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Transitive Delegation in Social Networks: Theory and Experiment

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Abstract. This paper develops a model of *delegative democracy* in which each voter can either vote directly or delegate her vote, together with the votes delegated to her, to another voter and analyzes the incentive for delegation and its impact on the quality of collective decision. A key finding is that as long as the delegation network is sufficiently ideologically homogeneous and large, voters are willing to delegate their votes even if they know neither who knows what nor who knows whom. I also show that delegation facilitates a better collective decision. The laboratory data confirm the theoretical predictions.

Keywords: Voting, Delegation, Democracy, Delegative democracy, Information aggregation

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1. Introduction

Delegative democracy is a voting system in which a voter has the option of delegating her vote to someone she trusts on top of the option of voting directly herself. That is, voters can either vote directly or delegate their vote to another voter, and moreover, they may also select different delegates for different issues. In this sense, delegative democracy lies between direct and representative democracy and is also called a “direct/proxy voting system.”¹

Although similar ideas had been discussed earlier, it was the advancement of the Internet and other technological developments that showed the possibility of implementing the idea and provoked imaginations and lively discussions.² Since the early 2000s, radicals such as pirate parties in Europe and some progressive parties in Latin America have been trying to improve their decision-making process using the new technologies. Digital activists and techno-progressives have developed platform apps such as *DemocracyOS* and *LiquidFeedback*, which help implement delegative democracy on a large scale.³ Proponents of delegative democracy argue that direct democracy is *genuinely better* but may be worse if implemented in reality than representative democracy because voters cannot make an informed decision on every issue. A solution to this problem may be the option to delegate the decision right to someone you can trust.

A key feature of delegative democracy, which is the focus of this study, is that a voter can delegate not only her own vote but also the votes delegated to her to a third person. In other words, delegation is *transitive*. Although delegative democracy has been lively discussed over years, little is known about the effects of transitivity of delegation on the incentive for delegation and the voting outcome.

¹ It is also known as liquid democracy. (See https://en.wikipedia.org/wiki/Liquid_democracy.) The term “direct/proxy voting system” is proposed by Green-Armytage (2015).

² In 1967, Gordon Tullock suggested that voters could choose their representatives or vote themselves in parliament “by wire”, while debates were broadcast by television. In 1969, James C. Miller favored the idea that everybody should have the possibility to vote on any question themselves or to appoint a representative who could transmit their inquiries. In 1970, Martin Shubik called the process an “instant referendum.” More can be found in https://en.wikipedia.org/wiki/Liquid_democracy.

³ For instance, German Pirate party adopted LiquidFeedback for their intra-party decision making, through which any registered member can propose an “initiative.” An initiative then must receive support of at least 10% of the registered users within a certain time span. If it succeeds, the proposal is discussed and modified. Lastly, members can vote on the final proposal directly or delegate their votes to someone whom they trust. For more about the apps see <http://democracyos.org/> (DemocracyOS) and <https://liquidfeedback.org/> (LiquidFeedback).

In this paper, I theoretically and experimentally examine a model of delegative democracy and analyze the incentive for delegation and its impact on the quality of collective decision. Specifically, I consider a situation where voters have to collectively decide whether to keep or replace a policy. If the current policy is better than the alternative, it should stay, but if not, it should be replaced. The problem is that a majority of voters do not know the true state of the world, i.e., whether the alternative policy is better than the current one. Everybody knows the delegation network structure. However, they do not know who knows the policies' relative merits, and furthermore, they may not know whether others have the same political preference with them.

Whether to delegate in this environment is a complex but interesting problem for a couple of reasons. First, the expected outcome depends on the network structure of potential delegations, the delegation decisions of others, and the information of not only who knows what but also who knows whom; You may be willing to grant your voting power to a person who knows what is best for you, i.e., a benevolent expert. Furthermore, you may also be willing to delegate your vote to a person who knows a benevolent expert or a person who knows a person who knows an expert, etc. Thus, even with the delegation option available, the option may not be used because voters are not sure to whom to delegate. Second, even without the delegation option, voters can collectively make a reasonably good decision because uninformed voters can strategically vote in order to make informed voters pivotal. That is, in principle, uninformed voters can make yes and no votes cancel each other out and let the informed decide. Thus, it is not clear whether the delegation option really brings additional benefit to the society and nor is whether people would utilize such an option.

Delegative democracy may improve the quality of collective decision because it allows voters to use more information. More specifically, in direct democracy, the only thing that voters need to know is the information of the relative merits of each alternative, which may be called the first-order information. In representative democracy, the second-order information, that is, the information of who is the most reliable representative, is more relevant. In contrast, higher-order information can be utilized in delegative democracy if voters have such information and are willing to use it. However, it is not straightforward whether and how such information can be used because such information is dispersed.

Key findings of this paper are as follows. First, as long as voters believe that the delegation network is sufficiently ideologically homogeneous (that is, connected voters are similar in

political preferences), they may be willing to delegate their vote to *anybody* in the network. This means that delegation is an attractive option even if voters cannot utilize any high-order information. It also implies that even if the uncertainty regarding the true state of the world is the main concern, the key information may not be that of who knows the true state or who knows the true expert but of who has the political preference same as yours. To understand this result intuitively, note that voters expect that their delegate will use the private information of whether informed or uninformed in the following way; by extending the chain of delegation if she is uninformed or by voting directly otherwise. If the network is well connected and sufficiently large, delegated votes will flow eventually to the informed with a high probability. Put differently, if voters can trust the network, they will delegate to it. However, high-order information would be of key importance if voters do not trust the network. That is, if the network is ideologically heterogeneous, a voter cannot delegate the vote to a random person in the network, and therefore she may prefer direct voting to delegation unless she knows an informed voter who shares the same political preference.

Another finding of this paper is that delegation facilitates information aggregation, and thus improves the quality of collective decision. As noted above, voters can collectively make a reasonably good decision even without the delegation option because uninformed voters can strategically make informed voters pivotal. However, without the delegation option, actions taken by uninformed voters and those by informed voters may be indistinguishable, which means the aggregated information is bound to be noisy. In a sense, the delegation option cleans up the aggregate information by separating the action of the uninformed from that of the informed. Furthermore, delegation may be a better option than abstention when a supermajority is required by institution or by a competition against another group of voters to implement the preferred policy because delegation increases the number of informed votes whereas abstention does not.

I also conducted a laboratory experiment which was based on the theoretical model. I find that (i) more people chose to delegate their vote in larger networks and (ii) subjects were reluctant to delegate their vote when they believe that someone in the network may have a political preference opposite to theirs. These results confirm the idea that key information may not be *substantial* (i.e., who knows the true state or an expert) but *ideological* (i.e., who has the political preference same as yours). Furthermore, the subjects collectively made better decisions and earned more money in the treatments with the delegation option than without.

Literature review

So far delegative democracy has been discussed mostly by computer scientists and engineers, and studies by economists and political scientists are surprisingly rare.⁴ Consequently, various technical issues in implementation (e.g., cryptography) have been the main focus of previous discussions, while agents' motivations have rarely been considered. Green-Armytage (2015) is the only exception that I am aware of.⁵ He develops a spatial model of voting with expressive utility to show that when a voter does not know her preference precisely, she can get better off by delegating the vote to someone else. Because voters with expressive utility do not care about the final outcome, his model concerns a single agent's decision-making; in this sense, it is a decision-theoretic model as opposed to game-theoretic. So, this is the first paper that investigates the incentives and behavior of voters in the mechanism allowing *transitive* delegation. This paper differs from Green-Armytage (2015) in many aspects. To name a few, first, I adopt the assumption more traditional in economics and political science, namely that voters strategically make decisions to influence the outcome. Second, I explicitly consider the delegation network and derive results in this regard. Third, I provide an empirical support for the theory using a lab experiment.

An issue that people have worried about, other than the technical issues, is that transitivity of delegation may result in too much voting power concentrated to a few voters. This concentration of power may be more problematic than its counterpart in representative democracy (i.e., the concentration of power to a few politicians) because these *super-voters* are less known to the public and thus may be less accountable.⁶ Kling et al. (2015) analyzes the data from LiquidFeedback to show that super-voters indeed exist, but mitigating the above mentioned concern, they vote in line with the majority.

The current paper is closely related to the literature on voting and information aggregation (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996, 1997; Bataglini et al., 2010; Morton and Tyran, 2011; Gerardi and Yariv, 2007; Goeree and Yariv, 2011 to name a few). A

⁴ Ford (2018) provides an overview.

⁵ See also Alger (2006) for a related discussion.

⁶ To address accountability issue, proposers of delegative democracy claim that canceling delegation should be allowed anytime the delegator wishes. However, unlimited canceling may make collective actions unstable, and thus it would have to be limited somehow.

typical study in this literature tries to characterize the conditions for Condorcet's jury theorem to hold. For example, earlier studies examine whether the theorem holds even when voters are strategic. More recent studies are concerned more about the effects of pre-vote communications on election outcome.

It is not difficult to see the *duality* between delegation and information transmission in social networks. On one side, decision right is flowing over the networks, seeking for information, while on the other side, information is.⁷ In this regard, Buechel and Mechtenberg (2019) is probably the most closely related paper. They explicitly consider private communication networks and show that pre-vote communication may undermine the efficiency and hence reduce the welfare if some voters have much more audiences than others. My paper differs from theirs in that first of all, I consider delegative democracy whereas their voting system is the usual direct/representative democracy. Also, I consider ring networks which allow transitive delegation whereas they consider pair-wise networks and star networks which do not allow a chain of information transmission.

This paper is also related to the experimental studies on delegation, which investigate delegator and delegate's behaviors in the dictator, ultimatum, gift-exchange, and public-good games. In the dictator and ultimatum games, delegation seems related to less pro-social behavior (Fershtman and Gneezy, 2001; Hamman et al., 2010; Bartling and Fischbacher, 2012). On the other hand, in the gift-exchange and public good games, delegation is associated with more generous and socially desirable behavior (Hamman et al. 2011; Charness et al. 2012; Iris et al., 2019). To my knowledge, there is no paper studying delegation in networks.

2. Theory

2.1 Model

Suppose that there are $N(\geq 3)$ voters who decide whether to implement a new policy or keep the current one. The voters can be divided into two groups who have the opposite

⁷ This means that delegation may be a substitute for information transmission. A more detailed discussion can be found at the end of the paper.

preferences. In particular, a voter is majority-type, denoted by M , with probability $1 - r$ and minority-type, denoted by m , with probability r . I assume that in expectation, the minority group is smaller, i.e., $r < 1/2$. However, since the types are stochastically realized, although unlikely, it is possible that the minority group is larger than the majority.

There are two alternatives: a , a new policy and b , the default policy. There are two possible states of the world, A and B : in state A , alternative a is preferred by the majority-type voters and in B , b is preferred. Assume that state A is more likely, i.e., $\pi > 1/2$ where π is the probability that the state is A . The payoff of a majority-type voter is given by:

$$v_M(a|A) = v_M(b|B) = 1$$

$$v_M(a|B) = v_M(b|A) = 0.$$

In words, the majority voters want to match the state. On the other hand, the minority voters want the opposite, i.e., $v_m(c|D) = 1 - v_M(c|D)$ for any $c \in \{a, b\}$ and $D \in \{A, B\}$. To implement the new policy, at least $q \leq N$ votes are required. If fewer than q votes are cast in favor of a , the default policy b is implemented. This implies that if everybody chooses to delegate their vote, the default is implemented.⁸

Voters are either informed of the true state or uninformed. They are ex-ante symmetric, but before their decision-making, randomly selected voters learn the true state of the world. Let $p \in (0,1)$ be the probability of a majority-type voter getting informed. I assume that minority-type voters are always informed of the true state, i.e., the probability that a minority-type voter gets informed is one. This assumption is to keep the model which provides the basis for a laboratory experiment as simple as possible, and it can be justified by the fact that what we could additionally learn from uninformed minority voters would not be substantial since uninformed voters, majority or minority, would make a decision in a similar way. Thus, below an informed majority voter will be referred to as an *informed*, an informed minority voter as a *minority*, and an uninformed majority voter as an *uninformed*.

To sum up, there exist three types of voters: uninformed, informed, and minority voters. Formally, the type space is $\Theta = \{M, m\} \times \{A, B, \emptyset\}$ where \emptyset means that the voter got no

⁸ Instead, I may assume that a (or a randomly selected policy) is implemented if everybody delegates. If I do so, however, then the problem may become less interesting as shown in Proposition 3, and we miss the opportunity to learn the working mechanism of delegative democracy.

signal, i.e., uninformed. A voter is informed with probability $p = \Pr(M, A) + \Pr(M, B)$, minority-type with probability $r = \Pr(m, A) + \Pr(m, B)$, and uninformed with probability $1 - p - r = \Pr(M, \emptyset)$. Uninformed minority-type voters do not exist, i.e., $\Pr(m, \emptyset) = 0$. These probabilities are common knowledge, but a specific voter's type is private information.

This setup can be interpreted in two ways. In the first, more straightforward, interpretation, the N voters constitute an independent body politic, and q is the institutionally required number of votes. An alternative interpretation is that the voters considered in the model are just socially connected, like-minded voters, and there are other voters outside the model who are not connected with but potentially compete against them. In this interpretation, q is the minimum number of votes required to win against the other group. Below I will consider situations where the delegation network is ideologically homogeneous, i.e., r is zero or very small. The assumption that $r = 0$ can be understood as follows. In the first interpretation, the voters are members of an organization, so they share the same goal, while in the second interpretation, they are socially connected, ideologically homogeneous citizens who may be facing a competition against another group of citizens.

Let us now turn our attention to the delegation network. To analyze a voter's optimal strategy, we would have to characterize the probability for the voter to be pivotal, and for that, we may have to consider all possible delegation decisions of others and the resulting distributions of voting power, which may easily become an overwhelming task. To make it tractable, I restrict my focus to the networks which satisfy the following conditions: (i) the voters are symmetric in terms of the degree in the network, (ii) each voter knows only one person to delegate to, and (iii) for any voters i and j , there exists a path which connects i and j . The first requirement is for the existence of a symmetric equilibrium, and the second substantially simplifies the delegation decision-making problem and the resulting distributions of voting power. The third requirement is not very restrictive. All in all, I assume that the delegation network is a ring network, a directed graph in which each node has one inward and one outward edges. If a voter chooses to delegate, the *next* voter on the ring can exercise the delegated voting power, or she too can delegate the delegated votes together with her own to the next. If everybody chooses to delegate their vote, the default is implemented.

The game is a simultaneous-move game with imperfect information where everybody makes

a single decision, knowing only their own type.⁹ So, a pure strategy s is a mapping from the type space $\Theta = \{M, m\} \times \{A, B, \emptyset\}$ to the action space $\mathcal{A} = \{a, b, d\}$ where a stands for “voting in favor of a ,” similar for b , and d is “delegation,” i.e., $s: \Theta \rightarrow \mathcal{A}$. A mixed strategy σ is a mapping from Θ to the set of probability distributions $\Delta(\mathcal{A})$, i.e., $\sigma(\theta) = (\sigma_a(\theta), \sigma_b(\theta), \sigma_d(\theta))$ for $\theta \in \Theta$.¹⁰

I focus on symmetric Nash equilibria; Voters of the same type play the same mixed strategy. In such an equilibrium, the informed and the minority know the true state, so play the dominant strategy, i.e., voting directly and sincerely. That is, $\sigma(M, A) = \sigma(m, B) = (1, 0, 0)$, and $\sigma(M, B) = \sigma(m, A) = (0, 1, 0)$. Naturally, the focus of the following discussion will be on the strategy of the uninformed. Note that the state and the number of each type of voters are uncertain. Thus, an uninformed voter is pivotal with a strictly positive probability, and hence the utility maximization problem is not trivial.

2.2 Example

Suppose that $N = 3$, and $q = 2$. In words, at least two out of three votes must be cast in favor of a to implement the new policy. Everybody has the same preference to match the state (and they know it), i.e., $r = 0$. The ex-ante probability that the state is A is strictly greater than half, i.e., $\pi > 1/2$. Voters know neither who nor how many are informed but do know that the probability of any other voter being informed is $p > 0$.

An informed voter will vote directly and sincerely, i.e., $\sigma(M, A) = (1, 0, 0)$, and $\sigma(M, B) = (0, 1, 0)$. In words, the informed play a in state A and b in state B . Below, for simplicity, I will use the notation σ_c for $\sigma_c(M, \emptyset)$ for $c \in \{a, b, d\}$. To find the optimal strategy of the uninformed, let us consider the actions in pair.

Suppose first that σ_d is zero. For a voter to be pivotal, the other two votes must split. Thus, the difference in the expected utilities are given by:

⁹ This is to make the model tractable. In sequential-move games, the strategy space would be vast, and calculating the pivotal probabilities and the expected utilities in each realized history would be intractable.

¹⁰ The strategy of an uninformed minority voter is irrelevant because such a voter does not exist by assumption. Just for the sake of completeness, I may assume that $\sigma(m, \emptyset) = (1, 0, 0)$.

$$\begin{aligned}
U(a|\emptyset) - U(b|\emptyset) &= 2\pi[p + (1-p)\sigma_a][(1-p)\sigma_b] - 2(1-\pi)[(1-p)\sigma_a][p + (1-p)\sigma_b]
\end{aligned} \tag{1}$$

where $U(c|\emptyset)$ is the expected utility of an uninformed voter when playing action c . From (1), two things are clearly noticed: (i) $\sigma_a = 1$ or $\sigma_b = 1$ cannot be an equilibrium strategy because $\sigma_a + \sigma_b = 1$. In other words, there is no such pure-strategy equilibrium. (ii) $\sigma_a > \sigma_b$ because $\pi > 1/2$. A mixed-strategy equilibrium may exist, which is characterized by

$$U(a|\emptyset) - U(b|\emptyset) = 0. \tag{2}$$

For it to be an equilibrium, however, there should not be an incentive to deviate to d . To check the deviation incentive, let us consider the utility difference between a and d , assuming that all the others are playing the mixed strategy given by (2):

$$U(a|\emptyset) - U(d|\emptyset) = \pi[p + (1-p)\sigma_a][(1-p)\sigma_b] - (1-\pi)[(1-p)\sigma_a][p + (1-p)\sigma_b] \tag{3}$$

which is zero when (2) holds because (3) is proportional to (1). Therefore, there is no profitable deviation. In short, if everybody else refrains from using the delegation option, nobody will delegate. However, this does not have to be the case if the voters believe that uninformed others will delegate their votes.

Let us now assume that $\sigma_d > 0$ and $\sigma_b = 0$. The three situations that an uninformed voter, labeled as “me”, is pivotal are depicted in Figure 1. There is no other situation in which the action of “me” matters. Note that delegation goes to the clock-wise direction and that *no arrow* from voter 1 means voter 1 is playing b .

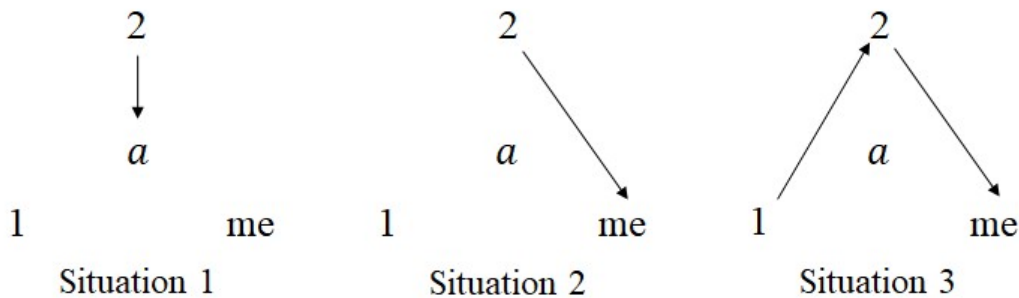


Figure 1. The situations that an uninformed voter, “me” is pivotal

In the first situation, the state is B , voter 1 is informed, and thus plays b . Voter 2 is uninformed

and plays a . This situation is realized with probability $(1 - \pi)p(1 - p)\sigma_a$. If the third voter, “me” plays a , the new policy will be implemented because voter 2 and “me” vote in favor of a , and if plays d (i.e., delegating her vote to voter 1), it will not be implemented because fewer than two votes are cast in favor of a . Thus, the third voter’s action is decisive.

Similarly, in the second situation, the state is B , voter 1 is informed, and voter 2 is uninformed. This time, voter 2 plays d , and thus “me” has two votes to cast and obviously is pivotal. This situation is realized with probability $(1 - \pi)p(1 - p)\sigma_d$. In the third situation, everybody is uninformed, so “me” can exercise voting power of three people. This situation is realized with probability $((1 - p)\sigma_d)^2$. Let us sum up these probabilities to see the relative payoffs:

$$U(a|\emptyset) - U(d|\emptyset) = -(1 - \pi)p(1 - p)\sigma_a - (1 - \pi)p(1 - p)\sigma_d + (\pi - (1 - \pi))((1 - p)\sigma_d)^2$$

In the first two situations, playing d is the right move, while playing a is the better option in the third situation. Recall that since $\sigma_b = 0$, $\sigma_a + \sigma_d = 1$. Therefore, the difference can be rewritten as:

$$U(a|\emptyset) - U(d|\emptyset) = (1 - p)[(2\pi - 1)(1 - p)\sigma_d^2 - (1 - \pi)p].$$

For “me” to be pivotal, at least one other voter must be uninformed. If everybody is uninformed and choose to delegate, “me” can choose the outcome single-handedly, so playing a increases the payoff by $2\pi - 1$. However, if her potential delegate, voter 1 is informed, and the state is B , then “me” will be better off by playing d . It is straightforward to show that the deviation to playing b is not profitable if p is not too small.

Thus, I reach to the following conclusion which foreshadows the theoretical findings presented below. If everybody else refrains from using the delegation option, nobody has an incentive to delegate. If p is neither too small nor too large, there exists a mixed-strategy equilibrium where

$$(\sigma_a^*, \sigma_b^*, \sigma_d^*) = \left(1 - \sqrt{\frac{(1 - \pi)p}{(2\pi - 1)(1 - p)}}, 0, \sqrt{\frac{(1 - \pi)p}{(2\pi - 1)(1 - p)}} \right).$$

If $p > (2\pi - 1)/\pi$, then a pure-strategy equilibrium exists where the uninformed delegate

their vote with certainty.

2.3 Homogeneous network

This subsection generalizes the discussion in the previous subsection. In particular, I assume that $r = 0$ and characterize the equilibria and expected utilities.

To establish a benchmark, I first assume that the delegation option is not available or just not used, i.e., $\sigma_d = 0$. For a voter to be pivotal, exactly $q - 1$ out of $N - 1$ votes must be cast in favor of a , which occurs with the following probabilities in each state:

$$\begin{aligned}\Pr(\text{pivotal}|A) &= \binom{N-1}{q-1} [p + (1-p)\sigma_a]^{q-1} [(1-p)\sigma_b]^{N-q} \\ \Pr(\text{pivotal}|B) &= \binom{N-1}{q-1} [(1-p)\sigma_a]^{q-1} [p + (1-p)\sigma_b]^{N-q}\end{aligned}\quad (4)$$

Therefore, the mixed-strategy equilibrium where nobody delegates is characterized by:

$$\pi [p + (1-p)\sigma_a^*]^{q-1} [(1-p)\sigma_b^*]^{N-q} = (1-\pi) [(1-p)\sigma_a^*]^{q-1} [p + (1-p)\sigma_b^*]^{N-q} \quad (5)$$

This establishes, first of all, a no-delegation benchmark. However, it is also an equilibrium of the game, as shown in the previous subsection; Given the strategy defined in (5), a voter has no incentive to deviate to playing d even if such an option is available. To see this, consider the situations where the $N - 2$ voters, those other than “me” and her potential delegate “voter 1”, are casting exactly $q - 1$ votes in favor of a , and her potential delegate (voter 1) is playing b , which happens with the following probabilities in each state:

$$\begin{aligned}\Pr(\text{pivotal}|A) &= \binom{N-2}{q-1} [p + (1-p)\sigma_a]^{q-1} [(1-p)\sigma_b]^{N-q} \\ \Pr(\text{pivotal}|B) &= \binom{N-2}{q-1} [(1-p)\sigma_a]^{q-1} [p + (1-p)\sigma_b]^{N-q}\end{aligned}$$

which is proportional to (4). In other words, given being pivotal, delegation is essentially same with playing b , and therefore they give the exactly the same expected utility, which in turn means that there is no incentive to deviate to playing d . In this equilibrium, the expected utility of a voter, a measure of the quality of collective decision is given by

$$\begin{aligned}
& \pi \sum_{k=q}^N \binom{N}{k} [p + (1-p)\sigma_a^*]^k [(1-p)\sigma_b^*]^{N-k} \\
& + (1-\pi) \sum_{k=0}^{q-1} \binom{N}{k} [(1-p)\sigma_a^*]^k [p + (1-p)\sigma_b^*]^{N-k}
\end{aligned} \tag{6}$$

where σ_a^* and σ_b^* are defined by (5). The discussion thus far is summarized in the following proposition.

Proposition 1. *Suppose $r = 0$. There exists a mixed-strategy equilibrium characterized by (5), in which nobody plays d . In the equilibrium, the expected utility of a voter is given by (6).*

This proposition shows that if nobody plays d , nobody plays d . However, if voters believe that $\sigma_d > 0$, they may be willing to delegate their vote too, hoping that it eventually flows over to an informed voter. Unfortunately, characterizing a mixed-strategy equilibrium where $\sigma_d > 0$ is not easy because there are too many irregular situations in which a voter is pivotal. Thus, I will just check whether there exists a pure-strategy equilibrium in which every uninformed voter plays d .

Suppose $\sigma_d = 1$. Let us again focus on voter “me.” If everybody else is uninformed, which happens with probability $(1-p)^{N-1}$, then by playing a instead of d , voter “me” can increase the payoff by $2\pi - 1$. If, on the other hand, she accumulates q votes, i.e., $q - 1$ consecutive voters *before* “me” are uninformed and play d , which happens with probability $(1-p)^{q-1}$, and if the state is B , and at least one voter is informed, then delegation is better than voting for a . Note that accumulating at least q votes is crucial for “me” to be pivotal because when the state is B , no one else will vote in favor of a . Summing up, we obtain the following:

$$\begin{aligned}
U(a|\emptyset) - U(d|\emptyset) &= (2\pi - 1)(1-p)^{N-1} - (1-\pi)(1-p)^{q-1}(1 - (1-p)^{N-q}) \\
&= (1-p)^{q-1}[(2\pi - 1)(1-p)^{N-q} - (1-\pi)(1 - (1-p)^{N-q})]
\end{aligned}$$

Therefore, $U(a|\emptyset) < U(d|\emptyset)$ if

$$(2\pi - 1)(1-p)^{N-q} < (1-\pi)[1 - (1-p)^{N-q}]$$

or equivalently,

$$N - q > \ln\left(\frac{1-\pi}{\pi}\right) / \ln(1 - p). \quad (7)$$

Because $\pi > 1/2$, playing b cannot be a better option than playing a . Thus, if (7) holds, playing d in pure strategy is indeed an equilibrium. The expected utility in this equilibrium is

$$1 - \pi(1 - p)^N. \quad (8)$$

Recall that the utility of a majority-type voter when matching the state is one. So, (8) implies that the collective decision-making is almost perfect; The only efficiency loss comes from the case in which everybody is uninformed.

Proposition 2. Suppose $r = 0$. If (7) holds, there exists an equilibrium where uninformed voters always delegate their vote. In the equilibrium, the expected utility of a voter is given by (8).

As the network size N increases, the probability that there exists at least one informed voter also increases, which makes delegation more attractive. As the required number of votes q increases, on the other hand, the probability of a voter being pivotal decreases, making the relative merit of playing d smaller.

The two propositions together imply that delegation exhibits strategic complementarity. To see this point, let us suppose that (7) holds. Proposition 1 states that even in this case, there exists an equilibrium where nobody delegates. However, Proposition 2 shows there exists another equilibrium where every uninformed voter delegates; If each uninformed voter believes that other uninformed voters would delegate their votes instead of voting directly, the delegated voting power is believed to be exercised by an informed voter, not by an uninformed voter.

The analysis thus far also shows that delegation facilitates information aggregation, and thus improves the quality of collective decision. When the delegation option is not exercised, as shown in (5), uninformed voters play both a and b , which means that they are trying to make informed voters pivotal and somehow incorporate the information into the collective decision. However, without delegation, actions taken by uninformed voters and those by informed voters may be indistinguishable, which means the aggregated information is bound to be noisy (see (6)). In contrast, with delegation, the collective decision-making is almost perfect (see (8)).

In fact, (8) is very close to the efficiency upper bound which is $1 - (1 - \pi)(1 - p)^N$. If nobody knows the true state, there is no way to make the right decision all the time. In such a

case the best thing to do collectively is to implement a because state A is more likely by assumption ($\pi > 1/2$). So, let us suppose for a moment that a is implemented if everybody delegates. Then, there is no downside of playing d ; If there exists at least one informed voter, delegation is the best action, and if there is no informed voter, there is no other way to improve the expected utility than just implementing a . Therefore, every uninformed voter will delegate their vote if the others also do so.¹¹

Proposition 3. Suppose that $r = 0$ and that policy a is implemented if everybody delegates. There exists an equilibrium where uninformed voters always delegate their vote. In the equilibrium, the expected utility is maximized subject to the information available in the network.

In a sense, the delegation option cleans up the aggregate information by separating the action of the uninformed from that of the informed. So, it would be instructive to compare delegation with abstention to which considerable attention has been paid in the literature of information aggregation and voter turnout (e.g., Feddersen and Pesendorfer, 1996). Abstention too provides a means to separate the actions of the uninformed from those of the informed. However, delegation may be a better option than abstention when q is sufficiently large because delegation increases the number of informed votes whereas abstention does not.¹²

According to the analysis thus far, delegative democracy seems truly remarkable. But, aren't there any the critical assumptions which may make these results less relevant to the reality? There are a couple of those.¹³ Notably, the assumption that the network is ideologically homogeneous may be too strong. In reality, there exists a spectrum of ideology, and it is unlikely that two people have exactly same political preferences. Moreover, others' ideology may not be fully known. This is crucial because for example, when a left-wing voter decides whether to delegate, she may have to worry about the possibility that her vote is delegated to a moderate left-wing voter, then to a centrist voter, moving to the right little by little, and eventually to a right-wing voter. This kind of misrepresentation of preference never happens if

¹¹ One may wonder why I did not assume that a is implemented in case of everybody's delegating in the first place. If I did so, however, the problem would have become less interesting, and we would have missed the opportunity to derive Proposition 2 which I believe more telling than Proposition 3. And, I think it is reasonable to assume that voters keep the current policy, that is b , when no one knows whether a is really better.

¹² Professors are encouraged to delegate their vote to the chair if unable to attend at the department meeting.

¹³ I discuss these at the end of the paper.

$r = 0$, and the voters know that it never happens. In the next subsection, I relax this assumption.

2.4 Heterogeneous network

Assume that $r \in (0, p]$. That is, now the network is heterogeneous, but still it is more likely that a socially connected informed voter has the majority-type preference than the minority-type. Now, there is a risk that the delegated power may be exercised by someone with the opposite ideology.

Let us first characterize the equilibrium where no uninformed voter delegates her vote. According to Proposition 1, there exists such an equilibrium even when $r = 0$. Delegation cannot become more attractive as a majority-type voter now believes that her vote may be delegated to a minority-type voter. Thus, there exists a mixed-strategy equilibrium if there exists a probability pair $(\sigma_a^{**}, \sigma_b^{**})$ where $\sigma_a^{**} + \sigma_b^{**} = 1$ solving the following equation which boils down to (5) if $r = 0$.

$$\begin{aligned} \pi[p + (1 - p - r)\sigma_a^{**}]^{q-1}[r + (1 - p - r)\sigma_b^{**}]^{N-q} \\ = (1 - \pi)[r + (1 - p - r)\sigma_a^{**}]^{q-1}[p + (1 - p - r)\sigma_b^{**}]^{N-q} \end{aligned} \quad (9)$$

If the following inequality, obtained by replacing σ_a^{**} in (9) by 1 and σ_b^{**} by 0, holds, on the other hand, there exists an equilibrium where every uninformed voter directly votes in favor of a with certainty.

$$\pi(1 - r)^{q-1}r^{N-q} > (1 - \pi)(1 - p)^{q-1}p^{N-q} \quad (10)$$

If $r \approx p$, and/or q is sufficiently large, (10) will hold. In such a case, the collective endeavor to make the informed voters pivotal does not work because now the minority voters will nullify the informed votes, and thus there is no better option than trying their best to implement a .

Proposition 4. If r is positive but small enough that (10) does not hold, then there exists a mixed-strategy equilibrium defined by (9). If (10) holds, there exists a pure-strategy equilibrium in which uninformed voters vote in favor of a .

This proposition shows that if r is large enough, the voters' ability to make an informed decision collectively may be significantly undermined. The next question is whether they can

do better if they utilize the delegation option. Suppose $\sigma_d = 1$. Note first that the utility difference between a and d can be written as follows:

$$U(a|\emptyset) - U(d|\emptyset) = \pi f(p, r) - (1 - \pi)g(p, r) + (2\pi - 1)(1 - p - r)^{N-1} \quad (11)$$

where $f(p, r)$ is the probability that (i) a is implemented if voter “me” votes directly in favor of a and (ii) b is implemented if she delegates as the votes are eventually delegated to a minority-type voter who votes in favor of b in state A . Similarly, $g(p, r)$ is the probability that (i) a is implemented if she plays a and (ii) b is implemented if she plays d as the votes are eventually delegated to an informed voter who votes in favor of b in state B . The last term is the utility difference generated in case that everybody delegates.

Even though it is extremely difficult to exactly characterize $f(p, r)$ and $g(p, r)$, we know the following.

Lemma 1. The following are true. (i) $f(p, r)$ and $g(p, r)$ are polynomials and thus continuous in p and r . (ii) $f(p, 0) = 0$ (iii) $f(p, r) = g(p, r)$ if $p = r$.

Proof. The first two claims are obvious, so let us move on to the last claim. Consider a situation where the state is A , and voter “me” is pivotal; (i) a is implemented if she votes directly for a and (ii) b is implemented if she delegates her vote as the votes are eventually delegated to a minority-type voter who votes in favor of b in state A . Keeping the uninformed voters untouched, replace all the minority-type voters by informed majority-type voters, and vice versa. Then, now the delegated voting power will be exercised by an informed voter, and “me” will be pivotal in state B . Summing up the probabilities of every such event, I obtain the following conclusion: $f(p, r) = g(r, p)$. Thus, if $p = r$, $f(p, r) = g(p, r)$. ■

From this lemma and (11), the following proposition is immediate. It shows that the availability of delegation does not help much if r is large enough.

Proposition 5. If r is small enough, there exists an equilibrium where uninformed voters always delegate their vote. On the other hand, if r is sufficiently large, they never delegate.

The discussion so far implies the following. Even if true state is the main concern, the key information may not be that of who knows the true state or who knows an expert but of who has the political preference same as yours. If voters can trust the network (i.e., r is small),

they will delegate to it. However, the information of who knows the true state or who knows an expert would be crucial if voters do not trust the network (i.e., r is large). That is, if the network is sufficiently ideologically heterogeneous, a voter cannot delegate the vote to a random person in the network; She may prefer direct voting to delegation unless she knows an informed voter who shares the same political preference with her.

3. Experiment

3.1 Design

The experiment was designed to test the following hypotheses.

- If r is small, as $N - q$ gets larger, more people will exercise the delegation option.
- More people will delegate their vote when r is smaller.
- If r is small, people make a better decision collectively when delegation is available than when unavailable.

So, I varied the decision environment in three dimensions: $N - q$, r , and the availability of the delegation option. In a session, (N, q) and r were fixed as Table 1 shows, so each participant experienced only one set of (N, q) and r . For instance, in T1, three participants were grouped, the number of votes required to implement the new policy was two, and the network was homogeneous.

Table 1. Between-subject treatments

	T1	T2	T3	T4
(N, q)	(3,2)	(3,2)	(9,5)	(9,5)
r	0	1/5	0	1/5
# participants	42	42	45	54

Note: $\pi = 2/3$ and $p = 1/3$ in all treatments

On the other hand, the delegation treatment was implemented in within-subject manner, i.e., all participants played both the games with and without the delegation option. In particular, participants played 20 rounds of the voting game in which π , the probability of the state being A , is $2/3$, and p , the probability that a voter is an informed majority-type voter is $1/3$. In the

first ten rounds, delegation was not allowed. In other words, the participants had to vote directly for either the new policy a or the existing one b . In the last ten rounds, the delegation option d was allowed. For convenience, I will call the game played in the first half *Game 1* and that in the second half *Game 2*.

At the beginning of each game, participants were assigned into a group of three or nine, depending on the N of the treatment and also assigned a number. The group and number remained the same over the game (i.e., for ten rounds). When a participant chose to delegate his/her vote in Game 2, it was delegated to the *next* participant; subject 1 delegates to subject 2, subject 2 delegates to subject 3, and the last one in line delegates to subject 1, which makes a full circle.

Because two or three types of players exist in the voting games, I adopted the strategy method to record their decisions. That is, subjects first submitted their strategy, i.e., an action plan in all possible realizations of the state and type, to their computer terminal, and then, the server computer aggregated the strategies and realized the state and each subject's type according to the parameters to determine the outcome of the game. Sample instructions can be found in the appendix.

All sessions were run in October 2019 at the laboratory managed by the Center for Research in Experimental and Theoretical Economics (CREATE) at Yonsei University, South Korea. The computer interface was built by o-Tree (Chen et al., 2016). In total, 183 undergraduate and graduate students with various majors participated in the experiment. The show-up payment was KRW 5,000 which was about USD 4.23. On top of that, I randomly selected one round in Game 1 (i.e., from the first half), and paid additional KRW 5,000 in case that they successfully implemented the *right* policy in the round, and did the same for Game 2 as well. So, the maximum payment per participant was KRW 15,000, and the average payment was about KRW 12,800.

3.2 Result

Table 2 shows the proportions of votes cast in favor of a in Game 1 in which delegation was not allowed. Since there were only two possible actions, the proportion of votes in favor of b is just the complementary frequency and thus omitted.

Table 2. Proportion of votes in favor of a (Game 1)

	T1	T2	T3	T4
Uninformed				
Benchmark	0.7808	1	0.5851	0.8117
Data	0.7929	0.7714	0.7378	0.7537
Informed in A	0.9833	0.9976	0.9978	0.9889
Informed in B	0.0095	0.0119	0.0044	0.0296
Minority in A		0.0119		0.0241
Minority in B		0.981		0.9537
# observations	420	420	450	540

The subjects voted in favor of a in about three fourths of their opportunities when uninformed, and the numbers do not vary much across the treatments (see the entries in the second row, “Uninformed-Data”). These numbers are greater than $1/2$ as predicted (because state A is more likely), and they are not terribly distant from the theoretical benchmarks (see the first row, “Uninformed-Benchmark”). When they were informed or had the minority-type preference, on the other hand, they did what they were supposed to do pretty well, which suggests that the participants well understood what was going on.

Table 3. Proportions of a and d (Uninformed, Game 2)

	T1	T2	T3	T4
Benchmark				
σ_a	0.2929	1	0	0.5675
σ_d	0.7071	0	1	0.4325
All rounds				
σ_a	0.3619	0.4643	0.22	0.4259
σ_d	0.5833	0.4548	0.7044	0.4352
Last 5 rounds				
σ_a	0.3238	0.5286	0.16	0.4407
σ_d	0.619	0.3905	0.7556	0.4296
# observations	420	420	450	540

Let us move on to Game 2 where delegation was allowed. In this game, there are two theoretical benchmarks. Recall that there always exists an equilibrium where nobody delegates her vote. On top of that, there may exist another equilibrium where uninformed voters exercise the delegation option. In the first rows of Table 3, I show the latter benchmark except for T2

where delegation is never optimal (nor is playing b).¹⁴

Table 3 shows how frequently uninformed voters played a and d . Their behavior is largely in line with the theoretical predictions. For instance, the theory predicts that when $r = 0$ (T1 and T3), the delegation option is more frequently exercised when $N - q$ is larger than when smaller, which turns out to be the case: 0.5833 (T1) < 0.7044 (T3). It is also confirmed that other things being equal, the possible presence of minority-type voters reduces the incentive for delegation: 0.5833 (T1) > 0.4548 (T2) and 0.7044 (T3) > 0.4352 (T4). These patterns are clearer in the last five rounds as shown in the last two rows of Table 3. Indeed, the numbers seem to converge to the theoretical benchmarks, which suggests that subjects have learned how to play the game over the rounds.

Table 4. Regressions on the indicator of delegation (Uninformed, Game 2)

	(1)	(2)	(3)	(4)	(5)	(6)
Network						
Homogeneous	0.2043*** (0.0593)	0.208*** (0.058)	0.2851*** (0.0599)			
Large	0.0478 (0.0594)	0.0731 (0.0589)	0.1173* (0.061)	0.1211 (0.0894)	0.1331 (0.0905)	0.1529* (0.0905)
Demographics						
Age		-0.004 (0.0114)	-0.0038 (0.0114)		0.0101 (0.0151)	0.0078 (0.0144)
Female		-0.1713*** (0.0588)	-0.2112*** (0.0604)		-0.0686 (0.0945)	-0.1025 (0.0935)
Econ. Major		0.0683 (0.0587)	0.0776 (0.0609)		-0.0271 (0.0902)	-0.0025 (0.0905)
Constant	0.4168*** (0.0507)	0.5389* (0.2855)	0.4892* (0.2903)	0.5833*** (0.0685)	0.3788 (0.3806)	0.4707 (0.3668)
Treatment	All	All	All	T1&T3	T1&T3	T1&T3
Round	>10	>10	>15	>10	>10	>15
# observations	1830	1830	915	870	870	435
R ²	0.0433	0.0779	0.1359	0.0160	0.0267	0.0374

Note: The dependent variable is the dummy variable for delegation. The numbers in the parentheses are the standard errors clustered at individual level. *** indicates statistical significance at 99% level, and * indicates 90% level.

¹⁴ As for the benchmark of T4, I relied on a simulation method which proceeds as follows. Set first $\sigma_a = \sigma_d = 0.5$, and pick one voter, say voter 1. Assuming that voter 1 plays a , and all the others follow the predesignated strategy (σ_a, σ_d) , run simulations of the voting game 100,000 times to calculate the average payoff of voter 1. Repeat the process assuming now that voter 1 plays d . Compare the average payoffs. If a gives a higher payoff, increase σ_a a little bit. Otherwise, increase σ_d a little bit. Repeat this until the two payoffs are *almost* the same or until σ_a reaches to zero or one.

Table 4 shows the results of regression analyses where the dependent variable is the dummy variable for delegation. For the first three columns, I use the data from all treatments to test whether the homogeneity of the network indeed increased the frequencies of delegation, while for the next three columns, I test whether the network size matters when $r = 0$. All standard errors are clustered at individual level.

The first three columns show that the heterogeneity of preference significantly reduced the tendency to delegate. The difference in the frequency of delegation between the homogeneous-preference treatments and the heterogeneous-preference treatments turns out to be larger than 20 percent points. The third column shows the results based only on the last five rounds, in which the difference got as large as 28 percent. On the other hand, the last three columns document that the effect of network size is a bit larger than 10 percent points and statistically significant only in the last five rounds.

To take into account subjects' heterogeneity, I control for age, gender, and whether the student's major is economics or related disciplines such as business administration and applied statistics. The results are reported in columns (2), (3), (5), and (6). It turns out, interestingly, that female subjects exercised the delegation option less often than male subjects. Moreover, this pattern is statistically significant in columns (2) and (3) but not in (5) and (6), which suggests that the presence of others with the opposite preference (i.e., $r > 0$) affected female subjects significantly more than male subjects. This observation seems consistent with the literature documenting that females are more risk-averse than males.

Lastly, to check whether and how much the delegation option improves the quality of collective decision, I run the following simulation: (i) Take the proportions of a , b , and d in the last-five-round data as a strategy $(\sigma_a, \sigma_b, \sigma_d)$. (ii) Run simulations of the voting game 100,000 times. (iii) Calculate the average payoff of a majority-type voter. Table 5 shows the results of such simulations.

Table 5. Frequencies of implementing the *right* policy

	T1	T2	T3	T4
Game 2 (w/ delegation)	0.8088	0.6494	0.9187	0.704
Game 1 (w/o delegation)	0.7356	0.6395	0.7547	0.6954
Difference	0.0732	0.0099	0.164	0.0086

Note: These are results simulated with the decisions in the last five rounds

Overall, the quality of the collective decision is higher with delegation than without. Most notably, in T3 ($(N, q) = (9, 5)$ and $r = 0$), the difference in the frequencies of implementing the right policy is as large as 0.164. To appreciate how large this is, recall that the frequency would be 0.6667 ($\approx \pi$) if everybody just vote for a no matter what. When $r = 1/5$ (T2 and T4), however, the numbers are smaller or slightly larger than 0.6667 in Game 1, and the improvement by the delegation option is not substantial.

4. Summary and Discussion

This paper is the first attempt to examine the new voting system so called delegative democracy, employing the standard methods of economics and political science. I show that uninformed voters are willing to delegate their vote if they believe that the delegation network is sufficiently homogeneous in terms of political preference and large (i.e., well-connected), which suggests that knowing exactly who is informed may be not as important as it appears at first glance. Moreover, delegation may significantly improve the quality of collective decision. The patterns found in the laboratory data are in line with these predictions.

As little is known about delegative democracy, many issues are open to fruitful future investigations. I discuss a few below.

Dispersed imperfect information

In this paper, I consider only the cases where the relevant information is either perfect and dispersed (i.e., informed voters know the true state) or imperfect and shared (i.e., everybody knows that the state is A with probability π). Of course, in reality, more often than not, information is imperfect and dispersed. In such a case, imperfectly informed voters may or may not directly cast their vote in delegative democracy. For example, imagine that each voter gets a signal correlated with the true state. However, the precision of the signal may vary across the voters; some precise and others less so. Then, there may exist a threshold in the precision of signal such that if the signal is above the threshold, the voter casts her ballot directly, and otherwise, she delegates. This implies that less precise information is abandoned all together, and the information aggregated in delegative democracy may be less precise than in the other

forms of democracy. This may be a serious problem if there exists a handful of experts well-known to the public, and the knowledge gap between the experts and the public is believed to be significant.

One possible solution to this problem is to allow partial (or fractional) delegation. In the partial delegation system, each voter has one vote and can delegate a fraction of it to another voter or to several other voters. Thus, voters can register in the system how confident they are by mixing direct voting and delegation. However, to incorporate the *confidence* information without a bias, a normalization of voting power may be required. For example, without a normalization, two voters who are equally confident about their signal may exercise different voting powers only because one is more highly connected in the network than the other. If so, people would legitimately concern the transparency of the system.

Communication

The analysis in this paper, especially Propositions 2, 3 and Table 5, shows that delegation may significantly improve the quality of collective decision. However, these results may exaggerate the potential gain from delegation because we are already using a substitute for delegation in social networks, namely communication in the networks. As mentioned in the introduction, transitive delegation can be regarded as a mirror image of information transmission in the network; instead of information, decision right is flowing over the network. Because we are already using the network as a medium of information exchange, the additional gain from using it as the basis for delegation may not be large, if exists at all.

I do not mean that communication and delegation are perfect substitutes to each other. They may differ in details, and details matter. For instance, delegated votes would be more easily tracked than transmitted information in the network. This is a merit of delegation; more often than not, information (or a rumor) is regarded as not credible because its source is unclear. The duality of communication and delegation would be worthy of more investigations.

Network structure

I believe that the main results of this paper would remain the same even if I consider different, more complex network structures. However, if information is dispersed and imperfect, it will

no longer be the case. Buechel and Mechtenberg (2019) show that the communication network structure matters when voters' private information is imperfect. In particular, pre-vote communication may undermine the efficiency and hence reduce the welfare if some voters have much more audiences than others because a handful of imperfect signals may get too much weight in the final decision.¹⁵

This will be a problem in delegative democracy, too. However, in contrast to the case of information sharing in social networks, delegation is more *controllable* in the sense that delegation should be verified by the centralized (or decentralized if a block-chain method is used) system. So, if necessary, we may be able to mitigate the problem by imposing a cap in the voting power that one can exercise. But of course, this will come at a price; an efficiency loss due to limiting the use of nearly perfect knowledge if it exists.

Information acquisition

Delegation is not just a substitute for information transmission also in that delegation may encourage voters to acquire information. Since Downs (1957), it has been well known that at least in theory, voters in a large election have little incentive to do a costly research about the issues at hand and to turn out to vote because the probability that they influence the election outcome is nearly zero. In contrast to the prediction, people turn out. However, this does not imply that voters are also willing to spend non-trivial time and effort to learn and evaluate the public issues.

In a companion paper (Kim, 2021), I investigate how delegation would affect the incentive for information acquisition. If voters coordinate successfully, delegation would facilitate information acquisition because those who accumulated enough voting power may have a strong incentive to learn the true state as they are sufficiently likely to be pivotal. For concreteness, consider a modified voting game which unfolds as follows. In the first stage, voters decide whether to delegate their vote to someone in their network. After observing the accumulated voting power, they decide whether to acquire information about the state of the world, which involves a cost. In the last stage, those who did not delegate their decision right

¹⁵ This echoes the fact that the Condorcet jury theorem does not hold if votes are highly correlated (e.g., Kaniovski and Zaigraev, 2011).

cast their ballots. It can be shown that in some parameter range, some voters acquire relevant information in delegative democracy but not in direct democracy.

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Appendix. Experimental instruction

Instruction for T1 and T3 (Homogeneous political preference)

Game 1

Thank you for participating in the experiment. Please read the following instructions carefully.

All participants' decisions in the experiment are anonymously collected and used only for research. No one knows what your decisions are in the experiment.

You will receive KRW 5,000 for participating in this experiment. On top of that, you will receive additional reward depending your choice.

You will be matched with other people in this room and play a voting game.

- You and two [eight in T3] other people in this room will form a group. (The members do not know each other's identity.)
- There are two policies, the existing one and a new one.
- If two [five in T3] or more people vote in favor of the new policy, it will be implemented.
- If the new policy is a good one and is implemented, everybody will receive KRW 5,000 additionally. If not implemented, everybody will receive KRW 0.
- If the new policy is a bad one and is implemented, everybody will receive KRW 0. If not implemented, everybody will receive KRW 5,000 additionally.

Some people may know whether the new policy is good or bad.

- The probability that the new policy is good one is $2/3$. The probability that it is a bad one is $1/3$.
- Each of the members learns whether it is good or bad with probability $1/3$. With probability $2/3$, you (and others) will have to vote without getting additional information.
- Who or how many got the information is unknown.

Let's take a look at an example.

Suppose that the new policy is a good one. So, if implemented, everybody will get additional KRW 5,000. If the old policy remains, everybody will receive KRW 0. At first, nobody knows this. But before the voting stage,

- The probability that player 1 receives the exact information is $1/3$.
- The probability that player 2 receives the exact information is $1/3$.
- The probability that player 3 receives the exact information is $1/3$.

- [...a bit more in T3]

Suppose only player 3 gets the information. In other words, player 3 learns that the new policy is a good one. However, the others do not know that player 3 knows it. Player 3 cannot speak to them. Players 1 and 2 have to vote knowing neither whether the new policy is a good one nor who has the information.

- If players 1 and 3 vote for the new policy and player 2 against it, then the new policy is implemented because two votes are in favor of the new policy.
- If players 1 and 2 vote against the new policy and player 3 for it, then the old policy remains since fewer than two votes are in favor of it.
- [Examples are slightly modified in T3]

The actual experiment unfolds as follows.

- You will first decide what to do in case that you learn whether the new policy is good or bad (called cases 1 and 2),
- And what to do in case that you do not learn it (called case 3).
- The server computer will decide whether the new policy is good or bad.
- Each member receive this information with probability 1/3. This too will be randomly decided by the server computer.
- Your actual voting follows the plan that you reported before.
- For instance, if player 1 does not receive the information, then the server computer counts the vote according to the plan for case 3.
- If, on the other hand, player 1 does receive the information, then the computer counts the vote according to the plan for case 1 or 2 depending on the realized state.

Please do not talk to each other and do not use a cell phone or the Internet until the experiment ends. You do not have to hurry if others finish early. If you have any questions, please raise your hand and wait for further instructions from the experimenter.

Game 2

We play the game again. But this time, you can have the option of delegation on top of voting for or against the new policy.

You will be matched with other people in this room and play a voting game.

- You and two [eight in T3] other people in this room will form a group. (The members do not know each other's identity.)

- There are two policies, the existing one and a new one.
- If two [five in T3] or more people vote in favor of the new policy, it will be implemented.
- If the new policy is a good one and is implemented, everybody will receive KRW 5,000 additionally. If not implemented, everybody will receive KRW 0.
- If the new policy is a bad one and is implemented, everybody will receive KRW 0. If not implemented, everybody will receive KRW 5,000 additionally.

Some people may know whether the new policy is good or bad.

- The probability that the new policy is good one is $2/3$. The probability that it is a bad one is $1/3$.
- Each of the members learns whether it is good or bad with probability $1/3$. With probability $2/3$, you (and others) will have to vote without getting additional information.
- Who or how many got the information is unknown.

You can cast your vote yourself or can delegate it.

- You can choose to vote for, vote against the new policy or delegate your vote.
- Each member will be assigned a number.
- If you decides to delegate your vote, your vote will be delegated to the next in line. That is, player 1 will delegate the vote to player 2, player 2 to player 3, and player 3 to player 1. [The chain is a bit longer in T3.]
- If you delegate and so does the one before you, the one next you will have three votes to cast. For instance, if players 1 and 2 choose to delegate, player 3 will exercise three votes. [Example is slightly modified in T3.]
- If everybody chooses to delegate, it means nobody votes for the new policy. So, the old policy remains.

Let's take a look at an example.

Suppose that the new policy is a good one. So, if implemented, everybody will get additional KRW 5,000. If the old policy remains, everybody will receive KRW 0. At first, nobody knows this. But before the voting stage,

- The probability that player 1 receives the exact information is $1/3$.
- The probability that player 2 receives the exact information is $1/3$.
- The probability that player 3 receives the exact information is $1/3$.
- [...a bit more in T3]

Suppose only player 3 gets the information. In other words, player 3 learns that the new policy is a good one. However, the others do not know that player 3 knows it. Player 3 cannot speak to them. Players 1 and 2 have to vote knowing neither whether the new policy is a good one nor who has the information.

- If players 1 and 3 vote for the new policy and player 2 against it, then the new policy is implemented because two votes are in favor of the new policy.
- If player 1 delegates, player 2 votes against the new policy, and player 3 votes for it, the old policy remains since player 2 votes against it with two votes.
- If player 1 votes against it, player 2 delegates, and player 3 votes for it, the new policy is implemented because player 3 votes in favor of it with two votes.
- If players 1 and 2 delegate, and player 3 votes for it, the new policy is implemented because player 3 votes in favor of it with three votes.
- If everybody delegates, the old policy remains because fewer than two votes are in favor of the new policy.
- [Examples are slightly modified in T3]

The actual experiment unfolds as follows.

- You will first decide what to do in case that you learn whether the new policy is good or bad (called cases 1 and 2),
- And what to do in case that you do not learn it (called case 3).
- The server computer will decide whether the new policy is good or bad.
- Each member receive this information with probability 1/3. This too will be randomly decided by the server computer.
- Your actual voting follows the plan that you reported before.
- For instance, if player 1 does not receive the information, then the server computer counts the vote according to the plan for case 3.
- If, on the other hand, player 1 does receive the information, then the computer counts the vote according to the plan for case 1 or 2 depending on the realized state.
- Depending on the outcome, the additional rewards are determined. After filling in a short survey (age, gender, etc.), you will receive the payment (KRW 5,000 for participation + reward from Game 1 + reward Game 2).

Please do not talk to each other and do not use a cell phone or the Internet until the experiment ends. You do not have to hurry if others finish early. If you have any questions, please raise your hand and wait for further instructions from the experimenter.

Instruction for T2 and T4 (Heterogeneous political preference)

Game 1

Thank you for participating in the experiment. Please read the following instructions carefully.

All participants' decisions in the experiment are anonymously collected and used only for research. No one knows what your decisions are in the experiment.

You will receive KRW 5,000 for participating in this experiment. On top of that, you will receive additional reward depending your choice.

You will be matched with other people in this room and play a voting game.

- You and two [eight in T4] other people in this room will form a group. (The members do not know each other's identity.)
- There are two policies, the existing one and a new one.
- If two [five in T4] or more people vote in favor of the new policy, it will be implemented.

Not everybody wants the same thing. The majority type people and the minority type people want the opposite.

- Each member knows his/her own type, but knows neither who is nor how many are which type.
- Each member becomes a majority type voter with probability $4/5$ and a minority type voter with probability $1/5$. Since the types are realized stochastically, it is possible (although very unlikely) that everybody becomes the minority type.
- If the new policy is a good one for the majority:
 - If implemented, a majority type voter will receive KRW 5,000 additionally. A minority type voter will receive KRW 0.
 - If not implemented, a minority type voter will receive KRW 5,000 additionally. A majority type voter will receive KRW 0.
- If the new policy is a good one for the minority:
 - If implemented, a minority type voter will receive KRW 5,000 additionally. A majority type voter will receive KRW 0.
 - If not implemented, a majority type voter will receive KRW 5,000 additionally. A minority type voter will receive KRW 0.

Some people may know which type of voter the new policy benefits.

- The new policy is good for the majority with probability $2/3$. It is good for the minority with probability $1/3$.
- Minority type people know which type of people the new policy benefits.

- With probability $1/3$, a vote becomes an informed majority type voter who knows the true nature of the policy. With probability $7/15 (= 1-1/3-1/5 = 0.4667)$, a voter becomes an uninformed majority type voter.
- Others do not know who or how many are informed.

Let's take a look at an example.

Suppose that the new policy is good for the majority. So, if implemented, every majority type voter will get additional KRW 5,000, and minority type voters will get KRW 0. If the old policy remains, every majority type voter will receive KRW 0, and minority type voters will get KRW 5,000. At first, nobody knows this. But before the voting stage,

- The probability that player 1 becomes an informed majority type voter is $1/3$. The probability to become an uninformed majority type voter is $7/15 (=0.4667)$. The probability to become an informed minority type voter is $1/5$.
- The probability that player 2 becomes an informed majority type voter is $1/3$. The probability to become an uninformed majority type voter is $7/15 (=0.4667)$. The probability to become an informed minority type voter is $1/5$.
- The probability that player 3 becomes an informed majority type voter is $1/3$. The probability to become an uninformed majority type voter is $7/15 (=0.4667)$. The probability to become an informed minority type voter is $1/5$.
- [...a bit more in T4]

Suppose everybody became majority type voters, and only player 3 received the exact information. In other words, player 3 learns that the new policy is a good one. However, the others do not know that player 3 knows it. Player 3 cannot speak to them. Players 1 and 2 have to vote knowing neither whether the new policy is a good one nor who has the information.

- If players 1 and 3 vote for the new policy and player 2 against it, then the new policy is implemented because two votes are in favor of the new policy.
- If players 1 and 2 vote against the new policy and player 3 for it, then the old policy remains since fewer than two votes are in favor of it.
- [Examples are slightly modified in T4]

The actual experiment unfolds as follows.

- You will first decide what to do in case that you are a minority type voter (called cases 1 and 2),
- What to do in case that you are an informed majority type voter (called cases 3 and 4),
- And what to do in case that you are an uninformed majority type voter (called case 5).
- The server computer will decide which type the new policy benefits.

- Each member's type is determined. This too will be randomly decided by the server computer.
- Your actual voting follows the plan that you reported before.
- For instance, if player 1 is an uninformed majority type voter, then the server computer counts the vote according to the plan for case 5.
- If, on the other hand, player 1 is an informed majority type voter, then the computer counts the vote according to the plan for case 3 or 4 depending on the realized state.

Please do not talk to each other and do not use a cell phone or the Internet until the experiment ends. You do not have to hurry if others finish early. If you have any questions, please raise your hand and wait for further instructions from the experimenter.

Game 2

We play the game again. But this time, you can have the option of delegation on top of voting for or against the new policy.

You will be matched with other people in this room and play a voting game.

- You and two [eight in T4] other people in this room will form a group. (The members do not know each other's identity.)
- There are two policies, the existing one and a new one.
- If two [five in T4] or more people vote in favor of the new policy, it will be implemented.

Not everybody wants the same thing. The majority type people and the minority type people want the opposite.

- Each member knows his/her own type, but knows neither who is nor how many are which type.
- Each member becomes a majority type voter with probability $4/5$ and a minority type voter with probability $1/5$. Since the types are realized stochastically, it is possible (although very unlikely) that everybody becomes the minority type.
- If the new policy is a good one for the majority:
 - If implemented, a majority type voter will receive KRW 5,000 additionally. A minority type voter will receive KRW 0.
 - If not implemented, a minority type voter will receive KRW 5,000 additionally. A majority type voter will receive KRW 0.
- If the new policy is a good one for the minority:

- If implemented, a minority type voter will receive KRW 5,000 additionally. A majority type voter will receive KRW 0.
- If not implemented, a majority type voter will receive KRW 5,000 additionally. A minority type voter will receive KRW 0.

Some people may know which type of voter the new policy benefits.

- The new policy is good for the majority with probability $2/3$. It is good for the minority with probability $1/3$.
- Minority type people know which type of people the new policy benefits.
- With probability $1/3$, a vote becomes an informed majority type voter who knows the true nature of the policy. With probability $7/15$ ($= 1 - 1/3 - 1/5 = 0.4667$), a voter becomes an uninformed majority type voter.
- Others do not know who or how many are informed.

You can cast your vote yourself or can delegate it.

- You can choose to vote for, vote against the new policy or delegate your vote.
- Each member will be assigned a number.
- If you decides to delegate your vote, your vote will be delegated to the next in line. That is, player 1 will delegate the vote to player 2, player 2 to player 3, and player 3 to player 1. [The chain is a bit longer in T4.]
- If you delegate and so does the one before you, the one next you will have three votes to cast. For instance, if players 1 and 2 choose to delegate, player 3 will exercise three votes. [Example is slightly modified in T4.]
- If everybody chooses to delegate, it means nobody votes for the new policy. So, the old policy remains.

Let's take a look at an example.

Suppose that the new policy is good for the majority. So, if implemented, every majority type voter will get additional KRW 5,000, and minority type voters will get KRW 0. If the old policy remains, every majority type voter will receive KRW 0, and minority type voters will get KRW 5,000. At first, nobody knows this. But before the voting stage,

- The probability that player 1 becomes an informed majority type voter is $1/3$. The probability to become an uninformed majority type voter is $7/15$ ($=0.4667$). The probability to become an informed minority type voter is $1/5$.
- The probability that player 2 becomes an informed majority type voter is $1/3$. The probability to become an uninformed majority type voter is $7/15$ ($=0.4667$). The probability to become an informed minority type voter is $1/5$.
- The probability that player 3 becomes an informed majority type voter is $1/3$. The

probability to become an uninformed majority type voter is $7/15$ ($=0.4667$). The probability to become an informed minority type voter is $1/5$.

- [...a bit more in T4]

Suppose everybody became majority type voters, and only player 3 received the exact information. In other words, player 3 learns that the new policy is a good one. However, the others do not know that player 3 knows it. Player 3 cannot speak to them. Players 1 and 2 have to vote knowing neither whether the new policy is a good one nor who has the information.

- If players 1 and 3 vote for the new policy and player 2 against it, then the new policy is implemented because two votes are in favor of the new policy.
- If player 1 delegates, player 2 votes against the new policy, and player 3 votes for it, the old policy remains since player 2 votes against it with two votes.
- If player 1 votes against it, player 2 delegates, and player 3 votes for it, the new policy is implemented because player 3 votes in favor of it with two votes.
- If players 1 and 2 delegate, and player 3 votes for it, the new policy is implemented because player 3 votes in favor of it with three votes.
- If everybody delegates, the old policy remains because fewer than two votes are in favor of the new policy.
- [Examples are slightly modified in T4]

The actual experiment unfolds as follows.

- You will first decide what to do in case that you are a minority type voter (called cases 1 and 2),
- What to do in case that you are an informed majority type voter (called cases 3 and 4),
- And what to do in case that you are an uninformed majority type voter (called case 5).
- The server computer will decide which type the new policy benefits.
- Each member's type is determined. This too will be randomly decided by the server computer.
- Your actual voting follows the plan that you reported before.
- For instance, if player 1 is an uninformed majority type voter, then the server computer counts the vote according to the plan for case 5.
- If, on the other hand, player 1 is an informed majority type voter, then the computer counts the vote according to the plan for case 3 or 4 depending on the realized state.

Please do not talk to each other and do not use a cell phone or the Internet until the experiment ends. You do not have to hurry if others finish early. If you have any questions, please raise your hand and wait for further instructions from the experimenter.