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Ex-ante Heterogeneity and Equilibrium Redistribution (Preliminary)

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Abstract

The standard models have difficulty in justifying the current labor income tax rate in the U.S. The optimal tax rate is much higher than the current rate (e.g., Piketty and Saez (2013)). A majority of the population should be also in favor of raising taxes, as the income distribution is highly skewed. We show that the political equilibrium is actually close to the current tax rate, once we take account the ex-ante heterogeneity in household earnings (e.g., Guvenen (2009)) and income-dependent voting behavior, (e.g., Mahler (2008)) into the standard incomplete-markets model.

Keywords: Optimal Tax, Ex-ante Heterogeneity, Voting Turnout Rates

1. Introduction

The standard models have difficulty in justifying the current labor income tax rate in the U.S. The optimal tax rate is much higher than the current rate (e.g., Piketty and Saez (2013)): the potential benefits from the tax increase well exceed the efficiency costs from distorting the labor supply. The majority of the population should also be in favor of fiscal reform to raise the tax rate, as the income distribution is highly skewed.

A larger literature built on utilitarian social welfare functions finds that the current tax rate in the U.S. is far from optimal (e.g., Floden and Linde, 2001; Piketty and Saez, 2013; Corbae et al. 2009; and Heathcote and Tsujiyama, 2016). By contrast, Conesa and Krueger (2006) argue that lowering the marginal tax rate for the rich along with increased deductions for the poor enhances social welfare.[1] [2]

In this paper we argue that the political equilibrium is actually close to the current tax rate, once we take account the ex-ante heterogeneity in household earnings (e.g., Guvenen (2009)) and income-dependent voting behavior, (e.g., Mahler (2008)) into the standard incomplete-markets model. Individual household's productivity consists of a permanent earnings ability (ex-ante heterogeneity) and pure luck (ex-post heterogeneity). When the household's income differences are largely driven by the permanent difference in earnings ability, the potential insurance benefit from the government's tax-and-transfer policy is small. Moreover, when households' voting turnout rates are positively correlated with income (as documented in Mahler (2008)), the tax rate chosen by the majority vote will be skewed toward the one preferred by the income income households. According to the simulation from our model, the tax rate chosen by the majority is close to the current average income tax rate in the U.S.

[1]The income distribution in their model is much more evenly distributed (e.g., 0.39 for the income Gini) than that in the data (e.g., 0.58 in the SCF (Díaz-Giménez et al. 2011) and 0.5 in the OECD Database (2015)).

[2]Lockwood and Weinzierl (2016), Chang et al. (2016), and Heathcote and Tsujiyama (2016) search for the Pareto weights that would justify the current tax rates. Golosov and Tsyvinski (2007) and Chetty and Saez (2010) demonstrate that the existence of private insurance lowers the optimal level of government intervention. Heathcote et al. (2016) introduce endogenous skill investment and the externality associated with public expenditures and argue that the optimal tax scheme is less progressive than the current one.

We feature three model economies that differ with respect to the *relative* size of ex-ante heterogeneity in the overall income dispersion: no ex-ante heterogeneity (NH), small ex-ante heterogeneity (SH), and large ex-ante heterogeneity (LH). By construction, all three economies match the overall income dispersions (Gini coefficients) in the data. The dispersion of permanent components (ex-ante heterogeneity) and that of stochastic components (ex-post heterogeneity) are estimated from the PSID following Guvenen (2009). In NH model, the entire income dispersion is driven by ex-post heterogeneity (luck or productivity shocks). In SH model, about one-third (31.3%) of income dispersion is driven by ex-ante heterogeneity (permanent ability differences). In LH model, slightly more than half (56.7%) of the income dispersion is driven by the ex-ante heterogeneity.

Due to the veil of ignorance, the optimal income tax rates based on the utilitarian social welfare function are much higher than the current tax rate but similar in all three economies (which exhibit similar overall income distributions by construction). The optimal tax rates are 37.3% (NH), 37.7% (SH) and 36.9% (LH), respectively. However, once we take into account the income-dependent voting turnout rates in the data (e.g., Mahler, 2008)—the turnout rate increases with income level– the political equilibrium vastly differs across the three economies. The tax rates chosen by effective voting in the three model economies are 35% (NH), 33% (SH), and 27% (LH). The tax rate chosen in LH (which is our preferred model economy) is not far from the current average tax rate (23.8%) in the U.S. As the permanent differences (ex-ante heterogeneity) becomes more important in total earnings, the insurance benefit from the government tax-and-transfer policy diminishes.

Our results are closely related to the large existing literature on the optimal income tax. Piketty and Saez (2013) summarize recent developments in optimal labor-income taxation. According to their optimal linear tax rate formula, based on the Mirrlees (1971) model, both the utilitarian and the median voter tax rates are much higher than the current one in the U.S. Heathcote et al. (2016) present an equilibrium model that features endogenous skill investment, flexible labor supply, and the externality linked to government purchases. Their model suggests that the optimal tax scheme is less progressive than the current one. Corbae et al. (2009) compute the utilitarian optimal tax rates and political outcome by a median voter: the optimal tax rates are higher than those in the data. Corbae et al. (2009) point to voter turnout rates as a possible cause of the gap between the model and the data.

Benabou and Ok (2001) present the prospect of upward mobility (POUM) hypothesis: the poor may not support a strong redistribution policy because they may be rich in the future. Alesina et al. (2011) find empirical support for the POUM hypothesis. Charité et al. (2015) report that according to their experiment, subjects' preferences for redistribution diminish significantly when they know their initial endowments than when they don't.

Mahler (2008) examines the electoral turnout rates and income redistribution among 13 developed countries and shows that turnout rates increase with income level in many countries. According to a meta analysis by Smets and Van Ham (2013), 21 out of 40 studies find a statistically significant relationship between income and turnout rates. Our paper quantitatively shows that the observed pattern in turnout rates can indeed justify the current tax rate as a voting outcome.

Chang et al. (2016) measure the Pareto weights that justify the current tax progressivity across 32 OECD countries through a unified framework and find that Pareto weights vastly differ across countries. Based on the Mirrlees (1971) framework, Lockwood and Weinzierl (2016) infer social weights from U.S. tax policies betwen 1979 and 2010. Heathcote and Tsujiyama (2016) compare different tax systems given the Pareto weights for the current tax rates. Other studies try to explain the current tax rate in different ways. Golosov and Tsyvinski (2007) and Chetty and Saez (2010) demonstrate that the existence of private insurance lowers the optimal level of government intervention. Weinzierl (2014) presents a survey report that many people prefer the principle of equal sacrifice over conventional utilitarian objectives. Lockwood and Weinzierl (2015) argue that preference heterogeneity can reduce the optimal redistribution under certain conditions.

The remainder of the paper is organized as follows. In Section 2 we build a model with both ex-ante and ex-post heterogeneity in earnings. In Section 3, we calibrate this economy based on the empirical estimates of each component of the income process based on the panel data. In section 4, we compute the optimal tax reforms under utilitarian social welfare function. We then introduce the income-dependent voting turnout rates to simulate effective voting in regard to various tax reforms. Section 5 is the conclusion.

2. Model

The model economy introduces the permanent difference in earnings ability—i.e., *ex ante* heterogeneity—into *a la* Aiyagari (1994) with endogenous labor supply.

Households: There is a continuum (measure one) of households that have identical preferences. An individual household's productivity at time t is z_t. When a household with productivity z_t supplies h_t hours in the market, its labor income is $w_t z_t h_t$, where w_t is the wage rate for the efficiency unit of labor. The household's productivity z_t consists of two components: $z_t = \psi \cdot x_t$, where ψ is a time-invariant deterministic component (e.g., ability) and x_t is a stochastic component (e.g., luck). The stochastic component evolves over time according to a common Markov process with a transition probability distribution function: $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. Households hold assets (claims on production capital) a_t that yield the real rate of return r_t. Both labor and capital incomes are subject to income taxes at the same rate τ. Households receive a lump-sum transfer T_t from the government. A household maximizes its lifetime utility:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{{c_t}^{1-\sigma} - 1}{1-\sigma} - B \frac{{h_t}^{1+1/\gamma}}{1+1/\gamma} \right)$$

subject to

$$c_t + a_{t+1} = (1-\tau)(w_t \psi x_t h_t + r_t a_t) + a_t + T_t,$$

$$a_{t+1} \geq \underline{a}$$

where c_t is consumption. Parameters σ and γ represent relative risk aversion and the Frisch elasticity of labor supply, respectively. Capital markets are incomplete in two senses: (i) physical capital is the only asset available to households to insure against stochastic shocks to their productivity and (ii) households face a borrowing constraint: $a_t \geq \underline{a}$ for all t. Households differ *ex-ante* with respect to their permanent productivity ψ and differ *ex-post* with respect to their productivity shock x_t and asset holdings a_t.

The economy-wide distribution of households is characterized by the probability measure $\mu_t(a_t, x_t, \psi)$.

Firm: A representative firm produces output according to a constant-returns-to-scale Cobb-Douglas production technology using capital, K_t, and effective units of labor, $L_t = \int h_t \psi x_t d\mu$. Capital depreciates at rate δ each period:

$$Y_t = F(L_t, K_t) = {L_t}^{\alpha} {K_t}^{1-\alpha}.$$

Government: The government operates a simple fiscal policy characterized by a flat income tax rate (τ) and a constant lump-sum transfer (T). According to Piketty and Saez (2013), a linear income tax simplifies the analysis but still captures the key equity-efficiency trade-off. For example, progressivity at the top is often counter-balanced by the fact that a substantial fraction of capital income receives preferential tax treatment under most income rules. Overall, all transfers (including various government spending) taken together are fairly close to a demogrant, i.e., they are about constant with income. Hence, the optimal linear tax model with a demogrant transfer is a reasonable first-order approximation of the actual tax system and is useful to understand how the level of taxes and transfers should be set. We also assume that the government spends tax revenues exclusively for lump-sum transfers to households, and balances the budget:

$$T_t = \int \tau \big\{ w_t \psi x_t h_t + r_t a_t \big\} d\mu(a_t, x_t, \psi)$$

Recursive Formulation: It is useful to consider a recursive equilibrium. Let $V(a, x, \psi)$ denote the value function of a household with asset holdings a, productivity shock x, and permanent ability ψ:

$$V(a, x, \psi) = \max_{c,h} \left\{ \frac{c^{1-\sigma} - 1}{1-\sigma} - B \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \beta \mathbb{E}\big[V(a', x', \psi) | x, \psi \big] \right\}$$

subject to

$$c + a' = (1 - \tau)(w \psi x h + r a) + a + T,$$

$$a' \geq \underline{a}$$

The inter-temporal first-order condition for optimal consumption is:

$$c(a,x,\psi)^{-\sigma} = \beta(1+(1-\tau)r)\mathbb{E}\big[c(a',x',\psi)^{-\sigma}\big].$$

The intra-temporal first-order condition for optimal hours worked is:

$$B \cdot h(a,x,\psi)^{1/\gamma} c(a,x,\psi)^{\sigma} = (1-\tau)w\psi x.$$

Equilibrium: A stationary equilibrium consists of a value function, $V(a,x,\psi)$; a set of decision rules for consumption, asset holdings, and labor supply, $c(a,x,\psi)$, $a'(a,x,\psi)$, $h(a,x,\psi)$; aggregate inputs, K, L; and the invariant distribution of households, $\mu(a,x,\psi)$, such that:

1. Individual households optimize: Given w and r, the individual decision rules $c(a,x,\psi)$, $a'(a,x,\psi)$, $h(a,x,\psi)$ and $V(a,x,\psi)$ solve the Bellman equation.

2. The representative firm maximizes profits:

$$w = \alpha(K/L)^{1-\alpha}$$
$$r + \delta = (1-\alpha)(K/L)^{-\alpha}$$

3. The goods market clears:

$$\int \big\{a'(a,x,\psi) + c(a,x,\psi)\big\} d\mu = F(L,K) + (1-\delta)K$$

4. The factor markets clear:

$$L = \int \psi x h(a,x,\psi) d\mu$$
$$K = \int a d\mu$$

5. The government balances the budget:

$$T = \int \tau\{w\psi x h(a,x,\psi) + ra\} d\mu$$

6. Individual and aggregate behaviors are consistent: For all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$

$$\mu(A^0, X^0, \Psi^0) = \int_{A^0,X^0,\Psi^0} \left\{ \int_{\mathcal{A},\mathcal{X},\Psi^0} \mathbf{1}_{a'=a'(a,x,\psi)} d\pi_x(x'|x,\psi) d\mu \right\} da' dx' d\psi$$

3. Quantitative Analysis

3.1. Calibration

We construct three model economies that differ with respect to the *relative* size of ex-ante heterogeneity in the income distribution. The first model economy features no ex-ante heterogeneity (denoted by NH). The second and third models feature small (SH) and large ex-ante heterogeneity (LH). While the three model economies differ in the relative size of the ex-ante component of household earnings, we choose the size of stochastic productivity shocks in each model so that all three economies exhibit the same degree of overall dispersion in earnings—i.e., the Gini coefficients of *realized* income distributions in all three models will be identical to that in the data.

Common Parameters The time unit is one year. The labor income share (α) is 0.64, and the annual depreciation rate of capital (δ) is 10%. Both the relative risk aversion (σ) and the labor supply elasticity (γ) are set to 1. Workers are not allowed to borrow in the benchmark: $\underline{a} = 0$. Table 1 summarizes the common parameters of the model economies.

Table 1: Common Parameters

Parameters	Values
Labor income share (α)	0.64
Depreciation rate of capital (δ)	0.10
Relative risk aversion (σ)	1.00
Labor supply elasticity (γ)	1.00
Borrowing constraint ($\underline{a}$)	0.00

Economy-specific Parameters We assume that the time-invariant individual earnings ability (ex-ante productivity) ψ is drawn from a log normal distribution: $\ln \psi \sim N(0, \sigma_\psi^2)$. In the no ex-ante heterogeneity (NH) model, $\sigma_\psi = 0$. The time-varying individual productivity shock (ex-post heterogeneity) x is assumed to follow an AR(1) process in logs: $\ln x' = \rho_x \ln x_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma_\epsilon)$. The persistence of the stochastic productivity shock is assumed to be $\rho_x = 0.946$ (which is a commonly used value in the literature). Given ρ_x, we choose $\sigma_\epsilon = 0.23$ to match the before tax/transfer income Gini coefficient of 0.5 in the data (the value for the U.S. in 2010, according to the 2015 OECD Database). Thus, in NH the entire income dispersion is generated by the stochastic shock to productivity as in Aiyagari (1994).

For the size of permanent ability in the small ex-ante heterogeneity (SH) and large ex-ante heterogeneity (LH) models, we follow Guvenen (2009), who decomposes individual earnings into ability and shock. He proposes two specification: restricted income profile (RIP) and heterogeneous income profile (HIP). We use his estimates based on a larger sample instead of his baseline estimates.[3] The details of the estimation are provided below.

For SH we set the relative size of ex-ante heterogeneity to that in the RIP of Guvenen (2009): $\sigma_\psi/\sigma_z = 31.3\%$. About one-third of the income differences across households are due to permanent differences in ability. Given the persistence of stochastic shocks ($\rho_x = 0.946$) based on our estimate of the RIP specification, we choose the size of productivity shocks ($\sigma_\epsilon = 0.213$) to generate the same before-tax income Gini. The resulting ex-ante heterogeneity in SH is $\sigma_\psi = 0.301$.[4]

For LH we set the ratio of ex-ante heterogeneity in the overall income dispersion ($\sigma_\psi/\sigma_z = 56.7\%$) also based on the HIP in Guvenen (2009), according to which a little over half of the income dispersion is accounted for by ex-ante differences in earnings ability. Note that the stochastic process of the productivity shock is now much less persistent ($\rho_x = 0.842$) because a large portion of earnings are captured by the permanent differences in individual ability. The required stochastic shocks to match the overall income Gini is $\sigma_\epsilon = 0.251$, and the value of ex-ante heterogeneity is $\sigma_\psi = 0.619$.

For each model, the time discount factor, β, is chosen so that the steady-state real interest rate is 4%. Those factors are 0.953 (NH), 0.955 (SH), and 0.96 (LH). Since the uninsurable income risk (ex-post heterogeneity) is the largest in NH, households in NH

[3]Since he reports only HIP estimates for a larger sample, we estimate parameters under the RIP specification based on his method and his labor earnings data.

[4]We assume that there are five groups for ψ.

have the strongest precautionary savings motive. Thus, a small discount factor is required to achieve the same real interest rate. The disutility from working, B, is chosen so that average hours worked in the steady state is 0.323 (OECD, 2015).[5]

The income tax rate, τ_0, is chosen to match the after-tax income Gini coefficient of 0.38 in the U.S. in 2010 (OECD, 2015). The calibrated income tax rate is τ_0, 23.8% in all three economies, which is almost identical to the tax-to-GDP ratio in 2010 (OECD, 2015). We can show that matching the two Ginis generates the same income tax rate in a linear tax system.[6] Table 2 summarizes the economy-specific parameters.

Table 2: Economy-Specific Parameters

	NH	SH	LH
Ex-Ante Heterogeneity Ratio (σ_ψ/σ_z)	0.000	0.313	0.567
SD of permanent component (σ_ψ)	0.000	0.301	0.619
SD of stochastic component (σ_x)	0.711	0.658	0.466
Persistence of shocks (ρ_x)	0.946	0.946	0.842
SD of innovation to shocks (σ_ϵ)	0.230	0.213	0.251
Time discount factor (β)	0.953	0.955	0.960
Disutility from working (B)	5.051	5.092	5.010

Estimating Ex-Ante and Ex-Post Heterogeneity (Guvenen, 2009) To empirically decompose the observed income differences into the two types of heterogeneity—*ex-ante* and *ex-post*— we assume that individual productivity (earnings or wages) consists of a deterministic age profile and stochastic shocks around the profile. We view the

[5]This is the average share of time devoted to working. According to the 2015 OECD database, working hours were 1,778 and total discretionary hours were 5,500 in the U.S. in 2010.

[6]Under a linear tax and lump-sum transfer system, the income tax rate τ is the same as the reduction rates in the Gini coefficients through taxes and transfers. Suppose that the Gini coefficient of income y is $G_y = \frac{1}{\mu}\int F(y)(1-F(y))dy$, where μ is average income, and F is the cumulative density function. Let disposable income be $z = (1-\tau)y + T$ (under a linear tax with lump-sum transfer). The Gini coefficient of disposable income is: $G_z = \frac{1}{\mu_z}\int F(z)(1-F(z))dz$, where μ_z is the average disposable income. Since z is a linear function of y, $F(z) = F(y)$. The government uses all tax revenues for transfers so that $\mu = \mu_z$. Then, $G_z = \frac{1}{\mu}\int F(y)(1-F(y))dz$ $= \frac{1}{\mu}\int F(y)(1-F(y))(1-\tau)dy$ (by the chain rule) $= (1-\tau)G_y$.

differences in the income profile as reflecting an ex-ante heterogeneity (ability) and the movement around the profile as reflecting an ex-post heterogeneity (shocks). We adopt the econometric procedure developed by Guvenen (2009), who considers two hypotheses regarding the shape of the income profile: restricted income profile (RIP) and heterogeneous income profile (HIP). The RIP assumes that workers have a common shape of the age-earnings profile except for an intercept. The HIP allows for differences in the slope (or wage growth) and in the intercept. According to Guvenen (2009), the log earnings of worker i at time t whose age is j (after controlling for a common age profile), $\hat{z}^i_{j,t}$, consist of a deterministic individual-specific age profile $\hat{\psi}^i_j$, persistent stochastic shocks $\hat{x}^i_{j,t}$, and i.i.d. measurement errors $\eta^i_{j,t}$:[7]

$$
\begin{aligned}
\hat{z}^i_{j,t} &= \hat{\psi}^i_j + \hat{x}^i_{j,t} + \xi_t \eta^i_{j,t} \\
\hat{\psi}^i_j &= \theta^i + \phi^i j \\
\hat{x}^i_{j,t} &= \rho_x x^i_{j-1,t-1} + \zeta_t \epsilon^i_{j,t}
\end{aligned}
$$

The individual-specific deterministic age profile ($\hat{\psi}^i_j$) consists of an intercept θ^i and slope ϕ^i. Stochastic income shocks ($\hat{x}^i_{j,t}$) follow an AR(1) process with persistence ρ_x and innovation $\epsilon^i_{j,t}$. ξ_t and ζ_t reflect any time effect on measurement errors and innovation to stochastic shocks, respectively. The RIP corresponds to the case where $\phi^i = 0$ for all i (i.e., no difference in the slope of the age profile).

The labor earnings data are obtained from Guvenen (2009) and are originally from the PSID 1968-1993. The main sample consists of male heads of household between ages 20 and 64. While Guvenen's baseline estimates restrict the samples to individuals whose labor-market experience is at least 20 years (1,270 individuals), we will use the estimates based on a larger sample—all workers whose labor market experience is 4 years or more (3,858 individuals). Table 3 compares our estimates based on the larger sample to those of Guvenen (2009).[8]

[7]More exactly, $\hat{z}^i_{j,t}$ are the residuals of regression of individual income on the polynomials of labor market experience – i.e., unexplained income components by a common trend of market experience.

[8]Guvenen (2009) provides only HIP estimates for a larger sample, and we estimate parameters under the RIP specification.

For the RIP specification, Guvenen's baseline estimate of persistence, $\rho_x = 0.988$, appears somewhat larger than those from other studies (e.g., Floden and Linde, 2001; Heathcote et al., 2008). Our estimate based on the larger sample, $\rho_x = 0.946$ is in line with those in the previous studies cited above.

For the HIP specification, the cross-sectional variance of the intercept of the age profile based on the larger sample, $\sigma_\theta^2 = 0.072$, is much bigger than that (0.022) in the baseline estimate of Guvenen (2009) because the larger sample includes younger workers. The variance of the slope parameter $100 \cdot \sigma_\phi^2 = 0.043$ is similar to that (0.038) in the baseline case. The correlation between the intercept and slope, $corr(\theta, \phi)$, is slightly more negative in the larger sample estimates (-0.33 vs. -0.23 in the baseline). The persistence of the stochastic component $\rho_x = 0.842$ is similar to the baseline estimate (0.821).

Table 3: Estimates of Earnings Process

Specification	σ_θ^2	$100 \cdot \sigma_\phi^2$	$corr(\theta, \phi)$	ρ_x	σ_ϵ^2	σ_η^2	Ratio $\left(\frac{E[\sigma_\psi]}{E[\sigma_\psi]+\sigma_x}\right)$
	Estimates in Guvenen (2009)						
RIP	0.058	-	-	0.988	0.015	0.061	0.233
HIP	0.022	0.038	-0.23	0.821	0.029	0.047	0.573
	Estimates based on Larger Sample						
RIP	0.077	-	-	0.946	0.039	0.009	0.313
HIP	0.072	0.043	-0.33	0.842	0.032	0.044	0.567

Note: Data are taken from Guvenen (2009) and are based on PSID 1968-1993. The baseline estimates are based on male heads of household between ages 20 and 64 whose labor market experience is at least 20 years (1,270 individuals). The estimates from the larger sample are based on all workers whose labor market experience is at least 4 years (3,858 individuals).

The dispersion of ex-ante heterogeneity is measured by the average standard deviation of $\hat{\psi}$: $E[\sigma_\psi] = E\left[\sqrt{\sigma_\theta^2 + 2\sigma_{\theta,\phi} j + \sigma_\phi^2 j^2}\right]$.[9] The dispersion of ex-post heterogeneity is measured by the standard deviation of $\hat{x}$: $\sigma_x = \sigma_\epsilon / \sqrt{(1-\rho_x^2)}$. Then, the relative importance

[9] We consider the market experience j from 1 to 35, when we calculate the average standard deviation of $\hat{\psi}$ ($E[\sigma_\psi]$)

of ex-ante heterogeneity in the total dispersion of the cross-sectional income distribution is $\frac{E[\sigma_\psi]}{E[\sigma_\psi]+\sigma_x}$. For the RIP, this ratio is 0.31 (0.23 in the baseline case). For the HIP, this ratio is 0.567 (0.573 in the baseline case).

For our quantitative analysis below, the estimated income process from the RIP (based on larger sample) is used for small ex-ante heterogeneity (SH) and that from HIP is used for large ex-ante heterogeneity (LH). No ex-ante heterogeneity (NH) corresponds to the case where the entire dispersion of the cross-sectional income distribution is driven by stochastic shocks (x), as in Aiyagari (1994).

3.2. Income and Wealth Distributions

Table 4 reports the steady-state wealth Gini and the relative incomes across the 5 income quintiles in the data (SCF) and the three model economies. By construction, the steady-state interest rate (4%), average hours worked (0.323), before-tax income Gini (0.5), and after-tax income Gini (0.38) are the same in all three models. Since the income Gini coefficients are identical, relative incomes across income quintiles are also very similar. For example, the relative income (before taxes and transfers) in the first income quintile is 18% in NH, 17% in SH, and 17% in LH. Relative incomes from the SCF are more dispersed because the before-tax Gini coefficient in the SCF (0.575) is slightly larger than our target (0.5 from the OECD Database). The wealth Ginis of NH (0.765) and SH (0.770) are somewhat larger than that of LH (0.69) because of the stronger precautionary savings motive and also because of more persistent productivity shocks ($\rho_x = 0.946$ vs. 0.842).

4. Tax Reform and Political Outcome

4.1. Utilitarian Optimal Tax

Starting from the current steady state, where the income tax rate $\tau_0 = 23.8\%$, we look for the optimal tax rate τ_0^* that maximizes the equal-weight utilitarian social welfare function (as in Aiyagari and MacGrattan (1998)):

$$\mathcal{W}(\tau^*, \tau_0) = \int V(a_0, x_0, \psi; \tau^*, \tau_0)\, d\mu(a_0, x_0, \psi; \tau_0),$$

Table 4: Income and Wealth Distribution

	Data (SCF)	NH	SH	LH
Wealth Gini	0.834	0.765	0.770	0.690
Relative Income				
Income Q1	0.140	0.176	0.172	0.165
Income Q2	0.335	0.379	0.390	0.375
Income Q3	0.565	0.651	0.644	0.653
Income Q4	0.915	1.089	1.086	1.115
Income Q5	3.045	2.705	2.709	2.692

Note: The SCF statistics are based on Díaz-Giménez et al. (2011).

where $V(a_0, x_0, \psi; \tau^*, \tau_0)$ is the discounted sum of the lifetime utility of a household with asset holdings a_0, stochastic productivity x_0, and deterministic ability ψ. The steady-state distribution of households over (a_0, x_0, ψ) under the current tax rate, τ_0, is denoted by $\mu(a_0, x_0, \psi; \tau_0)$. More specifically:

$$V(a_0, x_0, \psi; \tau^*, \tau_0) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c(a_t, x_t, \psi; \tau^*, \tau_0)^{1-\sigma} - 1}{1-\sigma} - B \frac{h(a_t, x_t, \psi; \tau^*, \tau_0)^{1+1/\gamma}}{1+1/\gamma} \right\}$$

This is a once-and-for-all change in the tax rate (permanent tax reform) from τ_0 to τ^*. In computing welfare, we include welfare during the transition from the current state to the new steady state. A detailed computational algorithm for the optimal tax rate is provided in Appendix A.2.

Table 5 summarizes the optimal tax rates and corresponding new steady states. In all three model economies, the optimal tax rates are much higher than the current rate (and similar to each other): 37.3% (NH), 37.7% (SH) and 36.9% (LH). The optimal tax rates are similar in all three economies (which exhibit similar overall income distributions by construction) regardless of the composition of ex-ante and ex-post heterogeneity. Owing to the social planner's veil of ignorance, whether the productivity difference is ex-ante or ex-post is not crucial.[10] This result (a very high optimal tax rate) is consistent with

[10]By construction, the dispersion of the income distribution is identical across the three models. Also, the stochastic process of SH and LH is based on the same PSID data. Since the overall dispersion and *average* persistence of total productivity (despite the different composition between ex-ante and ex-post) are similar across the models, the social planner would choose a similar optimal tax rate given the veil of ignorance.

Table 5: Optimal Tax Rates under Utilitarian Welfare Function

Model	NH	SH	LH
Current τ	0.238	0.238	0.238
Optimal τ^*	0.373	0.377	0.369
Interest rates	0.054	0.054	0.052
Hours worked	0.279	0.277	0.281
Before-tax Gini	0.525	0.527	0.524
After-tax Gini	0.329	0.328	0.331
Wealth Gini	0.781	0.787	0.714
Approval Rate	0.576	0.556	0.538

a majority of the literature based on a utilitarian social welfare function (for example, Piketty and Saez, 2013; Heathcote and Tsujiyama, 2016; and Chang et al., 2016).

Fiscal reform to adopt the optimal tax rate will hardly be Pareto improving—there will be winners and losers as a result of the reform. Thus, there is no guarantee that the optimal tax rate will be chosen as a political outcome. While the examination of a complicated political process to select a policy is beyond our ability and knowledge, we can still ask a simple question: would the majority of the population be better off from fiscal reform that adopts the utilitarian optimal tax rate? Table 5 shows the approval rates for optimal tax reform: 57.6% (NH), 55.6% (SH), and 53.8% (LH). As the relative size of ex-ante heterogeneity increases, the approval rate for a tax increase declines because the potential insurance benefit from a tax-and-transfer policy is small if income is largely driven by the permanent earnings ability (rather than by stochastic shocks). Nevertheless, in all three economies, the majority is in favor of raising the tax rate to effect optimal tax reform.

4.2. Successive Majority Voting

As an alternative to the utilitarian optimal tax, one may argue that the political equilibrium should be the tax rate that maximizes the welfare of the median instead of the average—the so-called median voter theorem (e.g., Romer (1975) and Roberts (1977)). However, the median voter theorem may not be easily applicable in our model where the median is not uniquely pinned down because households differ along multiple dimensions (e.g., asset holdings, permanent ability, and ex-post realization of the productivity shock) and the state of households changes over time. We address this issue by finding a close-to-optimal tax rate that would be approved by the majority from a series of successive binary voting.

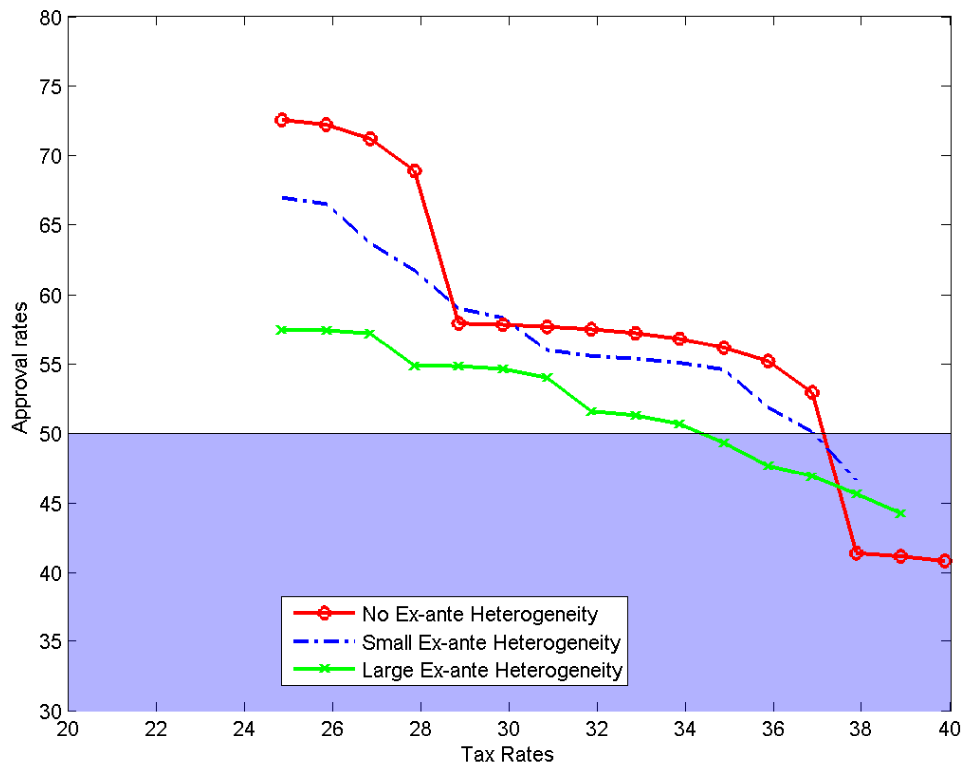
More specifically, starting with a newly proposed tax rate $\tau = \tau_0 + 1\%$, which is 1 percentage point higher (or lower) than the current tax rate τ_0, we simulate the binary voting between the current (e.g., $\tau_0 = 23.8\%$) and proposed tax rate (e.g., $\tau = 24.8\%$). If the proposed tax rate is approved by the majority, we immediately propose another tax rate ($\tau = \tau_0 + 2\%$) that is 2 percentage points higher (or lower) than the current one and simulate the binary voting between the previous winner (e.g., $\tau = 24.8\%$) and the new contender (e.g., $\tau = 25.8\%$) and so forth. We call the final winner of this successive binary voting, "Majority τ^M." Figure 1 illustrates the approval rates for a series of proposed tax rates τ's. First of all, the approval rate for tax increase is much lower in the economy with large ex ante heterogeneity (LH). For example, the proposal to increase percentage point increase in τ from the current rate receives the approval rate of 73% in NH whereas the approval rate for that proposal is 66% and 57% in SH and LH, respectively. The tax rate finally chosen by this successive voting is slightly lower than—but actually close to—the utilitarian optimal tax rate in all three models. For example, the tax rate chosen by the successive binary voting in NH is $\tau^M = 36.8\%$, only 0.5 percentage point lower than the optimal tax rate $\tau^* = 37.3\%$. The tax chosen by successive voting in LH (33.8%) is 3 percentage points lower than the optimal rate of 36.9%. As the ex-ante heterogeneity increases, the potential insurance benefit from a heavy tax-and-transfer policy diminishes.[11]

4.3. Income-Dependent Turnout Rates

We have shown that the tax rate chosen by either the utilitarian social planner or the successive majority voting is far from the currently observed rate, regardless of the com-

[11] Alesina et al. (2011) and Charité et al. (2015) report empirical evidence for the correlation between ex-ante heterogeneity and preferences for redistribution.

Figure 1: Approval Rates for the New Tax Reforms



position of the income process. If the majority of the population can improve upon the tax reform, why hasn't society adopted it? The "optimality" depends on the specification of the social welfare function. It is not obvious whether each government's goal is to maximize the equal-weight utilitarian welfare function, despite its popular use in quantitative macroeconomic analysis. There are many other alternative criteria. One may argue that it is desirable for a society to maximize the welfare of the poorest members instead of the average (i.e., Rawlsian). Society's choice for redistribution is also affected by various factors such as the externality of public expenditures (Heathcote et al., 2016), profession (Lockwood et al., 2016), the preference heterogeneity (Lockwood and Weinzierl, 2015), the reference point (Charité et al., 2015), benefit-based taxation (Weinzierl, 2016), or the equal sacrifice rule (Weinzierl, 2014). Moreover, the process under which policies are actually determined is much more complicated than the simple majority rule. For

Table 6: Tax Reform by a Successive Majority Vote

	NH	SH	LH
Current τ	0.238	0.238	0.238
Utilitarian τ^*	0.373	0.377	0.369
Majority τ^M	0.368	0.368	0.338
Effective Majority τ^{EM}	0.348	0.328	0.268

Table 7: Turnout Rates by Income Quintiles in the U.S.

Income Quintile	1st	2nd	3rd	4th	5th
Turnout Rates	50.6	55.4	66.0	72.6	86.7

Source: Mahler (2008)

example, the political equilibrium under a multi-party system can be different from that chosen by the median. These questions are immensely important but beyond the scope of this paper. Here, we suggest a rather simpler reconciliation: the income-dependent voting turnout rates. It is well known that in the U.S. low-income households are much less likely to participate in voting than are high-income households. According to Mahler (2008), the voting turnout rate of the lowest income quintile is only 50.6%, while that of the highest income quintile is 86.7% (Table 7).[12]

We incorporate this income-turnout rate profile in Mahler (2008) into our model. More specifically, we assume that the turnout rate $TR(z)$ depends on productivity z as:

$$TR(z) = \overline{TR} \cdot \exp(\omega) \cdot z$$

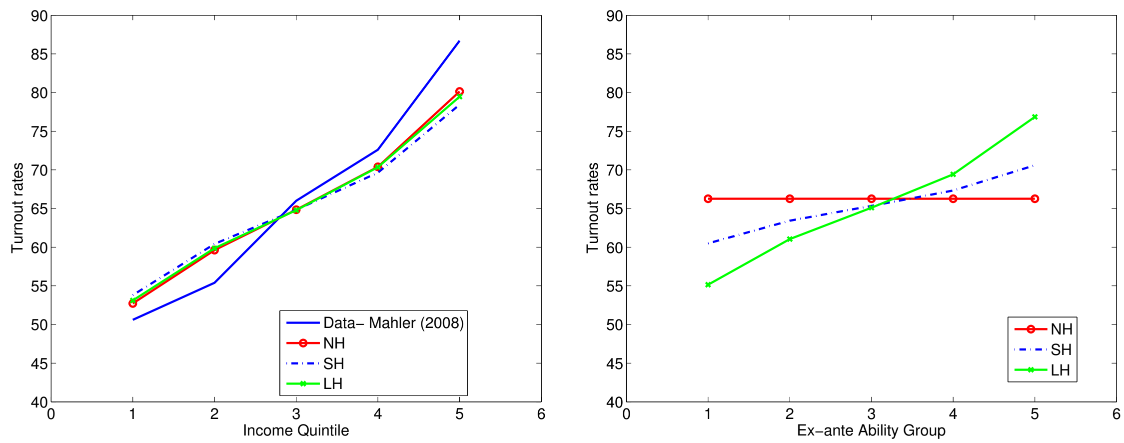
where ω and $\overline{TR}$ are constants. Two parameters ω (slope) and $\overline{TR}$ (average) are chosen to match the income quintile-turnout rates profile from Mahler (2008) in Table 7. The

[12]The voter turnout rates by income from Mahler (2008) are based on the Comparative Study of Electoral Systems (CSES) which conducted post-election surveys across countries in 1996 and 2000. The Mahler numbers are based on the 1996 survey. According to the voter turnout rate (casted votes divided the registered voters) by Institute for Democracy and Electoral Assistance (IDEA), the voter turnout rate for the presidential election was 82% and that of parliament was 67% in 1996.

left panel of Figure 2 shows the turnout rates by income quintiles from the model and the data (Mahler, 2008). The calibrated turnout rates in the three models are very similar (as we tie the turnout rate to realized income z). The turnout rate profiles in the model are slightly flatter than that in the data. Thus, our calibration is conservative. This is because we are constrained in choosing ω so that $TR(z_{max})$ cannot exceed 100%. The right panel of Figure 2 illustrates the turnout rates across 5 ex-ante productivity (ψ) groups. Table 8 reports the chosen values of ω and $\overline{TR}$ for each model.

Figure 2: Data and Model Turnout Rates

(a) Across Income Quintile (b) Across Ex-ante Group



We repeat the successive-voting simulation taking into account the upward-sloping profile of income-turnout rates. Figure 3 compares the approval rates under this income-dependent turnout rate (which we call "Effective Vote") to those under the assumption that all households vote (labeled as "Simple Vote"). With the income-dependent turnout rates, fiscal reform to raise the tax rate obtains much lower approval rates than under the simple vote assumption. The differences are not large in NH; the tax rates chosen under the effective majority voting ($\tau^{EM} = 34.8\%$) is only 2 percentage point lower than that chosen by the simple majority ($\tau^{M} = 36.8\%$). In SH, $\tau^{EM} = 32.8\%$ is 4 percentage points lower than $\tau^{M} = 36.8\%$. In LH, $\tau^{EM} = 26.8\%$ is now 7 percentage points lower than $\tau^{M} = 33.8\%$. The tax rate chosen by the majority (when the income-dependent turnout rate is taken into account) is now much closer to the current rate. As ex-ante productivity becomes more important in total earnings, the potential insurance benefit from a high tax-and-transfer policy diminishes.

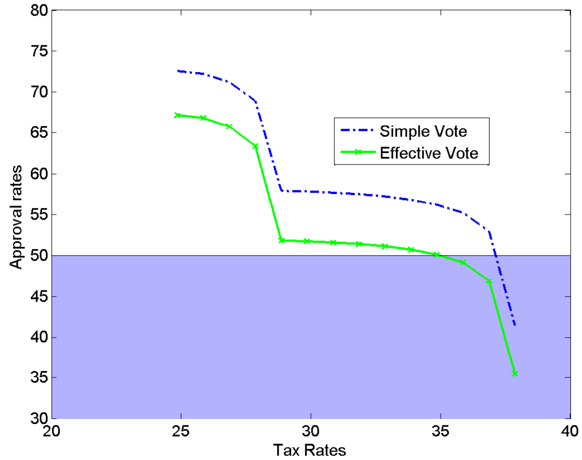
With large ex-ante heterogeneity, voting behavior is sharply divided among the population. Table 8 reports the approval rates for the chosen tax rates (τ^{EM}) across 5 income quintiles and those across 5 ex-ante productivity groups. In LH *all* households in the bottom two ex-ante productivity groups support this reform, while *all* households in the two highest ex-ante productivity groups oppose the tax reform.

Table 8: Approval Rates across Income Quintiles and Ex-ante Productivity Group

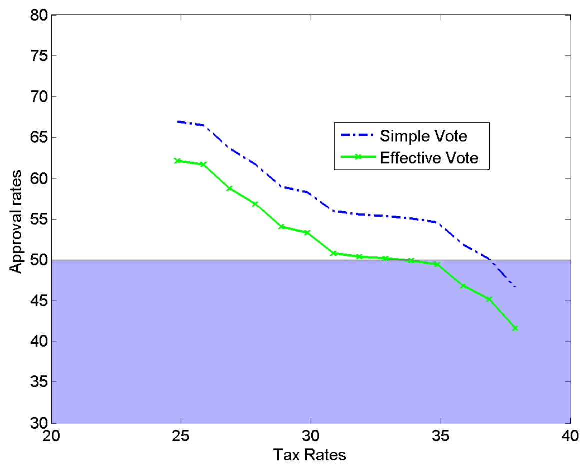
	NH	SH	LH
Ex-ante Heterogeneity Ratio	0.000	0.313	0.567
$\overline{TR}$	0.682	0.674	0.662
ω	0.204	0.180	0.191
Utilitarian τ^*	0.373	0.377	0.369
Simple Majority τ^M	0.368	0.368	0.338
Effective Majority τ^{EM}	0.348	0.328	0.268
Approval Rates			
- By Income Quintile			
1st	1.000	1.000	0.993
2nd	1.000	0.968	0.842
3rd	0.825	0.658	0.674
4th	0.058	0.278	0.333
5th	0.008	0.045	0.030
- By Ex-Ante Group			
1st	0.562	0.922	1.000
2nd	0.562	0.727	1.000
3rd	0.562	0.574	0.853
4th	0.562	0.402	0.006
5th	0.562	0.144	0.000

Figure 3: Approval Rates for New Tax Reforms

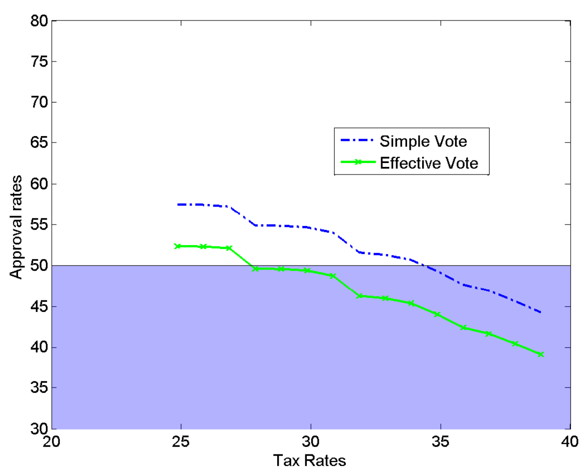
(a) No Ex-ante Heterogeneity



(b) Small Ex-ante Heterogeneity



(c) Large Ex-ante Heterogeneity



5. Summary....

According to the utilitarian social welfare function or the median voter theorem (a widely used criterion in the optimal taxation literature), the current rate of the labor-income tax in the U.S. is much lower than the optimal rate (see Piketty and Saez, 2013, for example). In this class of models, the insurance benefit from government tax dominates the cost from distorting labor supply.

We argue that the interaction between ex-ante heterogeneity and income-dependent voting behavior helps us to understand why the current tax rate is much lower than the utilitarian optimum. In our model, individual earnings consist of ability (ex-ante heterogeneity) and productivity shock (ex-post heterogeneity) whose structure is estimated from the panel data following Guvenen (2009). When household income depends largely on the permanent ability (rather than luck), a potential benefit from social insurance (by government tax-and-transfer) decreases significantly—especially for a high-ability group. When the model is calibrated to match the observed turnout rates by income quintile reported by Mahler (2008)—i.e., the voting turnout rate increases with income—the tax rate chosen by the majority drops to 27%, close to the average income tax rate in the U.S.

References

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics*, 109(3), 659-684.

Aiyagari, S. Rao, and Ellen R. McGrattan 1998. "The Optimum Quantity of Debt," *Journal of Economics*, 42(3), 447-469.

Alesina, Alberto, Paola Giuliano, A. Bisin, and J. Benhabib 2011. Preferences for Redistribution, *Handbook of Social Economics*, 93-132. North Holland.

Benabou, Roland J.M. and Efe Ok 2001. "Social Mobility and the Demand for Redistribution: The POUM Hypothesis," *Quarterly Journal of Economics*, 116(2), 447-487

Chang, Bo Hyun, Yongsung Chang, and Sun-Bin Kim 2016. "Pareto Weights in Practice: A Quantitative Analysis across 32 OECD countries," Working paper, University of Rochester.

Charité, Jimmy, Raymond Fisman, Ilyana Kuziemko 2015. "Reference Points and Redistributive Preferences: Experimental Evidence," *National Bureau of Economic Research Working Paper*.

Chetty, Raj and Emmanuel Saez 2010. "Optimal Taxation and Social Insurance with Endogenous Private Insurance," *American Economic Journal: Economic Policy*, 2(2), 85-116

Conesa, Juan Carlos and Dirk Krueger 2006. "On the Optimal Progressivity of the Income Tax Code," *Journal of Monetary Economics*, Vol. 53(7), 1425-1450

Corbae, Dean, Pablo D'Erasmo and Burhan Kuruscu 2009. "Politico Economic Consequences of Rising Wage Inequality," *Journal of Monetary Economics*, Vol. 56, 2009, p.43-61

Díaz-Giménez, Javier, Andy Glover, and José-Víctor Ríos-Rull 2011. "Facts on the Distributions of Earnings, Income, and Wealth in the United States: 2007 Update," *Federal Reserve Bank of Minneapolis Quarterly Review Vol.34, No 1, February 2011, pp-2-31*

Floden, Martin, and Jesper Linde 2001. “Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?” *Review of Economic Dynamics*, 4(2), 406-437.

Golosov, Mikhail and Aleh Tsyvinski 2007. “Optimal Taxation with Endogenous Insurance Markets,” *Quarterly Journal of Economics*, 122(2), pp.487-534.

Guvenen, Fatih 2009. “An Empirical Investigation of Labor Income Processes,” *Review of Economic Dynamics*, Vol. 12 (1), 58-79.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante 2008. “Insurance and Opportunities: A Welfare Analysis of Labor Market Risk,” *Journal of Monetary Economics*, 55(3), 501–525.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante 2016 “Optimal Tax Progressivity: An Analytical Framework,” Workign paper.

Heathcote, Jonathan and Hitoshi Tsujiyama 2016. “Optimal Income Taxation: Mirrlees Meets Ramsey,” Working paper.

Lockwood, Benjamin B. and Matthew Weinzierl, 2015. “De Gustibus non est Taxandum: Heterogeneity in preferences and Optimal Redistribution,” *Journal of Public Economics*. 2015;124 :74-80.

Lockwood, Benjamin B. and Matthew Weinzierl, 2016. “Positive and Normative Judgments Implicit in U.S. Tax Policy, and the Costs of Unequal Growth and Recessions,” *Journal of Monetary Economics*. 2016;77 :30-47.

Lockwood, Benjamin B., Charles G. Nathanson, and E. Glen Weyl, 2016. “Taxation and the Allocation of Talent,” *Journal of Political Economy*, Forthcoming.

Mahler, Vincent A. 2008. “Electoral turnout and income redistribution by the state: A cross-national analysis of the developed democracies,” *European Journal of Political Research*, Volume 47, Issue 2, pages 161183, March 2008

Mirrlees, James A. 1971. “An exploration in the theory of optimum income taxation,” *Review of Economic Studies*, pp. 175-208.

Organization for Economic Cooperation and Development 2015. OECD database in 2014 and 2015,” OECD.

Piketty, Thomas. and Emmanuel Saez 2013. “Optimal Labor Income Taxation,” NBER Working Paper No. 18521, *Handbook of Public Economics*, Volume 5, 391-474

Ríos-Rull, José-Víctor 1999. “Computation of Equilibria in Heterogeneous-Agents Models,” *Computational Methods for the Study of Dynamic Economies*, ed. Ramon Marimon and Andrew Scott, New York: Oxford University Press.

Roberts, Kevin W.S. 1977. “Voting Over Income Tax Schedules,” *Journal of Public Economics*, 8, 329-340.

Romer, Thomas 1975. “Individual Welfare, Majority Voting and the Properties of a Linear Income Tax,” *Journal of Public Economics*, 7, 163-88.

Smets, Kaat and Carolien van Ham 2013. “The embarrassment of riches? A meta-analysis of individual-level research on voter turnout,” *Electoral Studies*, 32(2), pp.344-359.

Tauchen, George 1986. “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions,” *Economics Letters* 20, 177-181.

Weinzierl, Matthew 2014. “The Promise of Positive Optimal Taxation: Normative Diversity and a Role for Equal Sacrifice,” *Journal of Public Economics*. 118, 128-142.

Weinzierl, Matthew 2016. “Revisiting the Classical View of Benefit-Based Taxation,” *National Bureau of Economic Research Working Paper*.

Appendix A: Computational Procedures

A.1. Steady-State Equilibrium

The distribution of households, $\mu(a, x, \psi)$, is time-invariant in the steady state, as are factor prices. We modify the algorithm suggested by José-Víctor Ríos-Rull (1999) in finding a time-invariant distribution μ. Computing the steady-state equilibrium amounts to finding the value functions, the associated decision rules, and the time-invariant measure of households. The proportional income tax rate τ_0 is taken from the improvement rate in the income Gini in the data. We search for (i) the discount factor β that clears the capital market at the given annual rate of return of 4%; (ii) the standard deviation of idiosyncratic productivity, σ_η, that matches the before-tax Gini coefficient; and (iii) the disutility parameter B to match the average hours worked, 0.323. The details are as follows:

1. Choose the grid points for asset holdings (a), ex-ante productivity (ψ) and idiosyncratic productivity ($z = \psi x$). The number of grids is denoted by N_a, N_ψ and N_z, respectively. We use $N_a = 490$, $N_\psi = 5$, and $N_z = 75$. Note that we set N_x $(= 15)$ grid points for idiosyncratic productivity in each ex-ante productivity, and the total number of grids for idiosyncratic productivity is $N_z = 75$. The asset holding a_t is in the range of [0, 90]. The grid points of assets are not equally spaced. We assign more points on the lower asset range to better approximate the savings decisions of households near the borrowing constraint.

2. Pick initial values of β, B, and σ_η. σ_ψ is set by ex-ante heterogeneous ratio $(\frac{\sigma_\psi}{\sigma_\psi+\sigma_\eta/\sqrt{(1-\rho_x^2)}})$. We construct five ex-ante productivity groups, whose ex-ante productivity, denoted by $\ln\psi_g$, is the median value of each quintile, where the lower and upper bounds are set to $\pm 2\sigma_\psi$. Those values are -1.42σ_ψ, -0.55σ_ψ, 0, 0.55σ_ψ and 1.42 σ_ψ, respectively. For idiosyncratic productivity, we construct five grid vectors of length N_x with respect to ψ_g. Elements in each vector, denoted by ln z_j's, are equally spaced on the interval $[\psi_g - 3\sigma_\eta/\sqrt{1-\rho_x^2},\ \psi_g + 3\sigma_\eta/\sqrt{1-\rho_x^2}]$. Then, we approximate the transition matrix of the idiosyncratic productivity using George

Tauchen's (1986) algorithm.

3. Start with an initial amount of government transfers T. Given β, B, σ_η, τ, and T, we solve the individual value functions V at each grid point for individual states. In this step, we also obtain the optimal decision rules for asset holdings $a'(a_i, x_j, \psi_g)$ and labor supply $h(a_i, x_j, \psi_g)$. This step involves the following procedure:

   (a) Initialize value functions $V_0(a_i, x_j, \psi_g)$ for all $i = 1, 2, \cdots, N_a$, $j = 1, 2, \cdots, N_x$, and $g = 1, 2, \cdots N_\psi$

   (b) Update value functions by evaluating the discretized versions:

$$V_1(a_i, x_j, \psi_g) = \max \Big\{ u\big((1-\tau_0)(wh(a_i, x_j, \psi_g)\psi x_j + ra_i) + a_i + T - a', h(a_i, x_j, \psi_g)\big) + \beta \sum_{j'=1}^{N_x} V_0(a', x_j', \psi_g))\pi_x(x_{j'}|x_j, \psi_g) \Big\},$$

   where $\pi_x(x_{j'}|x_j)$ is the transition probability of x, which is approximated using Tauchen's algorithm.

   (c) If V_1 and V_0 are close enough for all grid points, then we have found the value functions. Otherwise, set $V_0 = V_1$, and go back to step 3(b).

4. Using $a'(a_i, x_j, \psi_g)$ and $\pi_x(x_{j'}, x_j)$ obtained from step 3, we obtain the time-invariant measures $\mu^*(a_i, x_j, \psi_g)$ as follows

   (a) Initialize the measure $\mu_0(a_i, x_j, \psi_g)$.

   (b) Update the measure by evaluating the discretized version of a law of motion for each ψ_g:

$$\mu_1(a_{i'}, x_{j'}, \psi_g) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_x} \mathbf{1}_{a_{i'}=a'(a_i,x_j,\psi_g)} \mu_0(a_i, x_j, \psi_g)\pi_x(x_{j'}|x_j).$$

   (c) If μ_1 and μ_0 are close enough in all grid points, then we have found the time-invariant measure. Otherwise, replace μ_0 with μ_1 and go back to step 4(b).

5. Using decision rules and invariant measures, check the balance of the government budget. Total tax revenues are:

$$Rev = \int_{a,x,\psi} \tau_0(w\psi x h + ra) d\mu(a, x, \psi).$$

If Rev is close enough to T, then we have obtained the amount of government transfers. Otherwise, choose a new T and go back to step 3.

6. We calculate the real interest rate, Gini coefficient, individual hours worked using μ^* and decision rules. If the calculated real interest rate, average hours worked, and before-tax Gini coefficient are close to the assumed ones, we have found the steady state. Otherwise, we choose a new β, B, and σ_η, and go back to step 2.

A.2. Optimal Tax Rates and Voting Outcome

Individual utilities include those in the transition periods from the initial to the new steady state. To find optimal tax rates and voting outcome we compute the value functions and decision rules backwards and the measure of households forward. Computing the transition equilibrium amounts to finding the value functions, the associated decision rules, and measure of households in each period. The details are as follows:

1. Compute the initial steady state under the current tax rate (τ). Use the algorithm for the steady-state equilibrium.

2. Choose a new tax rate and compute all transition paths as follows:

   (a) Compute the final steady state under a new tax rate. Use the algorithm for steady-state equilibrium.

   (b) Assume that the transition is completed after $T - 1$ periods, and that the economy is in the initial steady state at time 1 and in the final steady state at T. Choose a T big enough so that the transition path is unaltered by increasing T.

   (c) Guess the capital-labor ratios $\{K_t/E_t\}_{t=2}^{T-1}$ and compute the associated $\{r_t, w_t\}_{t=2}^{T-1}$.

(d) Guess the path of government transfers $\{T\}_{t=2}^{T-1}$. Note that the amounts of government transfers are all different in each period, since decision rules and measures are different. Going backward, compute the value functions and policy functions for all transition periods by using $V_T(\cdot)$ from the final steady state. Using the initial steady-state distribution μ_1 and the decision rules, find the measures of all periods $\{\mu_t\}_{t=2}^{T-1}$.

(e) Based on the decision rules and measures, compute the aggregate variables and total tax revenues. If the total tax revenue is close to the assumed transfers, we obtain the amount of transfers. Otherwise, choose a new path of government transfers and go back to 2(d).

(f) Compute the paths of aggregated capital and effective labor and compare them with the assumed paths. If they are close enough in each period, we find the transition paths. Otherwise, update $\{K_t/E_t\}_{t=2}^{T-1}$ and go back to 2(c).

3. Choose the tax rate that yields the highest social welfare, which is the sum of individual values. This is the optimal tax rate under the utilitarian criteria.

4. For the voting outcome, start from a new tax rate 1 percentage point higher (or lower) than the current one ($\tau_1 = \tau_0 + 0.01$) . Using the above procedure, compute individual values from the current tax rate to a new one including transitions. If individual values under new tax reform are higher than the values under the current tax rate, i.e. $V(a, x, \psi|\tau_1, \tau_0) > V(a, x, \psi|\tau_0, \tau_0)$, then this individual is assumed to vote for the new tax reform. If a majority prefer this reform, increase (or decrease) the new tax rate by 1 percentage point ($\tau_2 = \tau_1 + 0.01$) further and compute individual values. If $V(a, x, \psi|\tau_2, \tau_0) > V(a, x, \psi|\tau_1, \tau_0)$, then this individual votes for the second reform. Keep increasing (decreasing) tax rates until a majority rejects a new tax reform. This is the outcome by a majority vote.