. The real total adjustment cost for a saver i is given by

$$AC_{w,t}^s(i) = \frac{\phi_w^s}{2}\left(\frac{W_t^s(i)}{W_{t-1}^s(i)} - 1\right)^2 w_t^s, \tag{16}$$

where ϕ_w^s is the wage adjustment cost parameter.

Let $\lambda_t^s(i)$ be defined as the multipliers on constraints (14). Similar to the borrowers, we suppose a symmetric equilibrium. Hence, The first order conditions of the savers are as follows:

$$U_{c,t}^s = \lambda_t^s, \tag{17}$$

$$q_t U_{c,t}^s = U_{d,t}^s + \gamma(1-\delta)E_t(U_{c,t+1}^s q_{t+1}), \tag{18}$$

$$\lambda_t^s = \gamma E_t\left(\lambda_{t+1}^s \frac{R_t}{\pi_{c,t+1}}\right), \tag{19}$$

where $U_{c,t}^s$ and $U_{d,t}^s$ are the marginal utilities of nondurables and durables respectively.

In equation (17), the savers' marginal utility of nondurable consumption is equated to the shadow value of budget constraints (14). Equation (18) requires that the savers to equate the marginal utility of nondurable consumption to the shadow value of durables. Equation (19) is a standard Euler equation.

2.2 Employment Unions

Borrowers and savers supply homogeneous labor services to the employment unions. We assume that there are two unions, one for each household pair. The unions differentiate labor services, offer labor services to intermediate good firms according to

$$N_t^b \equiv \left(\int_0^1 N_t^b(i)^{\frac{\varepsilon_w^b - 1}{\varepsilon_w^b}} di \right)^{\frac{\varepsilon_w^b}{\varepsilon_w^b - 1}}, \tag{20}$$

$$N_t^s \equiv \left(\int_0^1 N_t^s(i)^{\frac{\varepsilon_w^s - 1}{\varepsilon_w^s}} di \right)^{\frac{\varepsilon_w^s}{\varepsilon_w^s - 1}}. \tag{21}$$

Here, N_t^b and N_t^s are the amounts of the composite labor supplied by the aggregators, $N_t^b(i)$ and $N_t^s(i)$ are the labor supplies of household i and ε_w^b and ε_w^s are the elasticities of substitution among labor varieties of each household.

The first-order conditions for profit maximization yield the demand functions of each household's differentiated labor:

$$N_t^b(i) = \left(\frac{W_t^b(i)}{W_t^b} \right)^{-\varepsilon_w^b} N_t^b, \tag{22}$$

$$N_t^s(i) = \left(\frac{W_t^s(i)}{W_t^s} \right)^{-\varepsilon_w^s} N_t^s, \tag{23}$$

where $W_t(i)^b$ and $W_t(i)^s$ are the nominal wages of each household i respectively and W_t^b and W_t^s are the aggregate nominal wage indexes. These labor demands by firms are another constraints for each household.

Each household choose $W_t^b(i)$ and $W_t^s(i)$ in order to maximize each utility program above subject to the demand functions of each household as well as their budget constraints, respectively. Each household is endowed with the same initial condition and makes identical decision. This is why we prefer the Rotemberg mechanism to the Calvo mechanism to avoid heterogeneity in wages: in that way all the agents earn the same wage. Therefore, the first-order conditions are as follows:

$$\phi_w^b \left(\pi_{w,t}^b - 1 \right) \pi_{w,t}^b = \phi_w^b \beta E_t \frac{\lambda_{t+1}^b}{\lambda_t^b} \left[\left(\pi_{w,t+1}^b - 1 \right) \pi_{w,t+1}^b \frac{w_{t+1}^b}{w_t^b} \right] + (1 - \varepsilon_w^b) N_t^b + \varepsilon_w^b MRS_t^b \frac{N_t^b}{w_t^b}, \tag{24}$$

$$\phi_w^s\left(\pi_{w,t}^s-1\right)\pi_{w,t}^s=\phi_w^s\gamma E_t\frac{\lambda_{t+1}^s}{\lambda_t^s}\left[\left(\pi_{w,t+1}^s-1\right)\pi_{w,t+1}^s\frac{w_{t+1}^s}{w_t^s}\right]+(1-\varepsilon_w^s)N_t^s+\varepsilon_w^s MRS_t^s\frac{N_t^s}{w_t^s}, \tag{25}$$

where MRS_t^b and MRS_t^s are borrowers' and savers' marginal rates of substitution between leisure and consumption respectively.

It can be obtained from the following New Keynesian Wage Phillips curves by log-linearizing Equation (24) and Equation (25):

$$\widehat{\pi}_{w,t}^b=\beta E_t\widehat{\pi}_{w,t+1}^b+\frac{(\varepsilon_w^b-1)N^b}{\phi_w^b}\left(\widehat{MRS}_t^b-\widehat{w}_t^b\right), \tag{26}$$

$$\widehat{\pi}_{w,t}^s=\gamma E_t\widehat{\pi}_{w,t+1}^s+\frac{(\varepsilon_w^s-1)N^s}{\phi_w^s}\left(\widehat{MRS}_t^s-\widehat{w}_t^s\right). \tag{27}$$

where N^b and N^s are steady state labor supplies of each household.

2.3 Firms

This paper assumes the existence of a continuum of monopolistically competitive firms, indexed by $s\in[0,1]$, producing differentiated intermediate goods in each sector. A final good in each sector is produced by a perfectly competitive, representative firm. The firm produces the final good by combining a continuum of intermediate goods and sell it to the households.

2.3.1 Final goods firms

The final goods in each sector are aggregated by the constant elasticity of substitution (CES) technology:

$$Y_{j,t}=\left(\int_0^1 Y_{j,t}(s)^{\frac{\varepsilon_j-1}{\varepsilon_j}}ds\right)^{\frac{\varepsilon_j}{\varepsilon_j-1}}, \quad \varepsilon_j>1. \tag{28}$$

Let $Y_{j,t}(s)$ denote the inputs of intermediate good s in sector $j=c,d$ and let $Y_{j,t}$ be the quantity of the final good produced.

Profit maximization delivers the following equation for demand for intermediate goods:

$$Y_{j,t}(s)=\left(\frac{P_{j,t}(s)}{P_{j,t}}\right)^{-\varepsilon_j}Y_{j,t}, \tag{29}$$

where $P_{j,t}(s)$ is the price of intermediate good s and $P_{j,t}$ is the aggregate price level in sector j.

Finally, the zero-profit condition implies that:

$$P_{j,t} \equiv \left(\int_0^1 P_{j,t}(s)^{1-\varepsilon_j} ds \right)^{\frac{1}{1-\varepsilon_j}}. \tag{30}$$

2.3.2 Intermediate goods firms

Intermediate goods firms are monopolistically competitive. Inputs for the final goods are supplied by intermediate goods firms. Firms i in sector j has the following production function:

$$Y_{j,t}(s) = N_{j,t}(s), \tag{31}$$

where $N_{j,t}(s)$ denotes the amount of labor that is used by firm s in sector j. These firms have some price-setting power because intermediate goods are slightly differentiated. Nominal profits at period t are as follow:

$$\Pi_{j,t} = P_{j,t}(s)Y_{j,t}(s) - W_t N_{j,t}(s). \tag{32}$$

Intermediate goods firms are assumed to set nominal prices in a staggered fashion according to the stochastic time dependent rule proposed by Calvo (1983). Each firm in sector $j = c, d$ resets its price with the probability of $1 - \theta_j$ each period. Thus, a fraction θ_j of firms cannot re-optimize their prices.

An intermediate goods firm resetting its price in period t in sector $j = c, d$ will seek to maximize the profits using the nominal stochastic factor of the savers:

$$E_t \sum_{k=0}^{\infty} \theta_j^k \gamma^k \frac{\lambda_{t+k}^s}{\lambda_t^s} \frac{P_{c,t}}{P_{c,t+k}} \Pi_{j,t+k}, \tag{33}$$

subject to Equation (29) and Equation (31). The optimal reset prices, $P_{j,t}^*$, are as follows:

$$P_{c,t}^* = \frac{\varepsilon_c E_t \sum_{k=0}^{\infty} \gamma^k \theta_c^k \lambda_{t+k}^s \frac{W_{t+k}}{P_{c,t+k}} P_{c,t+k}^{\varepsilon_c} Y_{c,t+k}}{(\varepsilon_c - 1) E_t \sum_{k=0}^{\infty} \gamma^k \theta_c^k \lambda_{t+k}^s P_{c,t+k}^{\varepsilon_c - 1} Y_{c,t+k}}, \tag{34}$$

$$P_{d,t}^* = \frac{\varepsilon_d E_t \sum_{k=0}^{\infty} \gamma^k \theta_d^k q_{t+k} \lambda_{t+k}^s \frac{W_{t+k}}{P_{d,t+k}} P_{d,t+k}^{\varepsilon_d} Y_{d,t+k}}{(\varepsilon_d - 1) E_t \sum_{k=0}^{\infty} \gamma^k \theta_d^k q_{t+k} \lambda_{t+k}^s P_{d,t+k}^{\varepsilon_d - 1} Y_{d,t+k}}. \tag{35}$$

Finally, the equation describing the dynamics for the aggregate price level in sector $j = c, d$ is given by

$$P_{j,t} = \left[(1-\theta_j)\left(P^*_{j,t}\right)^{1-\varepsilon_j} + \theta_j P_{j,t-1}^{1-\varepsilon_j}\right]^{\frac{1}{1-\varepsilon_j}}. \tag{36}$$

By log-linearizing around a sectoral zero-inflation steady state, Equation (34), Equation (35), and Equation (36) take the form of forward looking New Keynesian Phillips curves respectively:

$$\widehat{\pi}_{c,t} = \gamma E_t \widehat{\pi}_{c,t+1} + \frac{(1-\theta_c)(1-\gamma\theta_c)}{\theta_c}\widehat{w}_t, \tag{37}$$

$$\widehat{\pi}_{d,t} = \gamma E_t \widehat{\pi}_{d,t+1} + \frac{(1-\theta_d)(1-\gamma\theta_d)}{\theta_d}\left(\widehat{w}_t - \widehat{q}_t\right). \tag{38}$$

2.4 Monetary Policy

It is assumed that monetary policy is conducted by means of a simple Taylor-type rule:

$$\frac{R_t}{R} = \left(\frac{\tilde{\pi}_t}{\tilde{\pi}}\right)^{\phi_\pi} \exp(\varepsilon_t), \qquad \phi_\pi > 1, \tag{39}$$

where $\tilde{\pi}_t \equiv \pi_{c,t}^{1-\tau}\pi_{d,t}^{\tau}$ is a composite inflation index and R and $\tilde{\pi}$ are the steady state levels of the nominal interest rate and the inflation index, respectively. The monetary policy shock, ε_t, evolves according to

$$\exp(\varepsilon_t) = \exp(\varepsilon_{t-1})^{\rho} u_t, \tag{40}$$

with $u_t \sim iid$ and $0 < \rho < 1$.

2.5 Market Clearing

Good market clearing requires that the production function of the final good consist of total households' expenditure:

$$Y_{c,t} = \omega C_t^b + (1-\omega)C_t^s, \tag{41}$$

$$Y_{d,t} = \omega I_t^b + (1-\omega)I_t^s, \tag{42}$$

where $Y_{j,t} = \int Y_{j,t}(s)ds$.

Bond market and labor market clearing requires as follows

$$\omega b_t = (1-\omega)s_t, \tag{43}$$

Table 1: Parameters

Parameter	Description	Value
ω	Share of borrowers	0.50
β	Discount factor borrowers	0.98
γ	Discount factor savers	0.99
δ	Depreciation rate durable	0.01
χ	Fraction of durable that cannot be used as a collateral	0.25
η	Elasticity of substitution between C_t and D_t	1.00
ε_c	Elasticity of substitution among nondurable varieties	6.00
ε_d	Elasticity of substitution among durable varieties	6.00
ε_w^b	Elasticity of substitution among labor varieties of borrowers	6.00
ε_w^s	Elasticity of substitution among labor varieties of savers	6.00
φ^b	Inverse elasticity of labor supply of borrowers	1.00
φ^s	Inverse elasticity of labor supply of savers	1.00
θ_c	Calvo parameter for nondurable goods firms	0.75
θ_d	Calvo parameter for durable goods firms	0.00
ϕ_w^b	Wage adjustment cost parameter for borrowers	260.08
ϕ_w^s	Wage adjustment cost parameter for savers	266.02
τ	Weight of durables in the composite inflation index in the monetary policy rule	0.00
ϕ_π	Coefficient on inflation in monetary policy rule	1.50
ρ	Persistence parameter monetary policy shocks	0.50

$$N_{c,t} + N_{d,t} = \omega N_t^b + (1-\omega) N_t^s, \tag{44}$$

where $N_{j,t} = \int N_{j,t}(s) ds$.

Since we assume that there is no difference between borrowers' and savers' labors, the wage for borrowers is identical to that of savers:

$$w_t^b = w_t^s \equiv w_t, \quad \pi_{w,t}^b = \pi_{w,t}^s \equiv \pi_{w,t}. \tag{45}$$

3 Calibration and Solution Method

The models are calibrated to a quarterly frequency and the calibration follows Monacelli (2009). Table 1 provides a summary of the key parameters. The parameters ν^b and ν^s are calibrated to normalized labor supply to 1 in the steady state for each type of households. The parameter α in the CES basket is the same for both borrowers and savers, and it is calibrated such that in the steady state, aggregate durable expenditures are 20% of aggregate total expenditures. The parameter τ reflecting the weight of durable goods in the composite inflation index in the monetary policy rule is set to 0, meaning that monetary policy only responds to inflation in the nondurable goods sector as following Sterk (2010).

This paper imposes the Calvo parameter of nondurable sector on four quarter stickiness($\theta_c = 0.75$). Since the degree of nominal rigidity in durable prices has important effects on the model dynamics, several values for θ_d are considered, ranging from full flexibility to four quarter stickiness. In addition, we set $\phi_w^b = 260.08$ and $\phi_w^s = 266.02$ that the value of the Calvo parameter for wage rigidity is about 0.70. This paper log-linearizes the model around the deterministic steady state and solve the recursive law of motion.

4 Analytical Discussion

In this section, if durable good prices are flexible, for convenience, we can analytically demonstrate that the response of a two-sector sticky price model to monetary policy shocks depends crucially on how we assume the degree of wage stickiness. In particular, whether durable spending is accelerated or decelerated by collateral constraints to a monetary policy shock depends on how labor markets are designed. This paper shows that nominal wage rigidity is crucial for collateral constraints to accelerate the effects of monetary policy shocks on durable expenditure in this analytical discussion.

4.1 Sticky Nominal Wages and Collateral Constraints

This study investigates how the collateral constraints play a role in a two-sector sticky price general equilibrium model with nominal wage rigidity. To begin with, it is necessary to express aggregate total output and aggregate nondurable consumption as functions of the shadow values of durables. In this paper, $V_{d,t}^b$ and $V_{d,t}^s$ are defined as the shadow value of durables for the borrowers and the savers respectively, and their durables optimality conditions (9) and (18) can be rewritten as follows:

$$V_{d,t}^b \equiv q_t U_{c,t}^b = \frac{U_{d,t}^b + \beta(1-\delta)E_t(V_{d,t+1}^b)}{1-(1-\chi)(1-\delta)\psi_t E_t(\pi_{d,t+1})}, \tag{46}$$

$$V_{d,t}^s \equiv q_t U_{c,t}^s = U_{d,t}^s + \gamma(1-\delta)E_t(V_{d,t+1}^s). \tag{47}$$

Recall that $V_{d,t}^b$ will increase when the collateral constraints are tightened, which refers that ψ_t increases, and $V_{d,t}^s$ is quasi-constant.

In the model, the log-approximation of the aggregate total output $\widehat{Y}_t$ is:

$$\widehat{Y}_t \equiv \frac{Y_c}{Y_c+Y_d}\widehat{Y}_{c,t} + \frac{Y_d}{Y_c+Y_d}\widehat{Y}_{d,t} = \frac{\omega N^b}{\omega N^b+(1-\omega)N^s}\widehat{N}_t^b + \frac{(1-\omega)N^s}{\omega N^b+(1-\omega)N^s}\widehat{N}_t^s$$

$$= \frac{\omega N^b}{\omega N^b+(1-\omega)N^s}\frac{1}{\varphi^b}\left(\frac{\phi_w^b}{(\varepsilon_w^b-1)N^b}\left(\widehat{\pi}_{w,t}-\beta E_t\widehat{\pi}_{w,t+1}\right)+\widehat{V}_{d,t}^b+\widehat{w}_t-\widehat{q}_t\right)$$
$$+\frac{(1-\omega)N^s}{\omega N^b+(1-\omega)N^s}\frac{1}{\varphi^s}\left(\frac{\phi_w^s}{(\varepsilon_w^s-1)N^s}\left(\widehat{\pi}_{w,t}-\gamma E_t\widehat{\pi}_{w,t+1}\right)+\widehat{V}_{d,t}^s+\widehat{w}_t-\widehat{q}_t\right). \tag{48}$$

Even though both wage inflation effect and wealth effect have influence on aggregate total output, wage inflation effect is much more important to dynamics in this model. This effect have a huge impact on households' labor supply. Wage inflation effect, $\frac{\phi_w^b}{(\varepsilon_w^b-1)N^b}\left(\widehat{\pi}_{w,t}-\beta E_t\widehat{\pi}_{w,t+1}\right)$ and $\frac{\phi_w^s}{(\varepsilon_w^s-1)N^s}\left(\widehat{\pi}_{w,t}-\gamma E_t\widehat{\pi}_{w,t+1}\right)$, consists of the current wage inflation and the discounted expected future wage inflation. To be specific, as the current wage inflation is higher than the discounted expected future one, households work more greatly. In addition, the degree of wage stickiness decides how much they work.

Aggregate total output decreases in the model adding sticky nominal wages with collateral constraints after a monetary tightening. In the model with collateral constraints, given that the real wage in units of durables $\widehat{w}_t-\widehat{q}_t$ is 0 and the shadow value of durables $\widehat{V}_t^s$ remains quasi-zero, aggregate total output almost perfectly follows the responses of the wage inflation effects, $\frac{\phi_w^b}{(\varepsilon_w^b-1)N^b}\left(\widehat{\pi}_{w,t}-\beta E_t\widehat{\pi}_{w,t+1}\right)$ and $\frac{\phi_w^s}{(\varepsilon_w^s-1)N^s}\left(\widehat{\pi}_{w,t}-\gamma E_t\widehat{\pi}_{w,t+1}\right)$, and the shadow value of durables for the borrowers $\widehat{V}_{d,t}^b$ when a monetary tightening occurs. Even though $\widehat{V}_{d,t}^b$ responds positively, the wage inflation effects responds much more negatively. As a result, aggregate total output $\widehat{Y}_t$ responds negatively.

In spite of the negative response of aggregate total output in the model without collateral constraints($\omega = 0$), when monetary tightening takes place, wage inflation effect in the model with collateral constraints is more powerful than that in the model without collateral constraints because collateral constraints make the inflation of durable prices respond more negatively. Hence, collateral constraints accelerate aggregate total output in the model adding sticky nominal wages after a monetary tightening.

Next, it can be similarly expressed that aggregate nondurable consumption $\widehat{C}_t$ as a function of $\widehat{q}_t$ and the shadow values of durables $\widehat{V}^b_{d,t}$, $\widehat{V}^s_{d,t}$ by using the durable optimality conditions (46), (47), that is, $\widehat{V}^b_{d,t} = \widehat{q}_t + \widehat{U}^b_{c,t} = \widehat{q}_t - \widehat{C}^b_t$, $\widehat{V}^s_{d,t} = \widehat{q}_t + \widehat{U}^s_{c,t} = \widehat{q}_t - \widehat{C}^s_t$:

$$\widehat{C}_t \equiv \frac{\omega C^b}{\omega C^b + (1-\omega)C^s}\widehat{C}^b_t + \frac{(1-\omega)C^s}{\omega C^b + (1-\omega)C^s}\widehat{C}^s_t$$

$$= \frac{\omega C^b}{\omega C^b + (1-\omega)C^s}\left(\widehat{q}_t - \widehat{V}^b_{d,t}\right) + \frac{(1-\omega)C^s}{\omega C^b + (1-\omega)C^s}\left(\widehat{q}_t - \widehat{V}^s_{d,t}\right). \tag{49}$$

After a monetary tightening, the relative durable price $\widehat{q}_t$ responds negatively. Savers reduce their nondurable purchases proportionally. This is because the shock hardly affects the utility they derive from owning extra durables, as $\widehat{V}^s_{d,t}$ is quasi-zero. However, to borrowers, there are reasons to substitute away from nondurables towards durables, which is reflected by a positivity in $\widehat{V}^b_{d,t}$. Given that the response of the relative durable price $\widehat{q}_t$ is almost the same in the models without and with collateral constraints, aggregate nondurable consumption $\widehat{C}_t$ must respond more negatively in the model with collateral constraints.

As a result, aggregate durable spending $\widehat{I}_t$ must respond more negatively. This is because first, the response of aggregate total output responds more negatively than that of nondurable consumption and second, the gap of the responses between aggregate total output and aggregate nondurable consumption in the model with collateral constraints is more larger than in the model without collateral constraints. That is, collateral constraints accelerate the procyclical response of durable spending in the sticky nominal wage model.

4.2 Flexible Nominal Wages and Collateral Constraints

As Sterk (2010) explains in the general equilibrium aspects, collateral constraints make the response of durable spending to monetary policy to decelerate. Specifically, when borrowing by the borrowers decreases as a consequence of tighter collateral constraints to monetary tightening, it is required that the savers save less to remain in equilibrium. The savers then buy durable goods as an alternative way of saving. This kind of behavior enfeeble intuition that collateral constraints help generating the negative response of durable spending by causing a decrease in durable purchases by the borrowers. Moreover, even though the shadow value of durables for savers is quasi-constant, the shadow value of durables for borrowers increases after a monetary contraction because durables works as collateral for borrowing and their borrowing constraints become tighter. Subsequently, the borrowers weaken the effect of the constrained ability to borrow on their durable purchases, which they achieve by substituting away from nondurables and working more. This makes aggregate output to increase. The borrowers also have stronger incentives to substitute away from nondurables and buy durables instead, which leads to a larger decrease of aggregate nondurable consumption. Being driven from aggregate total output, the aggregate durable expenditure increases more greatly.

As the assumption of flexible nominal wages is one special case of the sticky nominal wages model above, the log-linear approximation of aggregate total output $\widehat{Y}_t$ can be expressed as the following reduced equation ($\theta_w = 0$):

$$\widehat{Y}_t \equiv \frac{Y_c}{Y_c + Y_d}\widehat{Y}_{c,t} + \frac{Y_d}{Y_c + Y_d}\widehat{Y}_{d,t} = \frac{\omega N^b}{\omega N^b + (1-\omega)N^s}\widehat{N}_t^b + \frac{(1-\omega)N^s}{\omega N^b + (1-\omega)N^s}\widehat{N}_t^s$$

$$= \frac{\omega N^b}{\omega N^b + (1-\omega)N^s}\left(\frac{1}{\varphi^b}\left(\widehat{V}_{d,t}^b + \widehat{w}_t - \widehat{q}_t\right)\right) + \frac{(1-\omega)N^s}{\omega N^b + (1-\omega)N^s}\left(\frac{1}{\varphi^s}\left(\widehat{V}_{d,t}^s + \widehat{w}_t - \widehat{q}_t\right)\right). \quad (50)$$

Aggregate total output remains nearly constant in the model without collateral constraints and increases in the model with collateral constraints after a monetary tightening. To be specific, the real wage in units of durables $\widehat{w}_t - \widehat{q}_t$ is 0 because durable prices are set according to a constant markup over the nominal wage. Given that $\widehat{V}_{d,t}^s$ remains quasi-zero, aggregate total output almost perfectly follows the positivity in the shadow value of the borrowers $\widehat{V}_{d,t}^b$ when a monetary tightening takes place.

Since the response of nondurable consumption $\widehat{C}_t$ to a monetary tightening is analogous in the sticky nominal wages model, it responds more negatively when households have collateral constraints.

There is a clear reason on why collateral constraints lead to a more positive response of aggregate durable purchases. When the response of aggregate total output is more positive after adding collateral constraints, and nondurable consumption falls more greatly, then the aggregate resource constraint can only remain satisfied if production in the durable goods producing sector responds more positively. Consequently, collateral constraints decelerate the response of durable spending in the flexible nominal wage model.

5 Simulation Results

If the prices of durable good are sticky, we can numerically investigate that the response of a two-sector sticky price model with collateral constraints to monetary policy shocks.

Collateral constraints accelerate durable spending to monetary shocks in the model with sticky nominal wages regardless of durables price stickiness. Figure 1 plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable investment, and aggregate total output to a monetary tightening for the sticky nominal wages model. Aggregate nondurable consumption $\widehat{C}_t$ respond more negatively in the model with collateral constraints after a monetary contraction. Also, aggregate durable spending $\widehat{I}_t$ must respond more negatively. This is because first, the response of aggregate total output responds more negatively than that of nondurable consumption and second, the gap of the responses between aggregate total output and aggregate nondurable consumption in the model with collateral constraints is more larger than in the model without collateral constraints. That is, collateral constraints amplify the procyclical response of durable spending.

On the other hand, as Sterk (2010) shows, durable expenditure is decelerated by collateral constraints to monetary shocks in the model with flexible nominal wages regardless of the price stickiness of durable goods. Figure 2 plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable investment, and aggregate total output to a monetary tightening for the flexible nominal wages model. When the response of aggregate total output is more positive after adding collateral constraints, and nondurable consumption falls more greatly, then the aggregate resource constraint can only remain satisfied if production in the durable goods producing sector responds more positively. Consequently, augmenting the model with collateral constraints decelerate the response of durable spending.

6 Adding the Adjustment Cost in Durable Investment

Although the model with sticky nominal wages generate the negative response of durable spending in the initial period, the sensitivity of durable goods to a monetary contraction is too much larger than that of nondurables, which is at odds with empirical data. As Carlstrom and Fuerst (2010) shows that the assumption of sticky nominal wages leads to rapid over-shooting of durable purchase.

Thus, they add the adjustment costs in durable investment to solve such problem. Now, this paper considers the model with sticky nominal wages and adjustment costs in durable investment.

Durable investments increase the households' durable stocks according to:

$$D_t^b = F(I_t^b, I_{t-1}^b) + (1-\delta)D_{t-1}^b, \tag{51}$$

$$D_t^s = F(I_t^s, I_{t-1}^s) + (1-\delta)D_{t-1}^s, \tag{52}$$

where durable investment adjustment costs are given by

$$F(I_t^b, I_{t-1}^b) = \left[1 - S\left(\frac{I_t^b}{I_{t-1}^b}\right)\right] I_t^b, \tag{53}$$

$$F(I_t^s, I_{t-1}^s) = \left[1 - S\left(\frac{I_t^s}{I_{t-1}^s}\right)\right] I_t^s. \tag{54}$$

The function $S(\cdot)$ represents adjustment costs that are incurred when the level of durable investment changes over time. This research assumes that $S(1) = 0$, $S^{'}(1) = 0$, so that there are no adjustment costs in the steady state. $\kappa \equiv S^{''}(1) > 0$ is set in such a way that the the volatility of durable spending to monetary shocks is about four times larger than that of nondurable spending. In this model, equations (51) and (52) replace equations (6) and (15) respectively.

Figure 3 plots the IRFs for the nominal interest rate, aggregate nondurable consumption, aggregate durable investment, and aggregate total output to a monetary tightening for the sticky nominal wages model with durable investment adjustment costs. The model with adjustment costs in durable investment shows that over-shooting problem is solved. Also, like the previous model, total output decreases more greatly and both nondurable and durable purchases respond more negatively after a monetary tightening than the model without collateral constraints.

7 Conclusion

This paper has shown that collateral constraints on the household's side can play a critical role in a two-sector sticky price model. While many papers have stressed the role of credit frictions in an one-sector sticky price model, the two-sector sticky price model has been little explored. Although

Iacoviello (2005) and Iacoviello and Neri (2010) study the general equilibrium model with collateral constraints, they do not focus on how it has effect on durable goods spending. Monacelli (2009) emphasizes the role of collateral constraints for durable expenditure, but Sterk (2010) argues otherwise. As he shows, collateral constraints decelerate the response of durable spending to monetary shocks in the model with flexible nominal wages. However, this research shows that collateral constraints accelerate the effects of monetary policy shocks on durable spending in the model with sticky nominal wages. The main reason is that collateral constraints strengthen the wage inflation effect. It affects households' decision for labor supply significantly. Many research papers such as Kim (2000), Huang and Liu (2002), Christiano et al. (2005), and DiCecio (2009) have proved that nominal wage rigidity succeeds in capturing some key features of business cycles. Hence, it is legitimate to say that the role of collateral constraints can be important in the two-sector sticky price model as well as in the one-sector sticky price model.

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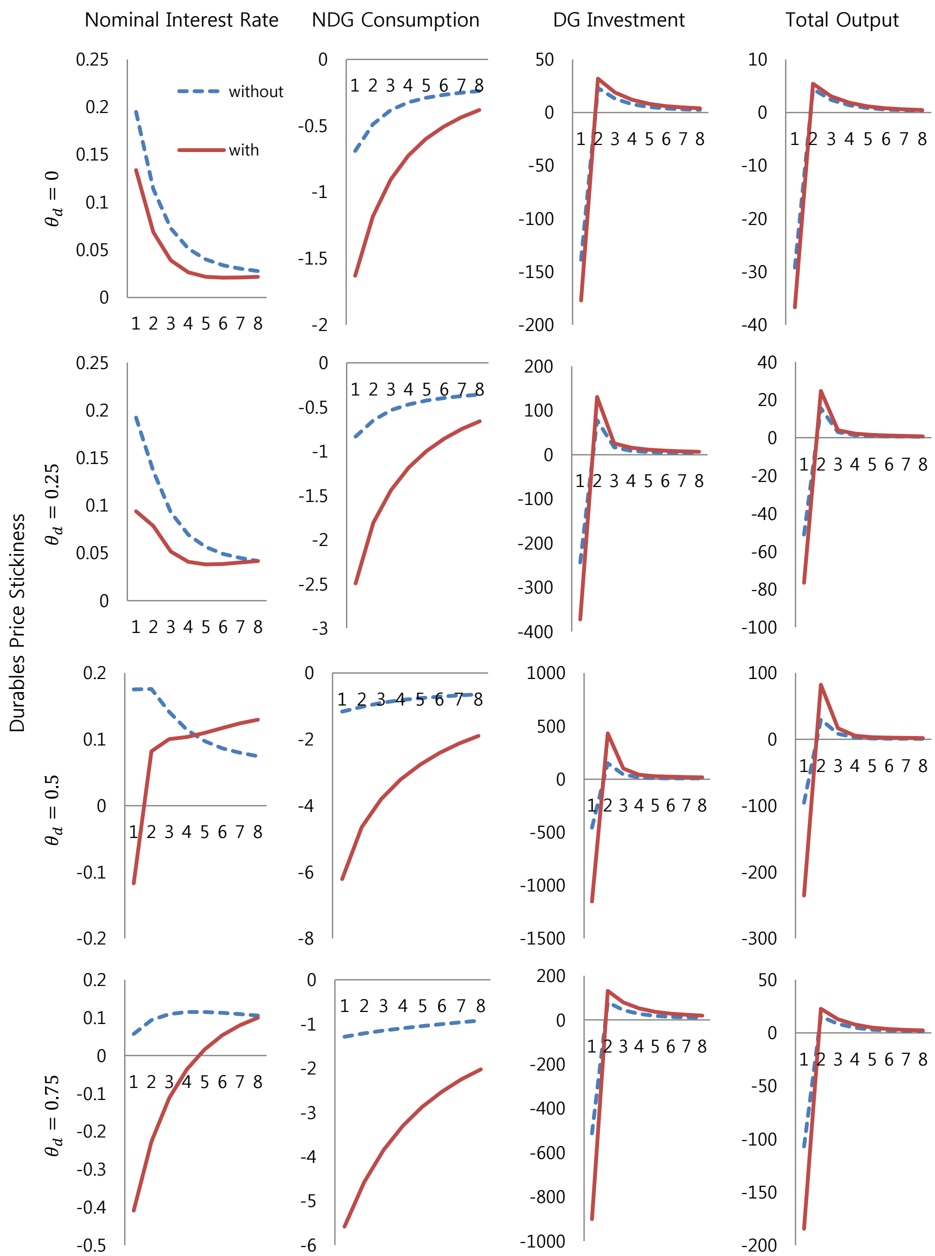

Figure 1: Responses to the Monetary Policy Shock in the Sticky Nominal Wages Model

Note: Responses to a monetary tightening (25 basis point innovation) in the sticky nominal wages model without and with collateral constraints. Response of nominal interest rate is plotted as deviation form steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.

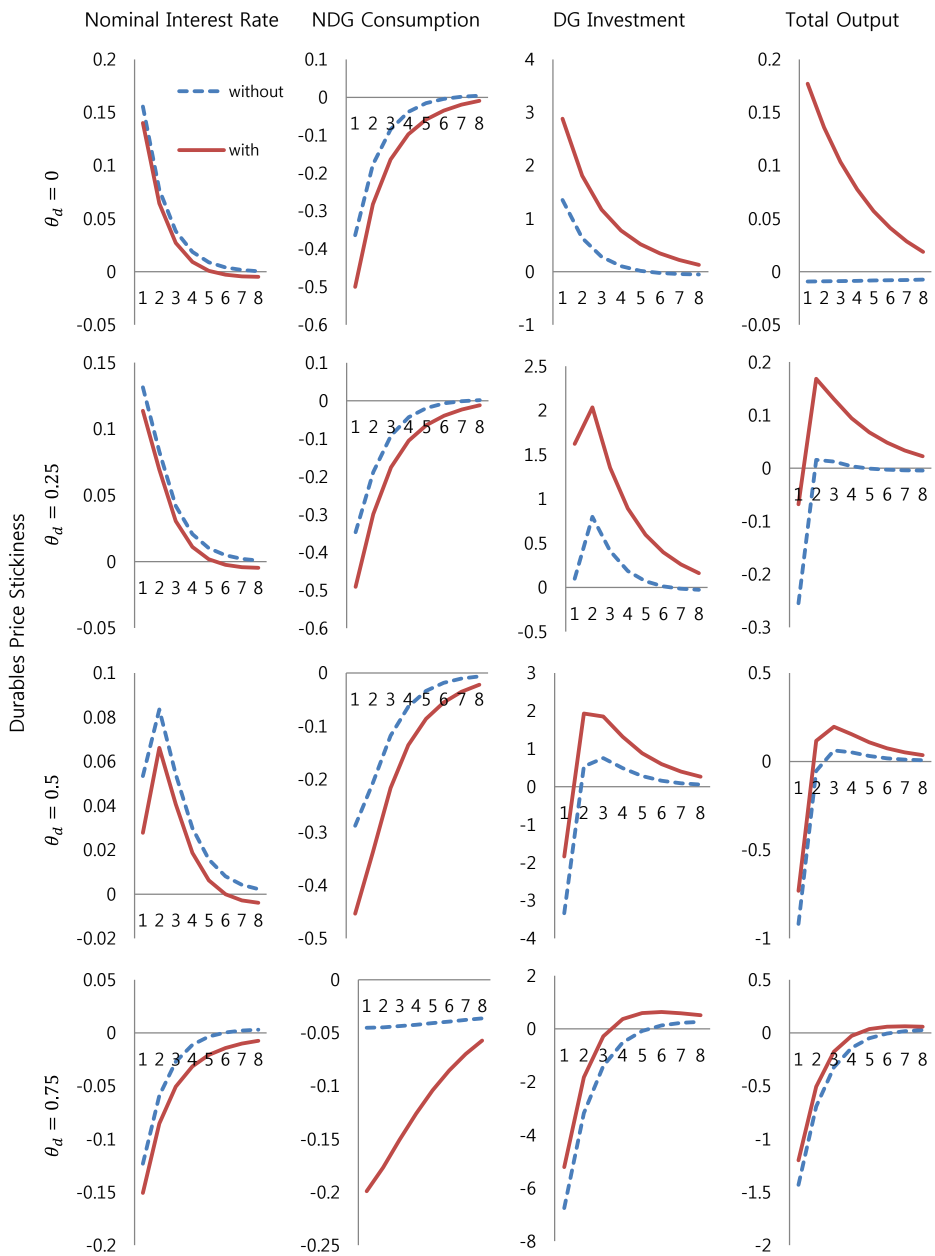

Figure 2: Responses to the Monetary Policy Shock in the Flexible Nominal Wages Model

Note: Responses to a monetary tightening (25 basis point innovation) in the flexible nominal wages model without and with collateral constraints. Response of nominal interest rate is plotted as deviation form steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.

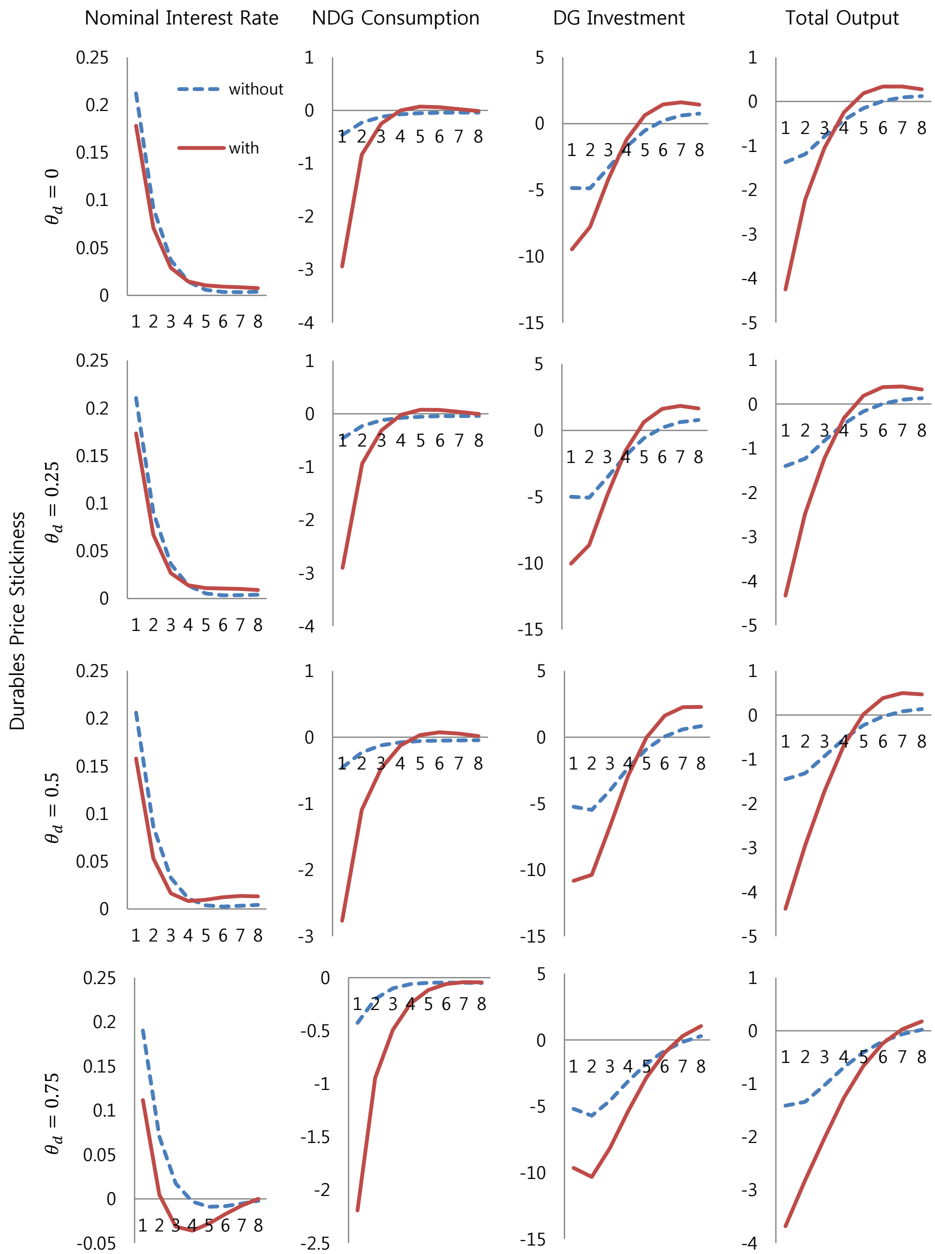

Figure 3: Responses to the Monetary Policy Shock in the Sticky Nominal Wages Model adding Durable Investment Adjustment Costs

Note: Responses to a monetary tightening (25 basis point innovation) in the sticky nominal wages model adding durable investment adjustment costs without and with collateral constraints. Response of nominal interest rate is plotted as deviation form steady state in percentage point, and other responses are plotted as percentage deviation from the steady state.