연세대학교 상경대학
경제연구소
Economic Research Institute
Yonsei Universit

서울시 서대문구 연세로 50
50 Yonsei-ro, Seodaemun-gu, Seoul, Korea
TEL: (+82-2) 2123-4065 FAX: (+82-2) 364-9149
E-mail: yeri4065@yonsei.ac.kr
http://yeri.yonsei.ac.kr/new

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Sangyup Choi
Yonsei University

Inkee Jang
The Catholic University of Korea

Kee-Youn Kang
University of Liverpool

Hyunpyung Kim
Yonsei University

July 2024
2024RWP-229

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Hyunpyung Kim¶
Yonsei University

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Abstract

Leveraging the universe of transaction-level data from the Korean tri-party repo market, we examine how repo contract terms interact with collateral quality. We find that both the haircut and the interest rate increase with collateral risk, consistent with previous findings. However, conditioning on the same collateral quality, we uncover the trade-off between the haircut and the interest rate: A 1% point increase in the interest rate is associated with a 1.3% point reduction in the haircut. Moreover, the rate at which the interest rate substitutes the haircut decreases in market uncertainty, suggesting that the relative importance of insurance instruments increases with default risk. We theoretically show that the interaction between lenders' incentives to acquire information about collateral quality and borrowers' incentives to default opportunistically in a collateralized debt market is key to rationalizing the positive (negative) unconditional (conditional) relationship between the haircut and the interest rate.

JEL Classification: D53, D8, E44, G23

Keywords: Repo market, collateralized debt, haircut, collateral quality, costly information acquisition, uncertainty

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*We are very grateful for comments and suggestions from our discussants (Jun Kyung Auh, Seungjun Baek, and Reiko Tobe), Gaetano Antinolfi, Narayan Bulusu, Chang-Koo Chi, Dong Beom Choi, Jaerim Choi, Armando Gomes, Bosung Jang, Youngwoo Koh, Jae Won Lee, Jongsub Lee, Jungho Lee, Fernando Martin, Nicolas Jacquet, Stephen Williamson, and seminar participants at the 2019 Yonsei Business-Economics Workshop, the Summer Finance Roundtable 2024, JSME-KMFA Joint Workshop 2024, 2024 Korea's Allied Economic Associations Annual Meeting, 2024 Korea Money and Finance Association Annual Meeting, 2024 Korean Econometric Society Annual Meeting, Bank of Korea, Korea Capital Market Institute, Seoul National University Graduate School of Business, Singapore Management University, University of Liverpool, and Yonsei University. This research was supported by the Yonsei Signature Research Cluster Program of 2024 (2024-22-0172) and the Yonsei Fellow Program, funded by Lee Youn Jae. Jeeyeon Phi and Seung Yong Yoo provided excellent research assistance.

†School of Economics, Yonsei University, sangyupchoi@yonsei.ac.kr

‡Department of Economics, The Catholic University of Korea, inkeejang@catholic.ac.kr

§Department of Economics, University of Liverpool, kkarikky@gmail.com

¶School of Economics, Yonsei University, hyunpyungkim@yonsei.ac.kr

1 Introduction

Repurchase agreements (repos) are vital instruments in short-term financing, utilizing collateral to mitigate risks and facilitate transactions. Despite its growing significance, the functioning of repo markets remains opaque because of their OTC (over-the-counter) nature, hindering comprehensive and timely empirical investigations. This is in sharp contrast to a large theoretical literature on the understanding of contract terms in collateralized loans.[1] Moreover, most empirical studies using detailed data on repo contract terms are focused on the U.S. and the European repo market, which calls for more studies on other markets to broaden our understanding.[2]

This study deepens our understanding of the repo market both empirically and theoretically. Empirically, we leverage a dataset comprising seven million contract-level observations from the Korean tri-party repo market, providing insights into how haircuts and interest rates interact with collateral quality.[3] Other than documenting the determinants of haircuts and interest rates, we pay particular attention to how they are jointly determined in a given contract. Despite the growing literature on what determines haircuts in the repo market, most studies have not answered this question. This is surprising given that participants negotiate both the haircut and the interest rate to compensate for various types of risk (Dang et al. (2013), Barsky et al. (2016), and Liu and Xie (2023)).

Moreover, even these few studies have not reached a consensus on the relationship between haircuts and interest rates. For example, Baklanova et al. (2019) documented suggestive evidence of the negative but weak relationship between the haircut and the interest rate using rather limited data on the U.S. bilateral repo market.[4] In contrast, us-

[1]See, for example, Bester (1985), Dang et al. (2013), Simsek (2013), Antinolfi et al. (2015), Fostel and Geanakoplos (2015), Barsky et al. (2016), and Gottardi et al. (2019).

[2]Notable exceptions are Julliard et al. (2022) for the United Kingdom and Suzuki and Sasamoto (2022) for Japan.

[3]Here, haircuts are the percentage difference between the collateral value and loan size. For example, suppose an agent borrows \$80 million and pledges \$100 million of bonds, then, the haircut is 20%[=(100-80)/100].

[4]The results in Baklanova et al. (2019) are obtained from three-day snaps of data from nine U.S. bank holding companies and limited to treasury bonds as collateral.

ing comprehensive transaction-level data in the Japanese tri-repo market, Suzuki and Sasamoto (2022) found a positive relationship between the two. Similarly, using bilateral repo transactions of 47 major European banks, Barbiero et al. (2024) found that a repo contract with a positive haircut tends to have a higher interest rate than a contract without a haircut, although this result is not statistically significant. Importantly, there has been no study on how the relationship between the haircut and the interest rate varies over time.

Compared to the existing studies on the relationship between the haircut and the interest rate in a repo contract, our analysis is much more comprehensive in both cross-sectional and time-series dimensions. Our dataset contains detailed information about contract terms, including the loan rate, haircut, loan sizes, maturity, and types of borrowers and lenders. Especially, we also have information on a unique identifier (International Securities Identification Number, ISIN) for collateral in every contract, which can be further merged into an independent security-level database. This unique information turns out to be the key to reconciling different findings about the relationship between the haircut and the interest rate.

First, we regress the transaction-level haircut and repo spread on standard determinants found in the literature, which are aimed at proxying collateral, counterparty, and market risks. The estimation results largely confirm the previous findings in the literature that both haircuts and spreads increase in the proxies for collateral risk (e.g., Hu et al. (2021); Auh and Landoni (2022); Julliard et al. (2022); Suzuki and Sasamoto (2022)). We then regress the haircut on the repo spread (the spread between the repo rate and the inter-bank rate of the same maturity) in addition to a set of standard control variables.[5] To sharpen identification and understand the source of variation in this equilibrium relationship, we flexibly introduce a different set of fixed effects.

Our main empirical findings are threefold. First, there is a seemingly positive relation-

[5]Throughout the paper, we use interest rates and spreads interchangeably for two reasons. First, all our empirical results are nearly identical whether using interest rates or spreads given the presence of time-fixed effects. Second, interest rates and spreads are the same in the model.

ship between the haircut and the interest rate when the quality of the underlying collateral is not conditioned. In other words, a repo contract with riskier collateral tends to have both a higher haircut and a higher interest rate, which is consistent with the findings in Auh and Landoni (2022), Suzuki and Sasamoto (2022), and Barbiero et al. (2024). The positive relationship continues to hold until we absorb collateral-specific fixed effects.

Second, once collateral risk is fully absorbed by exploiting data on unique collateral identifiers, a negative association between the haircut and the interest rate emerges. This trade-off is not only statistically significant but economically meaningful, as a 1% point increase in the spread is related to a 1.3% point decline in the haircut. While the trade-off was also documented in Baklanova et al. (2019), we confirm that this is a general phenomenon beyond a specific type of collateral or market participants. Importantly, a sequential absorption of various fixed effects sheds light on why previous studies found mixed results about this relationship. It is likely attributed to the failure to account for collateral quality due to the absence of rich security-level information in existing studies.

Third, we find that heightened uncertainty significantly influences the size of the trade-off. Once we interact the spread with the implied stock market volatility in our preferred baseline regression conditioning on the same collateral, this interaction term turns out to be positive and statistically significant. Specifically, a one standard deviation increase in stock market volatility reduces the rate at which the interest rate substitutes the haircut by 28%. Intuitively, as market uncertainty increases, the overall default risk of the contract also increases. This, in turn, raises the relative importance of the haircut compared to the interest rate since the haircut is a direct instrument to insure lenders against default.

To provide theoretical explanations for the three empirical findings, we build a model of collateralized debt characterized by two periods and two risk-neutral agents (a borrower and a lender). The borrower has liquidity needs in the first period and borrows goods from the lender, using assets as collateral. The asset provides stochastic dividends at the end of the second period, and information about the dividend state is revealed at the beginning of the second period before repayment (i.e., repurchase). Thus, the borrower may default opportunistically if the value of collateral assets turns out to be short of repayment. Both agents can acquire private information about the future value of assets in the first period at a cost before trading with each other.[6]

In the model, haircuts exist only when an incentive to acquire information exists, either in the form of the threat or actual acquisition of information about collateral quality. Actual information acquisition eliminates opportunistic default risk because lenders only provide loans if the collateral holds high value. In this case, the borrower maximizes the quantity of collateral and repayment to increase the loan size. Consequently, the set of optimal debt contracts is singular. On the other hand, when the threat of information acquisition exists, opportunistic default risk persists. Since this threat limits the loan size to prevent actual information acquisition, the interest rate and haircut do not affect the loan size directly. Thus, a continuum of pairs of interest rates and haircuts emerges, exhibiting a trade-off between the two. As interest rates rise, given the characteristics of agents and collateral assets, haircuts decrease.[7]

Our model also offers theoretical predictions that align with the other main empirical findings. First, as collateral risk, represented by the spread of the distribution of the tree's terminal values, escalates in the model, both interest rates and haircuts increase, explaining the unconditional positive relationship. Second, an elevation in the default risk of a

[6]For example, an agent can procure analytical reports that assess a firm's prospects with greater precision than conventional perceptions. This information can inform decisions regarding the purchase or sale of securities, or their use as collateral in secured loan contracts.

[7]Bester (1985), Kang (2021), and Kuong (2021) show that interest rates and haircuts may respond differently in response to a change in a certain parameter in their models. For example, Bester (1985) and Kuong (2021) show that safe firms are willing to pledge more collateral, which means a higher haircut given the fixed loan size, to signal their quality and lower interest rates, deriving a trade-off between interest rates and haircuts. Kang (2021) shows the interest rate increases and the haircut falls as the cost of faking the quality of collateral increases. However, in these models, a borrower type or the property of collateral is not controlled. Thus, within a group of borrowers with the same default risk given the same collateral assets, there is no trade-off between interest rates and haircuts in their models, in contrast to our model. One of our main theoretical contributions is to derive conditions for the trade-off between interest rates and haircuts, even after controlling for other contractual characteristics.

repo contract due to heightened uncertainty amplifies the significance of haircuts relative to interest rates in the lender's payoff. Consequently, the rate at which the interest rate substitutes the haircut falls.

The rest of the paper is organized as follows. Section 2 overviews the Korean repo market and presents the main empirical findings. A battery of robustness checks is also provided. Section 3 builds a theoretical model of collateralized debt to explain the findings. All omitted proofs are provided in Appendix B. Section 4 concludes the paper.

2 Empirical Evidence

In this section, we present empirical findings on repo contracts, drawing upon the universe of transaction-level data from the Korean tri-party repo market. The dataset encompasses detailed information on each repo contract, including the loan rate, haircut, loan size, maturity, a unique identifier for collateral (International Securities Identification Numbers), and type of cash borrowers (security sellers) and cash lenders (security buyers). This extensive dataset, available daily, enables us to effectively control for various characteristics in loan contracts, thereby shedding light on the determinants of haircuts and interest rates and the relationship between the two. In particular, information about a unique identifier for all collateral used in repo contracts creates an optimal environment where collateral quality can be fully conditioned.

2.1 Data

2.1.1 Market dynamics and regulatory framework

The Korean repo market comprises three primary categories: customer repo, institutional repo, and Bank of Korea (BOK) repo.[8] Our empirical analysis focuses on the tri-party in-

[8] Customer repos involve transactions between financial institutions and general customers, while BOK repos—designed for the conduct of monetary policy—are executed between the Bank of Korea and prime dealers as part of open market operations.

stitutional repo market in which the Korea Securities Depository (for over-the-counter trades) retains collateral until the maturity date. Over the years, the list of entities eligible to participate in the tri-party repo market steadily expanded, accompanied by a progressive relaxation of regulatory restrictions on participants and trade terms in the repo market.[9]

A noteworthy regulatory development occurred in May 2015 when non-bank financial institutions were restricted from accessing the call market (Suh (2016)), which led to a significant inflow of short-term funds into the repo market, predominantly from security companies. Consequently, the money market underwent a restructuring with the repo market at its core, resulting in a consistent increase in both the number of repo contracts and the size of loans, which governs the beginning of our sample period.

In an effort to enhance market efficiency and risk management, the government implemented a policy in July 2020 mandating borrowers to retain a portion of repo trades in cash-equivalent assets. This requirement is contingent upon transaction maturity, with the proportion decreasing as the maturity extends, thereby aiming to reduce reliance on overnight repos. More relevantly, the government issued guidelines for setting haircuts, taking into account collateral risk and borrower profiles. This notable structural break driven by government policy changes ends our sample under study. Thus, our baseline analysis spans from January 2015 to June 2020. For further details of institutional features of the Korean repo market, see Yun and Heijmans (2013)).

There are certain practices in the Korean tri-party repo market. For example, overnight repos continue to dominate the market, constituting over 95% of total transactions, which is higher than the share of overnight contracts (80%) in the U.S. tri-party repo market (Paddrik et al. (2021)). Positive haircuts are prevalent, even for government bonds. This is in sharp contrast to scarce non-zero haircuts in other repo markets during normal times (e.g., Julliard et al. (2022), Suzuki and Sasamoto (2022), Barbiero et al. (2024)) and makes

[9]For example, in December 2007, all restrictions on the types of underlying assets accepted as collateral in institutional repos were lifted.

the Korean repo market an ideal place to study the interaction between the haircut and the interest rate in contract terms. Our analysis focuses on the domestic currency (Won)-denominated repos only, as they account for over 99.99% of repo contracts.

Figure 1 illustrates the daily total loan size computed from the final sample used in regression analysis and the official daily trading volume obtained from the Korea Securities Depository. The monthly average is reported for both series. The trading volume computed from our final sample closely tracks that in the official statistics. Consistent with the aforementioned narrative, the Korean repo market has experienced substantial growth since 2015. By the end of our sample period, the daily average loan size reached 90 trillion Won (about 70 billion USD). The corresponding figure for the U.S. tri-party repo market is about 2.3 trillion USD.

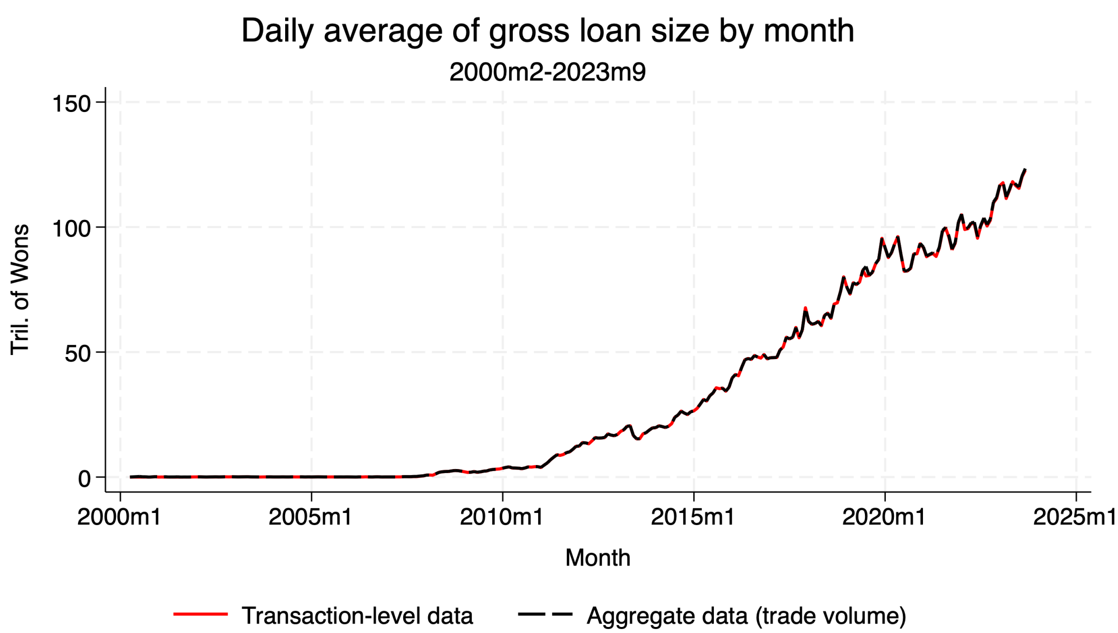

Figure 1: Size of the Korean tri-party repo market over time

2.1.2 Description of data

Table 1 provides a comprehensive summary of a haircut, spread, loan size, and maturity, along with the number of observations and unique collateral identifiers for each collateral class.[10] In our baseline analysis, we use spreads rather than loan rates to account for the level effect driven by confounding factors, such as monetary policy.[11] However, as will be shown later, both specifications deliver nearly identical results given the presence of maturity×time-fixed effects. Collateral classes are arranged in descending order based on the number of observations.

Table 1: Collateral class and loan terms

Collateral class	Avg. haircut (%)	Med. haircut (%)	Avg. spread (%)	Med. spread (%)	Avg. Principal (bil. of wons)	Med. Principal (bil. of wons)	Avg. maturity (days)	Med. maturity (days)	N. Obs.	Unique ISINs
Government bond	4.96	5	0.05	0.05	40.22	23.6	2.11	1	3,035,368 (41.05%)	539
Bank bond	4.96	5	0.05	0.05	66.13	39.5	12.47	1	1,830,212 (24.75%)	3,784
Monetary stabilization bond	4.96	5	0.05	0.04	46.13	28.5	2.13	1	694,515 (9.39%)	536
Special bond	4.95	5	0.04	0.04	67.67	37.2	22.78	1	693,176 (9.38%)	2,076
Financial bond	4.92	5	0.09	0.10	31.87	18.4	12.48	1	584,935 (7.91%)	4,806
Municipal bond	5.00	5	0.05	0.04	12.95	5.9	2.45	1	323,812 (4.38%)	1,537
Corporate bond	4.91	5	0.15	0.11	40.64	17	27.68	1	223,251 (3.02%)	4,993
ETF security (else)	23.89	30	0.49	0.38	45.66	47.8	18.51	7	4,045 (0.05%)	50
Equity	44.50	45	1.10	1.19	11.51	.98	46.90	30	3,598 (0.05%)	335
Unknown	5.00	5	0.27	0.13	9.79	9.3	1.17	1	555 (0.01%)	27
All collateral	4.99	5	0.06	0.05	47.91	26	8.25	1	7,393,467	15,895

The sample period is 2015m1-2020m6. The table shows the mean and median values of haircut, spread, principal, and maturity, as well as the number of observations and the unique identifier for collateral assets across various collateral classes. The percentages in the *N. Obs.* column represent the share of each collateral class in relation to the total number of observations.

Some observations merit attention. First, within the Korean tri-party repo market, most contracts involve safe assets as collateral, including government bonds, monetary stabilization bonds, and special bonds. These safe collateral classes collectively constitute 85% of all observations. Second, haircuts are clustered at 5% for most collateral types other than equities and ETFs. Third, both haircuts and spreads in our sample remain low, suggesting that this segment of money markets has been stable without material risk

[10]We report the median values in addition to the mean values because the former often carries more useful information than the latter given the discrete nature of a haircut and maturity.

[11]Following Auh and Landoni (2022), a spread is defined as the difference between the repo rate and the relevant inter-bank rate corresponding to the same maturity. Specifically, the repo rate is subtracted by the call rate, 1-week KORIBOR, 1-month KORIBOR, and 3-month KORIBOR for 1-day, 1-week, 1-month, and 3-month maturity, respectively. For other maturities (1-year and over 1-year), the spread is constructed by subtracting the repo rate from the 1-year KORIBOR.

during our sample period. This is consistent with the relative stability of tri-party repo markets compared to bilateral markets documented in the United States (Copeland et al. (2014) and Krishnamurthy et al. (2014)) and Europe (Mancini et al. (2016)). Table A-1 in Appendix A presents the same information only for the overnight maturity.

For a granular description of the Korean tri-party repo market, Tables A-2 and A-3 in Appendix A present distinct facets of market practices. In Table A-2, the distribution of the type of cash borrowers (sellers) and cash lenders (buyers) is delineated. Notably, collective investment schemes and security companies constitute the largest share of contracts (48.7% and 40.1%, respectively) as a cash borrower. On the cash lender side, collective investment schemes emerge prominently, accounting for 52.9%, with banks (trusts) as the second most frequent lenders at 20.3%. Table A-3 presents summary statistics of contractual terms for the top 10 frequent pairs of investors. While government bonds are the most used type of collateral for most investor pairs, bank bonds are most frequently used as collateral for a contract with a bank as a cash lender, implying inherent preferences for familiar assets as collateral.

2.2 Empirical analysis

2.2.1 Regression framework

Using an individual repo contract as a unit of observation, the main regression specification takes the following form:

$$Haircut_{B,L,M,c,t} = \beta Spread_{B,L,M,c,t} + \gamma Size_{B,L,M,c,t} + \Upsilon_{B,L,M,c,t} + \epsilon_{B,L,M,c,t}, \quad (1)$$

where a tuple $\{B, L, M, c, t\}$ consists of borrower type B, lender type L, maturity type M, unique collateral id c, and date of transaction t. Thus, $Haircut_{B,L,M,c,t}$ is the haircut applied to the repo contract on date t between borrower type B and lender type L over maturity type M using specific collateral c. $Spread_{B,L,M,c,t}$ denotes a spread for this contract defined above and $Size_{B,L,M,c,t}$ is the log of loan sizes. Importantly, $\Upsilon_{B,L,M,c,t}$ is a vector of fixed effects absorbing systematic variation across different dimensions exploiting the unique advantage of our data. Our main interest is the sign of β, which tells how the haircut and the interest rate are correlated after controlling for both observed and unobserved confounding factors. Note that our focus is not to estimate any causal effect of the interest rate on the haircut but the constellation of fixed effects largely alleviates endogeneity concerns.

In the baseline model, these fixed effects include the maturity type of a repo contract, borrower type, lender type, and unique identifier for collateral assets, which all interact with time-fixed effects. Thus, they allow us to control for a variety of individual repo contract characteristics that are not only constant but time-varying. They also capture time-variant components common to all repo contracts within a day, such as macroeconomic and financial market conditions and various policy actions. Standard errors are double clustered at the collateral level and date level but we also explore alternative clustering.

2.2.2 Determinants of repo trading terms

Before presenting the main findings regarding the relationship between haircuts and repo rates, we analyze the determinants of each of repo trading terms in the Korean tri-party repo market and check whether they are consistent with the previous findings obtained from other markets (e.g., Hu et al. (2021); Auh and Landoni (2022); Julliard et al. (2022); Macchiavelli and Zhou (2022); Suzuki and Sasamoto (2022)). To be specific, we estimate the following regression for both haircuts and spreads in turn:

$$Term_{B,L,M,c,t} = \gamma Size_{B,L,M,c,t} + I_B \Upsilon_B + I_L \Upsilon_L + I_M \Upsilon_M + I_c \Upsilon_c + \tau_t + \epsilon_{B,L,M,c,t}, \tag{2}$$

where $Term_{B,L,M,c,t}$ denotes either a haircut or a spread in a given contract.

The main difference from equation (1) is that we present the coefficients on each categorical variable capturing the characteristics of a given repo contract instead of saturating them via fixed effects. Time-fixed effects τ_t are still absorbed to control for time-varying aggregate confounding factors. Moreover, since we have full information on the ISIN of the collateral posted in every repo contract, we can match the security-level information obtained from FnGuide (a provider of financial information services in Korea) to our transaction-level data.

Although security-level data from FnGuide do not cover equities or ETFs, over 95% of our transaction-level data are still matched with this database. Exploiting this security-level information containing the issuance and maturity dates of each ISIN, we can not only classify each ISIN into precise asset classes but track the remaining maturity of the underlying collateral at any point in time. This security-level information is used to capture the role of collateral risk (i.e., the type and remaining maturity of collateral) in repo trading terms studied in the related literature (e.g., Copeland et al. (2014); Nyborg (2019); Hu et al. (2021); Auh and Landoni (2022)).

Table 2 summarizes the findings. The omitted baseline for each category is security companies and their trusts (for borrower type), domestic banks and their trusts (for lender type), overnight (for maturity), government bonds (for collateral class), remaining maturity less than or equal to 3 months (for remaining maturity of collateral).[12] Columns (1) and (2) report the results using haircuts as a dependent variable, whereas the dependent variable in Columns (3) and (4) is repo spreads. While Columns (1) and (3) employ the full sample, Columns (2) and (4) use the subsample that is matched with the security-level data from FnGuide, therefore excluding repo contracts using equities or ETFs. Since the matched sample with information on collateral assets is our preferred specification, we focus on discussing the results in Columns (2) and (4) with an exception for documenting that a repo contract using equity as collateral is associated with significantly higher

[12]Here, we aggregate some similar entities (e.g., banks and bank trusts) in each dimension of repo contract terms to streamline the analysis of haircut and interest rate determinants. In our main analysis, we use the most disaggregate-level entities in each dimension to control for unobserved heterogeneity as cleanly as possible.

Table 2: Preliminary analysis: determinants of haircut and spread

	Haircut		Repo rate	
	(1)	(2)	(3)	(4)
Size	-0.00631	0.00293***	-0.000704***	-0.000340*
	(-1.54)	(3.03)	(-3.08)	(-1.94)
Borrower type (baseline: Securities company and its trust)				
Collective investment scheme	0.0855***	0.0579***	0.0177***	0.0172***
	(7.12)	(16.70)	(25.78)	(26.47)
Government	0.0759***	0.0527***	0.0170***	0.0161***
	(8.11)	(13.86)	(18.35)	(16.57)
Specialized credit finance company	0.0865***	0.124***	-0.0360***	-0.0339***
	(4.04)	(10.94)	(-8.79)	(-7.90)
Domestic bank and domestic bank trust	0.220***	0.218***	-0.0703***	-0.0689***
	(13.68)	(14.44)	(-24.81)	(-24.71)
Foreign bank	-0.0628	0.0760***	-0.00199	0.00273
	(-0.95)	(5.45)	(-0.61)	(1.21)
Pension funds and insurance company	0.0833***	0.0574***	0.0116***	0.00899***
	(6.98)	(6.92)	(3.38)	(2.92)
Lender type (baseline: Domestic bank and its trust)				
Collective investment scheme	-0.0309***	-0.00368	-0.0184***	-0.0175***
	(-2.77)	(-1.62)	(-20.15)	(-20.60)
Security company and security company trust	-0.388***	-0.380***	0.0175***	0.0151***
	(-14.29)	(-14.84)	(10.08)	(9.09)
Pension funds and insurance company	-0.00185	0.00608*	-0.0186***	-0.0180***
	(-0.41)	(1.81)	(-21.87)	(-21.28)
Specialized credit finance company	0.0550**	-0.00857***	-0.0174***	-0.0194***
	(2.27)	(-3.39)	(-13.70)	(-19.01)
Government	-0.0288***	-0.0112*	-0.0163***	-0.0155***
	(-3.84)	(-1.94)	(-10.38)	(-10.07)
Foreign bank	0.0240*	0.0237*	-0.0304***	-0.0304***
	(1.69)	(1.78)	(-10.28)	(-10.19)
Maturity type (baseline: overnight)				
2-day to 1-week	0.0932**	0.00671	0.00995***	0.00462**
	(2.45)	(0.92)	(3.56)	(2.28)
1-week to 1-month	0.540**	0.0271***	0.0595***	0.0487***
	(2.18)	(4.40)	(7.46)	(13.87)
1-month to 3-month	2.092**	0.154***	0.0332	0.0490***
	(2.23)	(4.41)	(0.68)	(6.49)
Over 3 months	0.246***	0.164***	0.0219***	0.0189***
	(7.19)	(9.20)	(4.48)	(3.77)
Collateral type (baseline: Government bond)				
Bank bond	-0.00865	0.000641	0.00486***	0.00933***
	(-1.53)	(0.09)	(7.79)	(9.11)
Monetary stabilization bond	0.0149**	0.0213**	0.00146*	0.00621***
	(2.08)	(2.49)	(1.95)	(5.27)
Special bond	-0.00380	0.0201**	0.00683***	0.00974***
	(-0.29)	(2.13)	(8.77)	(9.33)
Financial bond	0.0178	0.0251**	0.0393***	0.0439***
	(1.45)	(2.15)	(34.71)	(35.49)
Municipal bond	0.0558***	0.0367***	0.00440***	0.00489***
	(9.12)	(8.63)	(5.65)	(6.19)
Corporate bond	0.0442**	0.0511***	0.106***	0.0991***
	(2.19)	(2.68)	(13.08)	(12.89)
Equity	38.40***		1.066***	
	(61.04)		(17.92)	
Remaining maturity type (base: $\leq$ 3-month)				
3-month to 6-month		0.0135***		0.00193***
		(2.67)		(3.93)
6-month to 1-year		0.0172***		0.00171***
		(2.64)		(3.10)
1-year to 2-year		0.0325***		0.00374***
		(4.43)		(5.04)
2-year to 5-year		0.0406***		0.00961***
		(4.83)		(8.05)
Over 5 years		0.0203*		0.00840***
		(1.68)		(5.06)
Date FE	Yes	Yes	Yes	Yes
Cluster Var.	Collateral	Collateral	Collateral	Collateral
	+ Date	+ Date	+ Date	+ Date
	(17,249)	(15,216)	(17,249)	(15,216)
N.Obs	7,393,437	7,005,499	7,393,437	7,005,499
Adj. R^2	0.728	0.090	0.965	0.971
Sample	Full	FnGuide	Full	FnGuide

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. Columns (1) and (2) have *haircut* as the dependent variable in contrast to Columns (3) and (4) whose dependent variable is *spread*. Different from Columns (1) and (3) employing the full sample, Columns (2) and (4) use the subsample that is matched with the security information data from FnGuide. The numbers in parentheses in the cluster variable row are the number of clusters.

haircuts and spreads.

In this table, types of collateral assets as well as those of borrowers and lenders are presented in the decreasing order in their shares in total observations. As shown in Column (2), there appears to be no clear pattern in the size of the haircut across the perceived risk of participants proxied by the types of borrowers and lenders. For example, the average haircut applied to the repo with government and domestic banks as a cash borrower is higher than that with securities companies, although the default risk of the former is unlikely larger than the latter. This finding is in contrast to the evidence found in the bilateral repo market (e.g., Baklanova et al. (2019) and Auh and Landoni (2022)) that haircuts systematically vary across the types of participants and highlights the difference between the tri-party and bilateral repo markets.

Regarding the role of contractual terms in determining a repo haircut, the haircut increases in the size of loans, which is consistent with Suzuki and Sasamoto (2022). A repo contract with longer maturity tends to have a higher haircut as in Julliard et al. (2022) and Suzuki and Sasamoto (2022). Importantly, unlike borrower or lender types, the results related to collateral quality show a clear pattern. Haircuts increase with the perceived riskiness of collateral assets, which is consistent with the previous findings (Gorton and Metrick (2012); Auh and Landoni (2022); Julliard et al. (2022); Suzuki and Sasamoto (2022)). For example, compared to government bonds (presumably the safest asset in our sample), a repo contract using riskier asset classes is associated with a higher haircut. Similarly, the longer the remaining maturity of a collateral asset is, the higher haircut is observed.

As in the case of haircuts, we do not find clear systematic variation in the repo spreads across the types of borrowers and lenders (see Column (4)). If anything, domestic banks and specialized credit finance companies borrow at a lower interest rate than others in the Korean repo market. Unlike the positive relationship between the haircut and loan size, the spread decreases in loan size, highlighting the distinctive nature between the two in terms of associated risk. Consistent with the findings in Hu et al. (2021) and Suzuki and

Sasamoto (2022), the spread tends to increase in the maturity of repo contracts.

Regarding collateral quality, the analysis of the repo spread paints a similar picture. While a repo contract with government bonds is associated with the lowest spread, the spread systematically increases with the perceived risk of collateral assets. For example, a repo contract using other kinds of safe assets, such as bank bonds, special bonds, and municipal bonds has a higher spread but the gap is only marginal. The difference in the spread is much larger for risky assets. Consistent with the findings on haircuts, a repo backed by collateral with a longer remaining maturity has a higher spread. A similar finding was also obtained in Auh and Landoni (2022).

2.2.3 Relationship between haircut and interest rate

Figure 2 shows the unconditional correlation between haircuts and spreads, revealing that a repo contract with a higher haircut is associated with a higher interest rate. This finding can also be anticipated from the findings in Table 2 in the previous section because both the haircut and the spread increase with the perceived risk of the collateral. This positive unconditional relationship is the first main empirical finding and it aligns with the intuition that increased collateral risk should be compensated by a higher haircut, a higher interest rate, or both. This finding is formally substantiated in a regression framework by estimating equation (1).

Table 3 presents the regression results where various fixed effects are sequentially incorporated to account for confounding factors.[13] As expected from Figure 2, the positive correlation is evident when none of the confounding factors is controlled (see Column (1)). When the interaction between various contract characteristic-fixed effects with date-fixed effects is successively absorbed from Columns (2) to (7), the positive relationship between the haircut and the spread and the negative relationship between the haircut and

[13]To ensure the robustness of our analysis, we exclude contracts with extreme repo rates (higher than 10%) or extreme haircuts (lower than 0% or higher than 200%), amounting to a total of only 29 contracts. This step aims to mitigate the impact of outliers on our results. The mean and standard deviation of haircuts are 4.99% and 1.17%, respectively, while for repo spreads, they are 0.06% and 0.96%, respectively.

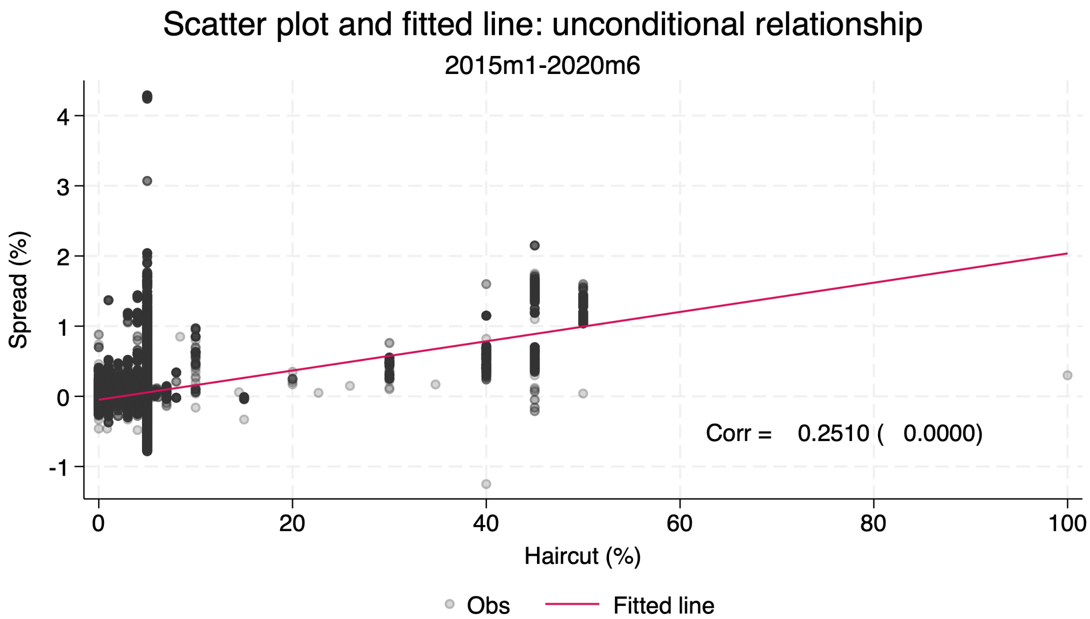

Figure 2: Unconditional relationship between haircut and spread

loan size continue to hold. Importantly, when the *collateral type*-fixed effect is absorbed (Column (7)), these relationships become much weaker and statistically insignificant.

To understand the positive relationship between the haircut and the spread, it is crucial to understand the connection between the quality of posted collateral, borrower default risk, and the lender's expected payoff. Higher collateral risk intensifies the borrower's incentive to default and default makes collateral the property of the lender. Consequently, the diminished expected value of the collateral, owing to poor quality, adversely affects the expected payoff of the lender. Lenders, therefore, are inclined to accept the contract terms only if they are favorable enough to compensate for the associated risk. This rationale is also evidenced by the results in the bottom panels of Table 2.

Intuitively, consider a scenario with only two assets available for a repo contract, in which one asset has higher quality (lower risk) than the other. If a borrower posts the lower-quality asset as collateral, a lender will accept it if the interest rate and/or haircut are higher than those in a contract involving the higher-quality asset for a given loan size.

Table 3: Main analysis: haircut, spread, and collateral quality

Dependent variable: Haircut	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Baseline (8)
Spread	2.989***	3.863***	2.637***	2.654***	3.276***	2.899***	0.349	-1.361***
	(5.72)	(5.72)	(4.28)	(4.25)	(4.83)	(4.88)	(0.83)	(-9.67)
Size	-0.0187***	-0.0172***	-0.0241***	-0.0196***	-0.0175***	-0.0134***	-0.00286	0.000429
	(-5.48)	(-4.90)	(-8.02)	(-6.22)	(-5.64)	(-4.97)	(-1.33)	(0.47)
Date FE		Yes						
Maturity type × Date FE			Yes	Yes	Yes	Yes	Yes	Yes
Borrower type × Date FE				Yes	Yes			
Lender type × Date FE					Yes			
Borrower type × Lender type × Date FE						Yes	Yes	Yes
Collateral type × Date FE							Yes	
Collateral × Date FE								Yes
Cluster Var.	Collateral + Date (17,249)	Collateral + Date (17,249)	Collateral + Date (17,248)	Collateral + Date (17,248)	Collateral + Date (17,248)	Collateral + Date (17,245)	Collateral + Date (17,241)	Collateral + Date (13,547)
N.Obs	7,393,438	7,393,438	7,393,146	7,392,903	7,392,008	7,388,320	7,387,971	6,162,332
Adj. R^2	0.064	0.087	0.490	0.500	0.533	0.606	0.828	0.647

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

Conversely, for a given haircut, the interest rate should be higher, or the loan size should be smaller, or both, for the lender to participate in the contract.

This poses an empirical challenge for identification. Failing to account for the quality of underlying collateral can introduce an omitted variable bias in estimating the relationship among contract terms. As shown in Column (7), controlling for asset classes cannot fully eliminate omitted variable bias, indicating that distinct collateral within the same asset class may possess different properties, such as the issuing entity and time-to-maturity. Consistent with our interpretation, a noteworthy reversal in these relationships occurs when collateral-specific fixed effects are included (Column (8)). Here, the coefficient for the haircut becomes negative, and that for loan size becomes statistically insignificant. In other words, the lack of consensus on the relationship between the haircut and the interest rate is likely driven by the lack of exact information on the quality of collateral and emphasizes the merit of our data with this information.

Once conditioning on the same collateral quality, there is a clear substitution effect between the interest rate and the haircut. This is our second main empirical finding that reconciles the contradicting findings in the literature. Intuitively, this scenario (i.e., conditioning on the same collateral quality) corresponds to a theoretical setup where only one asset can be used as collateral for a repo contract. In such an environment, a lender cannot simultaneously demand both a higher interest rate and a higher haircut at the expense of risk, as in the case of two assets. Consequently, the lender accepts a lower interest rate for a higher haircut, balancing the expected payoff for a given loan size.

Table 4 presents the baseline results again but separately for the full sample and the sample with one-day maturity only given that 95% of contracts in the institutional repo market are at this maturity. These results are presented in the first two columns and are quantitatively similar to the baseline results. In terms of economic magnitudes, a 1% point increase in the spread is related to a 1.3% point decline in the haircut when everything else is constant. This is not only a statistically significant but economically meaningful relationship. As expected from the inclusion of date-fixed effects, the results using the interest rate instead (Columns (3) and (4)) are nearly identical to the baseline.

In Table A-4, we conduct a test of dropping each group of fixed effects from the baseline model reported in Column (1) of Table 4. This exercise highlights that the key to uncovering the substitution effect in a repo contract is the collateral-fixed effect available from the unique feature of our data: The negative relationship between the haircut and the interest rate remains even without controlling for borrower and lender types, as long as collateral-specific fixed effects are absorbed. This finding suggests that the economic mechanism through which collateral risk influences repo terms can differ from the way repo terms are affected by risk associated with trading agents, which requires further theoretical investigation.

Table 4: Main analysis: full sample vs. 1-day maturity only

	Baseline		Repo rate	
Dependent variable: Haircut	(1) All maturities	(2) 1-day	(3) All maturities	(4) 1-day
Spread	-1.361***	-1.369***		
	(-9.67)	(-9.95)		
Repo rate			-1.361***	-1.369***
			(-9.67)	(-9.95)
Size	0.000429	0.000595	0.000429	0.000595
	(0.47)	(0.65)	(0.47)	(0.65)
Maturity type × Date FE	Yes	Yes	Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes	Yes	Yes
Collateral × Date FE	Yes	Yes	Yes	Yes
Cluster Var.	Collateral + Date (13,547)	Collateral + Date (13,347)	Collateral + Date (13,547)	Collateral + Date (13,347)
N.Obs	6,162,332	5,917,734	6,162,332	5,917,734
Adj. R^2	0.647	0.279	0.647	0.279

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

2.2.4 Robustness checks

To ensure the robustness of our main findings, we perform several sensitivity tests. First, we introduce additional multi-way fixed effects to account for potential confounding factors not absorbed in the baseline model. The nuanced nature of negotiations in the repo market implies that various facets of a contract may interact to determine its terms. For example, there might be systematic variations in the type of collateral used in repo contracts for a specific pairing of borrowers and lenders, possibly due to longstanding relationships and market practices. Such practices may vary over time as well. In this regard, the most demanding set of fixed effects is Collateral×Borrower×Lender×Date, which is designed to absorb such variations flexibly. In addition to the baseline fixed effects included in Column (1) of Table 4, we include a more demanding set of fixed effects in turn. As displayed in Table A-5 in Appendix A, the statistically significant negative relationship between the haircut and the spread conditional on collateral quality still remains in every case.

Second, to address the possibility that the trade-off is driven by a specific type of collateral, limiting the generalization of our findings, we systematically exclude each collateral type from the estimation in turn. Table A-6 presents the results (Columns (2) to (10)), confirming the resilience of our conclusions. Regardless of the type of excluded assets, the statistical significance and size of the main coefficients are remarkably stable.

Third, we present the estimation results using alternative standard error clustering methods. First, we single cluster standard errors at collateral×date level. Second, standard errors are double clustered at collateral and year×month and day×*dow* (where *dow* denotes the day of the week) levels. In the last two columns, standard errors are double clustered at collateral×(year×month) and day×*dow* levels. Irrespective of the level of standard error clustering, the statistically significant substitution effect between the haircut and the spread remains robust (see Table A-7 in Appendix A.)

2.2.5 Uncertainty and substitutability between haircut and interest rate

Now we assess the stability of the trade-off between the haircut and the interest rate over time. As market conditions change, they may influence the way haircuts and interest rates interact with each other in contract terms. To obtain a visual summary, we estimate equation (1) in a rolling window basis, employing a one-month window. The coefficients on the spread variable for each month, along with their corresponding 95% confidence intervals, are presented in Figure 3. The red horizontal line indicates the average of the rolling-window coefficients.

The consistently negative and statistically significant coefficients confirm the temporal stability of the negative relationship between the haircut and the spread. However, it appears that its slope changes over time in a systematic way. The (absolute) size of the slope (i.e., substitutability) tends to decrease during a period of heightened market uncertainty—proxied by the VKOSPI (implied volatility of the Korean stock market, corresponding to the VIX in the U.S. stock market). The correlation between the monthly coefficient β and the VKOSPI is 0.40 and highly statistically significant.

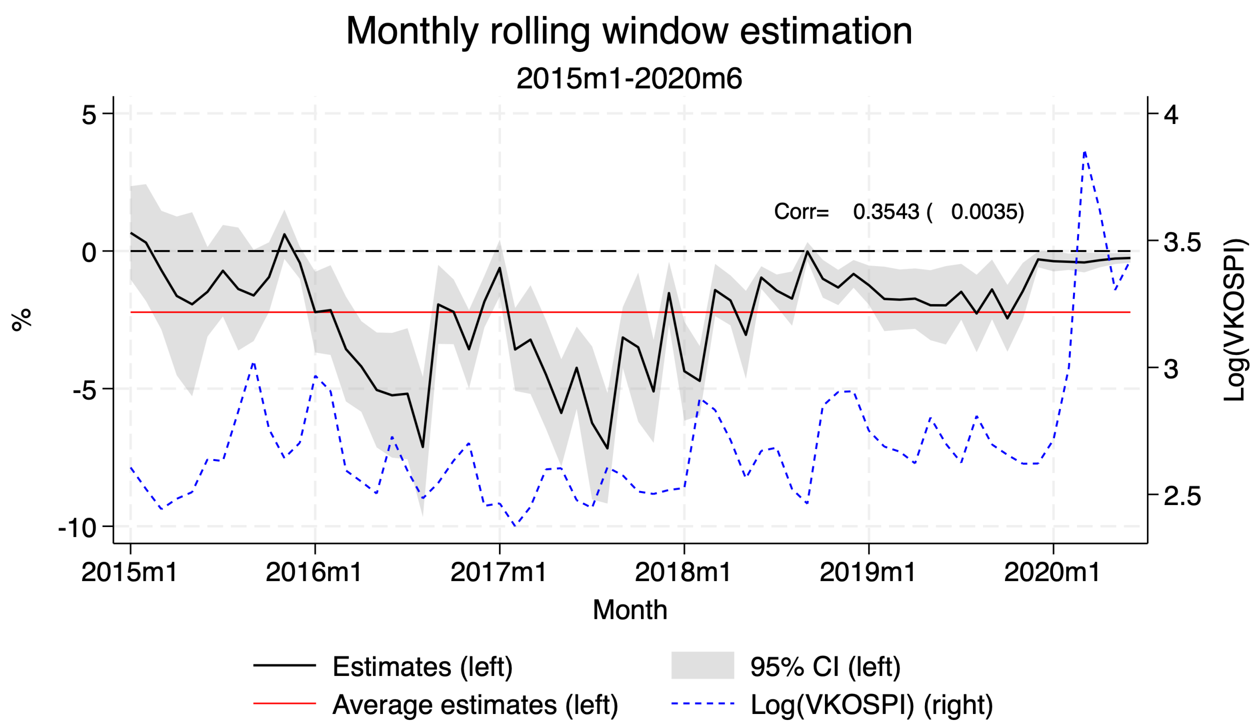

Figure 3: Time-varying relationship: rolling-window estimation

To draw a more robust conclusion about the haircut-interest rate relationship and uncertainty, we estimate the following regression:

$$
\begin{aligned}
Haircut_{B,L,M,c,t} =& \beta_1 Spread_{B,L,M,c,t} + \beta_2 Spread_{B,L,M,c,t} \times Unc_t \\
&+ \gamma_1 Size_{B,L,M,c,t} + \gamma_2 Size_{B,L,M,c,t} \times Unc_t + \Upsilon_{B,L,M,c,t} + \epsilon_{B,L,M,c,t}, \qquad (3)
\end{aligned}
$$

where the spread and loan size variables are further interacted with a time-varying measure of uncertainty. If the increased uncertainty indeed lowered the substitutability of the haircut to the interest rate in a given repo contract, we would observe a positive sign of the coefficient β_2.

Table 5 summarizes the results of estimating equation (3). As shown in the first column, the coefficient on the spread variable (β_1) is still negative, whereas that on the interaction term (β_2) is positive and highly statistically significant, lending further support to the suggestive evidence in Figure 3. A one standard deviation increase in uncertainty measured by VKOSPI reduces the sensitivity of the haircut to the spread by 28%, which is economically significant. This systematic variation in the substitutability between the haircut and the interest rate constitutes the third main finding, warranting further theoretical exploration.

To confirm the robustness of this finding, (i) we lag the VKOSPI variable (to mitigate reverse causality); (ii) use the monthly-averaged VKOSPI variable (to smooth excessive volatility at a daily frequency); (iii) employ the pre-COVID19 sample only (to minimize the influence of outliers); (iv) use the U.S. implied stock market volatility (VIX) instead (to address an endogeneity concern that the repo market development drives stock market volatility); (v) use interest rate volatility (to proxy uncertainty more relevant to bond prices); (vi) use alternative measures from the real economy (output growth) given the countercyclical nature of time-varying uncertainty (Bloom (2014)). As shown in Columns (2) to (7) in Table 5, the significant role of uncertainty in dampening the substitutability of the haircut to the interest rate is highly robust.

3 Theoretical Model

Our empirical analysis has yielded several intriguing findings about contract terms in the tri-party repo market. First, we observe an unconditional positive relationship be-

Table 5: Main analysis: uncertainty and haircut-spread relationship

	(1) Log(VKOSPI)	(2) 1-day lag	(3) Monthly average	(4) Before COVID	(5) Log(VIX)	(6) 10-year KTB volatility	(7) IP
Spread	-6.469***	-6.362***	-6.582***	-7.875***	-5.883***	-2.533***	-1.250***
	(-8.37)	(-8.33)	(-8.33)	(-4.11)	(-8.80)	(-3.66)	(-10.21)
Spread × Uncertainty	1.760***	1.722***	1.796***	2.240***	1.536***	0.990*	-0.171***
	(7.43)	(7.37)	(7.42)	(3.20)	(7.73)	(1.78)	(-7.16)
Size	0.000676	0.000460	0.000646	-0.00528	-0.000180	-0.00358	0.000502
	(0.21)	(0.14)	(0.18)	(-0.67)	(-0.07)	(-1.36)	(0.61)
Size × Uncertainty	0.0000137	0.0000923	0.0000255	0.00235	0.000316	0.00339*	0.0000679
	(0.01)	(0.09)	(0.02)	(0.82)	(0.39)	(1.78)	(0.53)
Maturity type × Date FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Collateral × Date FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Cluster Var.	Collateral + Date (13,547)	Collateral + Date (13,547)	Collateral + Date (13,547)	Collateral + Date (12,039)	Collateral + Date (13,547)	Collateral + Date (13,547)	Collateral + Date (13,547)
N.Obs	6,162,332	6,162,332	6,162,332	5,211,428	6,162,332	6,162,332	6,162,332
Adj. R^2	0.649	0.649	0.649	0.651	0.649	0.648	0.649

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. The columns *Log(VKOSPI)*, *1-day lag*, and *Monthly average* use the logarithm of VKOSPI of the current value, 1-day lagged value, and monthly averaged value for *uncertainty*, respectively. The column *Before COVID* uses the logarithm of VKOSPI of the current value for *uncertainty* focusing on a subsample from 2015m1 to 2019m12. The column *Log(VIX)* uses the logarithm of VKOSPI of the current value for *uncertainty*. The column *10-year KTB volatility* uses the logarithm of the conditional standard deviation of daily change in market yields of 10-year KTB estimated by GARCH(1,1) for *uncertainty*. The column *IP* uses the year-over-year growth rate of industrial production for *uncertainty*.

tween the repo haircut and the interest rate (or spread). To the extent that this positive relationship continues to hold when further controlling for types of maturities, borrowers, lenders, and collateral, one may conclude that the haircut and the interest rate are complements not substitutes in repo contract terms.

Second, however, we uncover a trade-off between the haircut and the interest rate after fully controlling for collateral risk owing to the advantage of our dataset. This finding confirms and extends the preliminary evidence found in Baklanova et al. (2019) and provides a rationale for why other studies without precise security-level information (e.g., Suzuki and Sasamoto (2022); Barbiero et al. (2024)) often found different results.

Third, we further document that the substitutability of the haircut to the interest rate decreases in the degree of market uncertainty. In other words, for a given increase in the repo rate, the reduction in the haircut in a repo contract is smaller in bad times. This finding has not been documented in the existing literature and sheds light on how the trade-off between the haircut and the interest rate responds to default risk.

In this section, we construct a model of collateralized debt to explore the economic mechanisms underlying our empirical findings within a unified framework.

3.1 Environment

We consider an exchange economy that consists of two risk-neutral agents, namely a borrower (b) and a lender (l), and two periods, $t \in \{0, 1\}$. The utility of each agent is

$$U^b = mc_{b0} + c_{b1}$$
$$U^l = c_{l0} + c_{l1},$$

where c_{it} is the consumption of goods by the agent $i \in \{b, l\}$ in period t and m is the marginal utility of the borrower from consuming goods in period 0. We assume that $m > 1$, which can be interpreted as the borrower having liquidity needs in period 0.

There is a single nondurable consumption good in each period and its endowment process is as follows. In period 0, the lender is endowed with e units of consumption goods and the borrower does not receive consumption goods. In period 1, the lender receives nothing, while the borrower is endowed with e units of consumption goods with a probability of $1 - \omega$ and receives nothing with the complementary probability ω.

In addition to consumption goods, the borrower is endowed with a units of the divisible Lucas tree at $t = 0$. Trees are subject to a shock: with probability $1 - \sigma \in (0, 1]$, a tree yields y units of consumption goods at the end of period 1. Otherwise, it pays nothing. Letting $\overline{y} = (1 - \sigma)y$, we keep $\overline{y}$ constant throughout the paper. Hence, an increase in σ

implies a mean-preserving spread of the trees, so σ captures the risk of the borrower's asset.

In period 0, there are gains from trading due to the borrower's liquidity needs, and we assume that the borrower makes a take-it-or-leave-it offer to the lender in period 0. However, unsecured credit is not feasible because of a limited commitment problem. Therefore, trees are necessary as a medium of exchange for trade to occur in period 0. The borrower can finance their liquidity needs in period 0 using trees in one of two ways. On the one hand, they can sell a certain quantity of trees to the lender in exchange for consumption goods (an asset sale). On the other hand, they can pledge trees as collateral to borrow consumption goods from the lender (collateralized debt). We assume that $e > ya$ so that the borrower has no restriction on trading all the trees she has if she wishes.

Costly information acquisition An economic agent may desire to acquire more information about the future value of an asset before purchasing or selling it. For example, an agent may obtain analytical reports regarding a firm's financial statements and prospects, which could provide more precise insights into the value of its equity share before making a transaction, even if she holds a common perception about the expected value of the firm's equity. Even for safe assets such as government bonds, their prices—particularly of long-term securities—can change significantly in response to macroeconomic shocks. Thus, an agent still has incentives to acquire forecasts of aggregate economic conditions, the monetary policy stance, or any market conditions that could affect security prices.

Since it is costly to purchase or produce such information, we assume that the lender can obtain private information regarding the quality of the tree at a cost before trading with the borrower in period 0, similar to Dang et al. (2013) and Gorton and Ordoñez (2014). Specifically, the lender must incur $\gamma > 0$ units of disutility in period 0 to obtain this information. In principle, the borrower may also desire to acquire information about the quality of her assets at a cost. However, in Appendix C, we demonstrate that the borrower never acquires such information at a cost. Thus, we simply assume that the borrower cannot obtain information about the tree's quality at a cost without loss of generality.

Defaults on collateralized debt In practice, a borrower may default on a collateralized debt for two reasons. First, she is unable to repay the loan because of insufficient resources. In the model, the borrower does not receive any consumption goods in period 1 with probability ω, and thus she cannot make a repayment on the loan. Thus, ω represents the probability of exogenous default on collateralized debt.

Second, borrowers may default on collateralized debt even though they have sufficient resources because it is profitable to do so. For example, in the real world, on the payment due date of collateralized debt, a borrower can observe the current market price of the collateral asset in spot markets and may default opportunistically if the collateral value is significantly lower than the repayment amount. To introduce this type of default into the model in a simple way, we assume that the information about the dividend state is revealed at the beginning of period 1 before the settlement of any debts. Thus, the borrower can decide whether to repay the loan in period 1 based on the revealed information, similar to the approach in Barro (1976).[14]

3.2 Bargaining problem in period 0

In this subsection, we focus on collateralized debt and later compare it with asset sales. Collateralized debt consists of three terms, (q, p, a'). In period 0, the borrower receives

[14]One may argue that this opportunistic default rarely occurs because trades among the market participants are not a one-shot game in real repo markets: Participants in these markets repeatedly enter into repo contracts over time. Specifically, if we construct our model such that the bargaining game between borrowers and lenders is infinitely repeated, and agents exhibit sufficient patience, then an equilibrium without opportunistic defaults can be achieved, given that the borrower's payoff from the lender's perpetual minmax strategy is zero. However, even in such a repeated game setup, an equilibrium still exists wherein the borrower defaults opportunistically whenever feasible. More importantly, the absence of default in repo markets in Korea does not necessarily imply there is no risk of potential opportunistic default. It is still possible that opportunistic default occurs, particularly when asset prices plunge during a severe financial crisis. Furthermore, what matters for our analysis is the existence of potential opportunistic default risk, not ex-post default. Later, we discuss how to modify the model with a continuum of agents so that defaults do not occur ex-post, while the potential default risk still affects the terms of the contract.

q units of the consumption goods from the lender and promises to repay p units of the consumption goods in period 1. At the same time, the borrower posts a' units of the trees as collateral in period 0. Incomplete payment entitles the lender to possess the collateral trees that belonged to the borrower.

The lender's payoff from accepting a contract (q, p, a') depends on whether the lender acquires costly information about the quality of the trees before trading with the borrower. Specifically, if the lender does not acquire the information, her expected payoff, π_{IIS}, is

$$\pi_{IIS} = -q + (1-\omega)(1-\sigma)p + \omega\overline{y}a', \tag{4}$$

and the lender's expected payoff, π_{IS}, with the information acquisition is given as

$$\pi_{IS} = (1-\sigma)\left[-q + (1-\omega)p + \omega y a'\right] - \gamma. \tag{5}$$

Note that when the lender does not check the collateral quality, the borrower makes the repayment p only if the borrower receives the consumption goods and the tree is good, as the borrower defaults opportunistically if the dividend state is bad. On the other hand, if the lender monitors the tree's dividend state in advance, the borrower cannot default opportunistically because the lender accepts the borrower's offer only if the dividend state is good.

Assuming that the lender's participation constraint holds, if $\pi_{IIS} \geq \pi_{IS}$, the lender accepts the borrower's offer without information acquisition, and if $\pi_{IIS} < \pi_{IS}$, the lender acquires the costly information and decides whether to accept the borrower's offer based on that information. Following Gorton and Ordoñez (2014), we label that collateralized debt is information sensitive (IS) if it triggers information acquisition by the lender and information insensitive (IIS) otherwise.

The borrower's maximized surplus from IIS collateralized debt, V_{IIS}, is given by

$$V_{IIS} = \max_{q,p,a'} \{mq - (1-\omega)(1-\sigma)p - \omega\overline{y}a'\} \tag{6}$$

subject to

$$-q + (1-\omega)(1-\sigma)p + \omega\overline{y}a' \geq 0 \tag{7}$$

$$-\sigma q + \gamma \geq 0 \tag{8}$$

$$ya' - p \geq 0 \tag{9}$$

$$a - a' \geq 0 \tag{10}$$

$$q, p, a' \geq 0. \tag{11}$$

Here, the inequality (7) is the lender's participation constraint without information acquisition. (8) is the constraint to avoid information acquisition, which means that the lender's payoff with information acquisition (5) should not be higher than the payoff without information (4), i.e., $\pi_{IIS} \geq \pi_{IS}$. (9) is the borrower's incentive constraint to make repayment when she has the resources to make repayment and knows that tree yields dividends. Finally, (10) and (11) are the feasibility constraints.[15]

Next, the borrower's maximized value from IS collateralized debt, V_{IS}, is given by

$$V_{IS} = \max_{q,p,a'} \{(1-\sigma)[mq - (1-\omega)p - \omega ya']\} \tag{12}$$

[15] The lender does not receive any endowment in period 1, so p must be non-negative.

subject to

$$(1-\sigma)\left[-q+(1-\omega)p+\omega ya'\right]-\gamma \geq 0 \tag{13}$$

$$\sigma q-\gamma \geq 0 \tag{14}$$

$$ya'-p \geq 0 \tag{15}$$

$$a-a' \geq 0 \tag{16}$$

$$q, p, a' \geq 0. \tag{17}$$

The inequality (13) is the lender's participation constraint with information acquisition. (14) is the constraint that induces the lender to acquire information about the collateral quality. Constraints (15) - (17) are equivalent to (9) - (11).

The optimal collateralized debt is then obtained by comparing V_{IIS} and V_{IS}. The next proposition describes the specific contract terms.

Proposition 1 *Define the cutoff levels of the information acquisition cost as*

$$\gamma^* \equiv \sigma\overline{y}a \text{ and } \gamma^{**} \equiv \frac{(m-1)\gamma^*}{m(1+\sigma)-1}. \tag{18}$$

Then, γ^* and γ^{**} increase in σ, and the optimal collateralized debt is as follows:

1) *[Fully-IIS] If $\gamma^* \leq \gamma$, then the information is not produced, $q=\overline{y}a$, $p=ya$, and $a'=a$.*
2) *[Partially-IIS] If $\gamma \in [\gamma^{**}, \gamma^*)$, then the information is not produced, $q=\frac{\gamma}{\sigma}$, and a pair $(p,a') \in \left[\max\left\{0, \frac{\gamma-\omega\gamma^*}{(1-\omega)\sigma(1-\sigma)}\right\}, \frac{\gamma}{\sigma(1-\sigma)}\right] \times \left[\frac{\gamma a}{\gamma^*}, \min\left\{a, \frac{\gamma a}{\omega\gamma^*}\right\}\right]$ is determined by*

$$\frac{\gamma}{\sigma}=(1-\sigma)[(1-\omega)p+\omega ya']. \tag{19}$$

3) *[IS] If $\gamma \in [0, \gamma^{**})$, then the information is produced, $q=ya-\frac{\gamma}{1-\sigma}$, $p=ya$, and $a'=a$.*

Proof. See Appendix B. ■

If the information acquisition cost, γ, is sufficiently low such that $\gamma < \gamma^{**}$, then it is optimal for the borrower to let the lender produce information about the dividend state. In this case, a trade occurs only if the dividend state turns out to be good. Thus, there is no informational friction, and the borrower pledges all trees as collateral, namely $a' = a$, and sets $p = ya$ to maximize the loan size q. However, the borrower must compensate for the information acquisition cost γ, which reduces q. The loan size also decreases with σ because a larger σ implies a lower probability of trade and the lender's information acquisition cost is covered only when a trade occurs.

On the other hand, if $\gamma \geq \gamma^{**}$, then it is too costly to check the dividend state in advance. Thus, the contract type is IIS. The specific terms of IIS collateralized debt depend on whether the constraint to avoid information acquisition (8) binds or not. First, if $\gamma \geq \gamma^{*}$, the constraint (8) does not bind (Fully-IIS). In this case, the borrower pledges all trees as collateral, namely $a' = a$, and sets $p = ya'$ to maximize the loan size q. Second, if $\gamma < \gamma^{*}$, the constraint (8) binds, which limits the trade volume. It implies that if the borrower increases the size of the loan by an additional unit, the borrower will likely acquire information. Consequently, collateralized debt is partially-IIS. An increase in σ (or a decrease in γ) intensifies the lender's information acquisition incentive, causing the loan size to decrease in σ while increases in γ.

An important property of partially-IIS collateralized debt is that there are infinite numbers of partially-IIS collateralized debt with different contract terms: The repayment value p and quantity of collateral trees a' are simultaneously determined by equation (19), so there are infinite pairs of (p, a'). The intuitive explanation is as follows. In general, p and a' affect the loan size q through their effects on the lender's payoff; however, q of partially-IIS contract is pinned down by the binding constraint to avoid information acquisition (8). (p, a') only affects the borrower's expected repayment. Thus, the borrower is indifferent about the choice of (p, a') as long as it gives a zero surplus in expectation to the lender, given the loan size $q = \frac{\gamma}{\sigma}$. However, when $\omega\gamma^{*} \geq \gamma$, the loan size is low enough that a'

cannot exceed $\frac{\gamma a}{\omega\gamma^*}$, as p would otherwise become negative. On the other hand, if $\omega\gamma^* < \gamma$, the loan size is sufficiently high, so p must be at least $\frac{\gamma-\omega\gamma^*}{(1-\omega)\sigma(1-\sigma)}$ because a' cannot exceed the borrower's asset holding a.

Optimality of collateralized debt Thus far, our focus has been on characterizing the optimal collateralized debt contract. However, the borrower can also raise funds by selling trees on the spot: sell a'_s units of the trees to the lender in exchange for q_s units of the consumption goods in period 0. The terms of optimal tree sales (q_s, a'_s) can be obtained by imposing the condition that $\omega = 1$ into the terms of the optimal collateralized debt, because if $\omega = 1$, then the lender always seizes the collateral trees, the same as asset sales. In the proof of Proposition 1, we derive V_{IIS} and V_{IS} explicitly and show that they are independent of ω, as are γ^* and γ^{**}. This implies that collateralized debt and tree sales are equivalent to the borrower in terms of trade surplus.[16]

In reality, financial institutions often need to hold specific assets for hedging purposes or to comply with regulatory requirements. Otherwise, they may face fees for regulatory non-compliance or incur costs for failing to hedge against risks. Now suppose that the borrower incurs $\varepsilon > 0$ units of disutility from failing to hold trees in period 1. Then, collateralized debt contracts dominate asset sales because the borrower relinquishes ownership of pledged trees only in the event of default under the collateralized debt contracts. Furthermore, as long as ε is sufficiently small, the terms of optimal contracts are equivalent to the optimal collateralized debt contract described in Proposition 1. Specifically, we can interpret the result in Proposition 1 as the limiting case with $\varepsilon \to 0$.

3.3 Model's predictions about haircuts and interest rates

We now explore how our model's predictions align with empirical findings about the repo market documented in section 2. For this purpose, we extend the model as follows. We

[16]The equivalence result between collateralized debt and asset sales was also shown by Lagos (2011) under symmetric information and by Rocheteau (2011) under asymmetric information.

assume that there is a continuum of borrowers (b) and lenders (l), each with unit mass. Their preferences and endowment processes remain consistent with the baseline model. In period 0, each borrower holds a units of trees. Borrowers exhibit heterogeneity regarding the exogenous default probability ω and the riskiness of their trees σ. Specifically, each borrower randomly draws ω and σ from finite sets $\Omega \subset [0, 1]$ and $\Sigma \subset [0, 1]$, respectively, at the beginning of period 0. Similarly, each lender draws γ from a finite set $\Gamma \subset \mathbb{R}_+$ at the beginning of period 0, so they differ in terms of information acquisition costs.

In period 0, there are bilateral meetings between a borrower and a lender wherein the borrower offers a contract as in the baseline model.[17] Note that there are $|\Omega| \times |\Sigma| \times |\Gamma|$ different groups of bilateral meetings and each meeting is characterized by $(\omega, \sigma, \gamma) \in \Omega \times \Sigma \times \Gamma$. Here, ω, σ, and γ capture the characteristics of the borrower, lender, and collateral trees, respectively. We assume that both the borrower and the lender in a meeting know (ω, σ, γ) of their meeting. Then, given (ω, σ, γ) in each meeting, the results of Proposition 1 delineate the optimal contract between the matched borrower and lender. We assume that if there are multiple optimal contracts for a trade, the borrower randomly chooses one of them.

Discussion: defaults at the aggregate level In the extended model, equilibrium predicts a positive mass of borrowers experiencing defaults, while defaults are rare in the real repo market: our sample period showed no incidents. To see how to resolve this disparity, suppose that when the macroeconomy is good, all borrowers receive consumption goods in period 1. Otherwise, borrowers' incomes depend on their individual shocks. Similarly, assume that all trees yield dividends when the overall financial market conditions are good, while the dividend state of each borrower's tree depends on their individual shocks when the financial market conditions are not good.

[17]Random matching is often assumed in models of over-the-counter markets. Examples include Duffie et al. (2005), Berentsen et al. (2014), Mattesini and Nosal (2016), and Herrenbrueck and Geromichalos (2017). These studies focus on matching frictions in decentralized markets and assets' liquidity, while we focus on the (potential) informational frictions in decentralized markets.

Introducing these aggregate shocks does not change the analysis of the optimal contract in each meeting.[18] However, in this modified environment, when the macroeconomy and overall financial market conditions are good, all borrowers receive consumption goods in period 1, and all trees yield positive dividends. Consequently, there are no exogenous or opportunistic defaults ex-post, aligning the model's prediction with actual experiences in the Korean repo markets.[19]

Haircuts and interest rates Given a collateralized debt contract (q, p, a'), an interest rate is defined as $r = \frac{p-q}{q}$. A haircut is the percentage difference between the value of the collateral tree and loan size, defined as $\theta = \frac{v-q}{v}$, where v is the expected value of collateral in period 0. In what follows, we use $r(\omega, \sigma, \gamma)$ and $\theta(\omega, \sigma, \gamma)$ to represent the equilibrium interest rate and the haircut, respectively, in a meeting with (ω, σ, γ). Additionally, $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ denote the average interest rate and haircut, respectively. The next proposition describes the interest rate and the haircut for each type of collateralized debt.

Proposition 2 *Given (ω, σ, γ), the interest rate r and the haircut θ of each type of collateralized debt are given as follows:*

1) *$r = \frac{\sigma}{1-\sigma}$ and $\theta = 0$ if collateralized debt is fully-IIS.*
2) *r and θ are simultaneously determined by*

$$1 = (1-\omega)(1-\sigma)(1+r) + \frac{\omega}{(1-\theta)}, \tag{20}$$

in the range of $r \in \left[\max\left\{0, \frac{1-\frac{\omega\gamma^*}{\gamma}}{(1-\omega)(1-\sigma)}\right\} - 1, \frac{1}{1-\sigma} - 1\right]$ and $\theta \in \left[0, 1 - \max\left\{\frac{\gamma}{\gamma^*}, \omega\right\}\right]$ if collateralized debt is partially-IIS.

3) *$r = \frac{\gamma}{\bar{y}a-\gamma}$ and $\theta = \frac{\gamma}{\bar{y}a}$ if collateralized debt is IS.*

[18] Introducing these aggregate shocks modifies only the probability that each borrower receives endowments and the probability that each tree yields dividends.

[19] During our sample period studied in section 2, the Korean economy did not experience dramatic shocks in the real or financial sectors that could lead to defaults in the repo market. Thus, the absence of default in our data can be interpreted as a result of the realization of good economic conditions during the sample period.

Proof. See Appendix B. ■

Given the loan size q, a triple (q, r, θ) represents an alternative expression of the contract terms (q, p, a'). Therefore, we refrain from extensively examining the interest rate r and haircut θ for each contract type here, as we have already thoroughly analyzed the terms of contract (q, p, a') in subsection 3.2. Nevertheless, it is worth exploring the factors that influence the presence of haircuts, which we will delve into.

Under fully-IIS debt contracts, the lender has no incentives to acquire information about the quality of the collateral asset. The only problem stemming from informational frictions is opportunistic default. However, the interest rate fully compensates for the risk of opportunistic default, in line with conventional finance theory, which posits that default risk is integrated into interest rates. Thus, haircuts do not come into play, suggesting that the risk of opportunistic default alone cannot account for the presence of haircuts.

In contrast, under the partially-IIS and IS contracts, the lender's incentive for information acquisition matters. Specifically, in the partially-IIS debt contract, the lender does not obtain information about the collateral quality, but the incentive constraint preventing the lender from acquiring information is binding. This implies that the lender will monitor the collateral quality once the borrower increases the loan size, so the binding incentive constraint limits the loan size. Next, under an IS debt contract, the lender indeed acquires the information, and the loan size accounts for the information acquisition cost. Thus, the lender's incentive to acquire information about the collateral quality shapes partially-IIS and IS debt contracts, and it is only when this incentive problem exists that a haircut emerges as an equilibrium outcome.

The result that haircuts are present only when there is a threat or actual acquisition of information about collateral quality is consistent with the findings in previous studies that haircuts account for the risk associated with the future value of the collateral asset.[20]

[20]There are papers, such as Kiyotaki and Moore (1997), Venkateswaran and Wright (2013), and Williamson (2016), that study the macroeconomic implications of haircuts by assuming limited pledgeability of collateral assets, rather than explicitly incorporating informational friction about collateral quality into the model. However, in such models, the haircut is solely determined by a parameter capturing this

For example, Dang et al. (2013), Kang (2021), and Madison (2024) have demonstrated that haircuts can exist in the presence of (potential) asymmetric information regarding asset quality. Additionally, Simsek (2013), Fostel and Geanakoplos (2015), and Barsky et al. (2016) have illustrated the presence of haircuts when there are disagreements regarding the belief in the future value of collateral assets. Kuong (2021) shows that a borrower's moral hazard can also result in haircuts in collateralized loan markets. However, in those models, the set of optimal debt contracts is a singleton. This makes it challenging to explain the trade-off between the haircut and the interest rate conditioning on collateral quality, a topic we explore further below.

In the rest of the paper, we elaborate on how our model can offer a narrative explanation for the main empirical findings in the following order: (i) the negative relationship between the haircut and the repo rate conditional on collateral quality, (ii) the unconditional positive relationship between the two, and (iii) the decrease in the rate at which the repo rate substitutes the haircut during periods of heightened market uncertainty or economic downturn.

In a tri-party repo market where most collateral is a safe asset and most contracts are short-term, lenders may be less inclined to bear the additional costs of verifying the true value of collateral assets, although their incentive to check collateral quality can still influence contract terms. Therefore, IS debt contracts are less likely to be prevalent. Given this rationale, we exclude the existence of IS debt contracts in the following analysis by imposing the following parametric assumption on γ.

Assumption 1 $\gamma > \frac{(m-1)\sigma_{\max}\overline{y}a}{m(1+\sigma_{\max})-1}$, *where* $\sigma_{\max} = \max \Sigma$.

limited pledgeability, failing to elucidate the underlying economic mechanism for its existence. We can also incorporate limited pledgeability into our model. In this case, a fully-IIS debt contract also includes positive haircuts. However, the size of haircuts simply depends on parameters capturing the extent of limited pledgeability and does not provide additional insights. Therefore, we opt not to include limited pledgeability, as it would only complicate the analysis without providing additional insight into the determinants of haircuts.

Trade-off conditioning on collateral quality Column (8) of Table 3 shows that the haircut and the interest rate exhibit a negative correlation when all confounding factors, including borrower traits, lender attributes, and collateral asset characteristics are controlled. From our model's perspective, the above result entails analyzing the relationship between $r(\omega, \sigma, \gamma)$ and $\theta(\omega, \sigma, \gamma)$ within a group of bilateral meetings sharing the same (ω, σ, γ).

Consider a group of bilateral meetings characterized by (ω, σ, γ). If the contract type is partially-IIS in those meetings, participants in each bilateral meeting randomly select a pair (r, θ) that satisfies (20). Because the right-hand side of (20) increases with both r and θ, what we observe in equilibrium is the negative correlation between r and θ when the contract type is partially-IIS. We emphasize that this negative relationship stems from the substitution effects between the interest rate and haircut in the negotiation of repo terms, rather than from any heterogeneity in other factors, as we hold (ω, σ, γ) constant.

The intuition is in line with our earlier observation. A choice of (r, θ) does not affect the loan size q of partially-IIS collateralized debt because q is determined by the binding incentive constraint (8). Thus, the choice of (r, θ) only affects the composition of the lender's payoff. An increase in r means an increase in the repayment p (i.e., the payoff in the non-default state). To break even, the lender's payoff in the default state must decrease. This is achieved by pledging less collateral a', which implies a lower haircut θ. As long as the lender receives zero surplus in expectation, the borrower is indifferent between partially-IIS collateralized debt with high r/low θ and that with low r/high θ.

As explained earlier, under both partially-IIS and IS contracts, the lender's incentive for information acquisition affects contract terms, and both contracts feature a positive haircut. However, only a partially-IIS debt contract involves the substitution effect. The key distinction between these contract types is the existence of the risk of opportunistic default: There is an opportunistic default risk in addition to the threat of information acquisition in the partially-IIS contract, whereas there is no opportunistic default risk in the IS contract because the trade occurs only if the dividend state is good. The substitution effect arises only when both the incentive for information acquisition and the risk of opportunistic default are present.[21]

In our data, the trade-off still remains without controlling for the characteristics of borrowers and lenders, as long as the characteristics of the posted collateral are controlled via ISIN-fixed effects (see Table A-4). To ascertain whether the model provides a comparable prediction, we analyze the relationship between $r(\omega, \sigma, \gamma)$ and $\theta(\omega, \sigma, \gamma)$ within a group of bilateral meetings sharing the same σ but with varying sets of (ω, γ). The subsequent proposition lays the foundation for equilibrium outcomes that align with this observation.

Proposition 3 *1) When ω changes, only one or neither of $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ changes. 2) If both $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ respond to a change in γ, they change in the opposite directions.*

Proof. See Appendix B. ■

The results of Proposition 3 imply that for a given σ, $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ do not move in the same direction simultaneously in response to changes in ω and γ. Combined with the property of partially-IIS debt contracts explained earlier, the results of Proposition 3 provide a basis for the negative relationship between $r(\omega, \sigma, \gamma)$ and $\theta(\omega, \sigma, \gamma)$ after controlling for collateral risk σ but without controlling for ω and γ in the model economy.

To understand the intuition behind this result, consider partially-IIS contracts. An increase in γ raises the loan size, which requires an increase in the repayment p, the quantity of collateral assets a', or both to ensure the lender breaks even. If $\gamma < \omega\gamma^*$, the initial loan size is sufficiently low and the borrower has idle trees that are not used as collateral. In this case, p and a' proportionally increase when the loan size increases, so the pairs (r, θ) for partially-IIS contracts remain unchanged. On the other hand, if $\gamma \geq \omega\gamma^*$, all trees are

[21] Furthermore, if the borrower is unaware of the dividend state before making the repayment decision in period 1, and thus cannot default opportunistically, the interest rate and the haircut of the partially-IIS debt contract affect the loan size. Consequently, the partially-IIS debt contract no longer exhibits the substitution effect.

pledged for the contract with the highest a' and the lowest p. Thus, the borrower can only increase p for this contract when the loan size increases. Consequently, interest rates increase on average, while haircuts decrease. Next, a change in ω only affects the lender's expected payoff, not the loan size of partially-IIS contracts. When ω changes, the borrower adjusts a' for each p when she has idle trees and adjusts p for each a' when she does not, as shown in Proposition 1. Thus, r and θ do not respond simultaneously to changes in ω.

Now consider fully-IIS contracts. Here, r and θ do not depend on ω and γ. However, a decrease in γ can switch the contract type from fully-IIS to partially-IIS. Note, from Proposition 2, that (r, θ) of the partially-IIS contract with the zero haircut is the same as that of fully-IIS as $(r, \theta) = \left(\frac{\sigma}{1-\sigma}, 0\right)$. Then, because r falls along the curve (20) as θ increases, haircuts increase and interest rates fall when the contract type switches from fully-IIS to partially-IIS, generating a negative relationship.

Unconditional positive relationship Another key empirical finding from Table 3 is that there exists a positive relationship between the haircut and the interest rate if the collateral quality is not conditioned via the ISIN-fixed effect. To generate the unconditional positive relationship in the model, it is crucial that both the average interest rate, $\hat{r}(\omega, \sigma, \gamma)$, and the average haircut, $\hat{\theta}(\omega, \sigma, \gamma)$, change in the same direction in response to variations in σ. The next proposition demonstrates that this criterion is satisfied under particular conditions.

Proposition 4 *Suppose that* $\omega < \frac{(m-1)2\sigma}{m-1+(2m-1)\sigma}$. *If both* $\hat{r}(\omega, \sigma, \gamma)$ *and* $\hat{\theta}(\omega, \sigma, \gamma)$ *change in response to alterations in* σ *for a given* (ω, γ)*, they do so in a consistent manner.*[22]

Proof. See Appendix B. ■

If the contract type is partially-IIS, a decrease in σ reduces the lender's information acquisition incentive, thereby increasing the loan size. Thus, if there is a partially-IIS contract in which the borrower pledges all trees, i.e., $a' = a$, then the haircut of this contract,

[22] When the contract type is partially-IIS and $\gamma \approx \gamma^{**}$, if $\omega \geq \frac{(m-1)2\sigma}{m-1+(2m-1)\sigma}$, then changes in σ can cause $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ to shift in opposite directions. However, if γ is sufficiently higher than γ^{**} when the contract type is partially-IIS, then $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ respond in the same way to a change in σ.

and thereby the average haircut, must decrease. However, the repayment of that contract does not need to increase by the amount of an increase in the loan size. This is because a decrease in σ improves the lender's expected payoff by mitigating the opportunistic default risk, in contrast to an increase in γ, which only boosts the loan size. Consequently, the average interest rate also falls. On the other hand, if there are idle trees that are not used as collateral, i.e., $a' < a$, for all partially-IIS contracts, then a' increases in proportion to an increase in the loan size. Consequently, haircuts remain unchanged. The average interest rate decreases because the repayment increases less than the loan size when σ decreases due to the opportunistic default risk channel.

Next, a decrease in σ diminishes the opportunistic default risk, which in turn reduces the interest rate of the fully-IIS contract.[23] Proposition 1 demonstrates that the contract type is fully-IIS for all $\sigma \leq \frac{\gamma}{\overline{y}a}$ and partially-IIS for $\sigma > \frac{\gamma}{\overline{y}a}$ given the assumption 1. Moreover, when $\sigma = \frac{\gamma}{\overline{y}a}$, the average interest rates and haircuts of the fully-IIS and partially-IIS contracts are equal. Then, because the average interest rates of both contract types increase with σ, the average interest rate of the fully-IIS contract is lower than that of the partially-IIS contract. Additionally, the haircut for the fully-IIS contract is zero. Therefore, when the contract type shifts between fully-IIS and partially-IIS in response to changes in σ, both interest rates and haircuts move in the same direction.

The result of Proposition 4 establishes a fundamental basis for deriving the unconditional positive relationship between the haircut and the interest rate. However, the result of Proposition 4 alone is not sufficient to generate an unconditional positive relationship. Even though $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ move in the same direction when σ changes, the model can still yield a negative correlation due to the substitution effect of the partially-IIS debt, provided the variation of σ is sufficiently low given (ω, γ).

To illustrate this, we conduct a numerical simulation with specific parameter values: $a = \overline{y} = 1$, $m = 1.1$, $\gamma = 0.02$, $\omega = 0.01$. Additionally, we choose two sets of σ as $\Sigma_1 =$

[23] Because we keep the expected dividend $\overline{y} = (1 - \sigma)y$ constant, an increase in σ implies an increase in y. Thus, the repayment $p = ya$ of fully-IIS contract increases with σ.

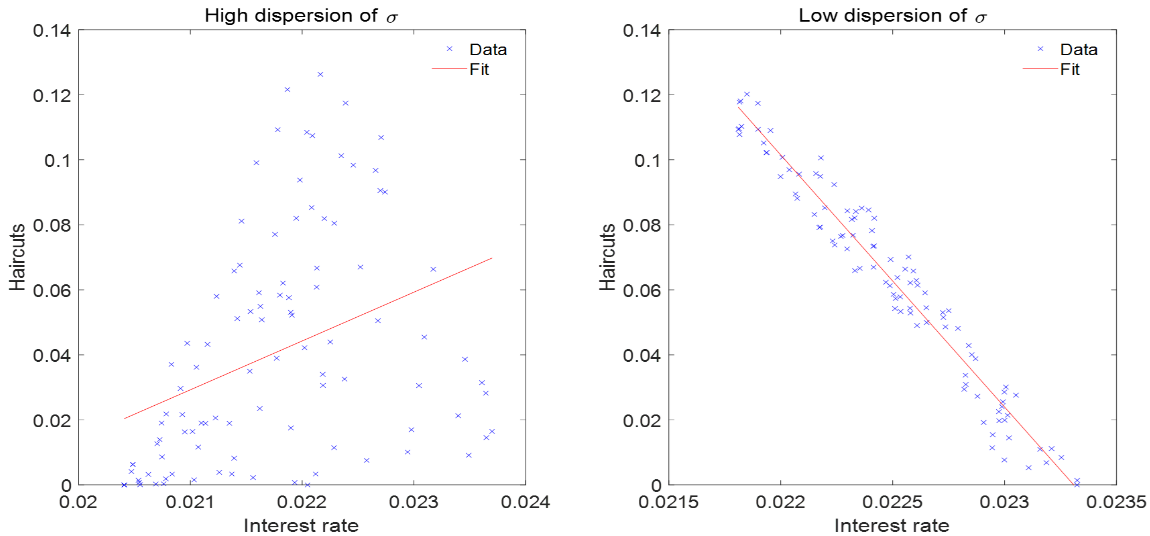

Figure 4: Dispersion of σ and unconditional relation between r and θ

$[0.02, 0.0234]$ and $\Sigma_2 = [0.0225, 0.0228]$ such that for all $\sigma \in \Sigma_i$ and $i \in \{1, 2\}$, the optimal contract type is partially-IIS, given our choices for other parameter values. For each $i \in \{1, 2\}$, we randomly select 100 samples of σ from Σ_i, and then for each σ, we randomly select a pair of (r, θ) that satisfies (20). The left panel of Figure 4 depicts the case when we draw σ from Σ_1, while the right panel illustrates the case when we draw σ from Σ_2. Note that when σ is drawn from Σ_1, it is more widely dispersed than when it is drawn from Σ_2 because $\Sigma_2 \subset \Sigma_1$.

As shown in (20), an increase in σ shifts up the iso-curves of (r, θ) that satisfy (20) given (ω, γ). Thus, when the dispersion of σ is high, the distances among iso-curves of (r, θ) are large. As a result, scatter plots of (r, θ) exhibit a positive unconditional relationship between r and θ, as depicted in the left panel of Figure 4.[24] On the other hand, if the dispersion of σ is low, then the iso-curves of (r, θ) with different triples of (ω, σ, γ) are close to each other. As a result, a negative unconditional relationship between r and θ can still manifest as illustrated in the right panel of Figure 4. These results imply that the positive unconditional relationship tends to exist when the characteristics of collateral

[24] Furthermore, as the dispersion of σ increases so that fully-IIS contracts emerge in meetings for a certain set of (ω, σ, γ), the positive unconditional relation is intensified given the results of Proposition 2.

assets are more diverse.

Uncertainty and trade-off between haircut and interest rate We have demonstrated, as shown in Figure 3 and Table 5, that the rate at which the repo rate substitutes the haircut tends to fall during a period characterized by heightened uncertainty and economic downturn. Through the lens of our model, we can interpret an escalation of uncertainty in financial markets and economic downturn as an increase in σ and ω, respectively. The coefficient for the spread in column (8) of Table 3 is represented by $\frac{\partial \theta}{\partial r}$ in each type of debt contract for a given (ω, σ, γ).

By taking total derivative of (20), we obtain

$$\frac{\partial \theta}{\partial r} = -\frac{(1-\omega)(1-\sigma)(1-\theta)^2}{\omega}, \tag{21}$$

for partially-IIS contracts. Thus, an increase in σ or ω reduces $\left|\frac{\partial \theta}{\partial r}\right|$ consistent with empirical finding in Table 5.

The interest rate r and the haircut θ serve as compensation to a lender in the non-default and default states, respectively. Intuitively, the default probability increases with σ or ω, leading to a higher chance that the lender is compensated with the collateral than repayment. This results in a decrease in the relative contribution of the interest rate to the lender's payoff compared to the haircut, as illustrated in (4). Thus, a larger increase in the interest rate is required to reduce the haircut.

4 Conclusion

In this paper, we undertake an empirical and theoretical exploration of the determination of interest rates and haircuts in a collateralized debt contract. Leveraging transaction-level data from the Korean repo market, we present three key empirical findings: (i) a positive correlation between the haircut and the interest rate when the quality of the underlying collateral is not controlled, (ii) a trade-off between the two, conditioning collateral quality by utilizing data on unique collateral identifiers, and (iii) heightened uncertainty reduces the rate at which the interest rate substitutes the haircut in this trade-off.

To elucidate the economic mechanisms driving these empirical observations, we develop a model of collateralized debt incorporating costly information acquisition and opportunistic default. Specifically, our model demonstrates that when both the lender's incentive to acquire information about collateral quality and the borrower's incentive to default opportunistically affect contract terms, the conditional trade-off between interest rates and haircuts emerges in equilibrium. In addition, an increase in default risk stemming from a rise in uncertainty about collateral quality makes insurance against the default state particularly valuable. As a result, changes in the haircut become less sensitive to the same change in the interest rate.

Specifically, our demonstrates that when lenders refrain from acquiring information about the quality of collateral assets, while the threat of information acquisition restricts the loan size, the conditional trade-off between interest rates and haircuts emerges in equilibrium.

Our findings based on sharpened identification not only reconcile the contrasting findings from the existing empirical studies but offer a new perspective on how market conditions influence the trade-off between the haircut and the interest rate in a repo contract. Testing rich predictions of the model using detailed contract-level data from other markets would be a fruitful area of research to broaden our understanding of how the repo market works.

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Online Appendix

Appendix A: Robustness checks and additional results

Table A-1: Collateral class and loan terms: overnight maturity

Collateral class	Avg. haircut (%)	Med. haircut (%)	Avg. spread (%)	Med. spread (%)	Avg. Principal (bil. of wons)	Med. Principal (bil. of wons)	Avg. maturity (days)	Med. maturity (days)	N. Obs.	Unique ISINs
Government bond	4.96	5	0.05	0.05	39.60	23.3	1.00	1	2,951,543 (41.71%)	539
Bank bond	4.96	5	0.06	0.06	60.03	37.6	1.00	1	1,735,405 (24.52%)	3,764
Monetary stabilization bond	4.96	5	0.05	0.04	45.10	28.2	1.00	1	676,718 (9.56%)	536
Special bond	4.95	5	0.05	0.05	56.44	32.9	1.00	1	640,495 (9.05%)	2,048
Financial bond	4.92	5	0.10	0.10	24.93	17.1	1.00	1	551,956 (7.80%)	4,728
Municipal bond	5.00	5	0.05	0.05	12.95	5.9	1.00	1	317,845 (4.49%)	1,536
Corporate bond	4.90	5	0.17	0.11	26.18	14	1.00	1	199,832 (2.82%)	4,887
ETF security (else)	5.07	5	-0.05	-0.09	38.00	50	1.00	1	1,852 (0.03%)	16
Equity	33.22	30	0.56	0.33	2.62	.2	1.00	1	29 (0.00%)	7
Unknown	5.00	5	0.26	0.13	9.68	9.3	1.00	1	539 (0.01%)	27
All collateral	4.96	5	0.06	0.05	43.94	24.9	1.00	1	7,076,214	15,334

The sample period is 2015m1-2020m6. The table shows the mean and median values of overnight contract's haircut, spread, principal, and maturity, as well as the number of observations and the unique identifier for collateral assets across various collateral classes. The percentages in the *N. Obs.* column represent the share of each collateral class in relation to the total number of observations.

Table A-2: Types of cash borrowers and cash lenders

Type of investors	Borrower type (%)	Lender type (%)
Collective investment scheme	48.70	52.97
Securities company	40.13	3.42
Government	3.79	0.49
Securities company (trust)	3.49	6.97
Specialized credit finance company	2.98	4.00
Domestic bank	0.72	5.63
Foreign bank	0.10	0.25
Insurance company	0.09	3.52
Foreigner	0.00	0.00
Pension funds and guarantee	0.00	2.38
Bank (trust)	.	20.32
ETC (trust)	.	0.04
Credit union and thrift institution	.	0.01
Other financial institutions	.	0.00

The sample period is 2015m1-2020m6. The table separately shows the share of each borrower type and lender type in all observations. The shares are calculated based on the number of repo contracts.

Table A-3: Top 10 investor pairs and loan terms

Investor pair: Borrower type	Investor pair: Lender type	Most-used collateral type	Spread (%)	Haircut (%)	Principal (bil. of wons)	Maturity (days)	Share (%)
Collective investment scheme	Collective investment scheme	Government bond (40.86%)	.06	5	12.5	1	28.28
Securities company	Collective investment scheme	Government bond (47.95%)	.03	5	15	1	20.22
Securities company	Bank (trust)	Bank bond (43.60%)	.04	5	100	1	8.93
Collective investment scheme	Bank (trust)	Bank bond (44.82%)	.07	5	50	1	8.81
Securities company	Securities company (trust)	Government bond (32.56%)	.06	5	34.5	1	3.19
Collective investment scheme	Domestic bank	Government bond (57.76%)	.09	5	36.7	1	3.08
Collective investment scheme	Specialized credit finance company	Bank bond (34.14%)	.08	5	21.7	1	2.51
Government	Collective investment scheme	Government bond (51.09%)	.07	5	15.1	1	2.19
Securities company	Domestic bank	Government bond (74.53%)	.07	5	60	1	2.12
Collective investment scheme	Insurance company	Government bond (37.40%)	.06	5	23.3	1	1.80

The sample period is 2015m1-2020m6. The table shows the median value of haircut, spread, principal, and maturity by top 10 investor pairs. The percentages in the *Most-used collateral type* column are the share of repo contracts that the corresponding pair of investors agree to set a collateral of the corresponding most-used collateral type. All shares in the table are calculated based on the number of contracts.

Table A-4: Robustness check: elimination of fixed effects

Dependent variable: Haircut	Baseline (1)	Elimination of fixed effects (2)	(3)	(4)
Spread	-1.361***	-1.291***	-1.459***	2.899***
	(-9.67)	(-9.47)	(-9.37)	(4.88)
Size	0.000429	0.000260	-0.00312***	-0.0134***
	(0.47)	(0.28)	(-3.06)	(-4.97)
Maturity type × Date FE	Yes		Yes	Yes
Borrower type × Lender type × Date FE	Yes	Yes		Yes
Collateral × Date FE	Yes	Yes	Yes	
Cluster Var.	Collateral + Date (13,547)	Collateral + Date (13,549)	Collateral + Date (13,552)	Collateral + Date (17,245)
N.Obs	6,162,332	6,162,662	6,167,994	7,388,320
Adj. R^2	0.647	0.655	0.585	0.606

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. The numbers in parentheses in the cluster variable row are the number of clusters.

Table A-5: Robustness checks: controlling for additional fixed effects

	(1)	(2)	(3)	(4)
Dependent variable:	All	All	All	All
Haircut	maturities	maturities	maturities	maturities
Spread	-1.361***	-1.380***	-0.742***	-0.676***
	(-9.67)	(-9.20)	(-6.38)	(-5.93)
Size	0.000429	0.000842	0.000540	0.000445
	(0.47)	(0.91)	(0.74)	(0.60)
N.Obs	6,162,332	5,651,717	4,764,059	4,606,518
Adj. R^2	0.647	0.669	0.706	0.765
	(5)	(6)	(7)	(8)
Dependent variable:				
Haircut	1-day	1-day	1-day	1-day
Spread	-1.369***	-1.396***	-0.762***	-0.701***
	(-9.95)	(-9.53)	(-6.72)	(-6.31)
Size	0.000595	0.000954	0.000636	0.000555
	(0.65)	(1.04)	(0.87)	(0.75)
N.Obs	5,917,734	5,501,590	4,647,036	4,497,358
Adj. R^2	0.279	0.296	0.319	0.441
Borrower type × Collateral × Date FE		Yes	Yes	
Lender type × Collateral × Date FE			Yes	
Borrower type × Lender type × Collateral × Date FE				Yes

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. *Maturity type × Date, Borrower type × Lender type × Date*, and *Collateral × Date* fixed effects are all included in each regression unless they are included in the multi-way fixed effect. The standard errors are clustered at the collateral level.

Table A-6: Robustness checks: excluding each of collateral types

	Baseline	Elimination of collateral types							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent variable: Haircut	All maturities	All maturities	All maturities	All maturities	All maturities	All maturities	All maturities	All maturities	All maturities
Spread	-1.361***	-1.234***	-1.138***	-1.242***	-1.394***	-1.425***	-1.359***	-1.561***	-1.417***
	(-9.67)	(-7.25)	(-7.31)	(-8.87)	(-9.56)	(-9.67)	(-9.65)	(-10.18)	(-10.12)
Size	0.000429	-0.00216**	0.000783	0.000767	0.000631	0.000447	0.000546	0.000432	0.000509
	(0.47)	(-2.37)	(0.74)	(0.75)	(0.68)	(0.48)	(0.59)	(0.47)	(0.56)
N.Obs	6,162,332	3,203,216	4,640,994	5,477,794	5,676,212	5,875,472	6,044,211	6,052,153	6,159,944
Adj. R^2	0.647	0.797	0.686	0.675	0.653	0.652	0.650	0.650	0.266
	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
Dependent variable: Haircut	1-day	1-day	1-day	1-day	1-day	1-day	1-day	1-day	1-day
Spread	-1.369***	-1.275***	-1.167***	-1.256***	-1.405***	-1.434***	-1.367***	-1.573***	-1.372***
	(-9.95)	(-7.72)	(-7.69)	(-9.23)	(-9.85)	(-9.95)	(-9.94)	(-10.52)	(-9.97)
Size	0.000595	-0.00183**	0.000909	0.000952	0.000792	0.000620	0.000705	0.000582	0.000596
	(0.65)	(-2.04)	(0.85)	(0.92)	(0.85)	(0.66)	(0.76)	(0.63)	(0.65)
N.Obs	5,917,734	3,042,630	4,483,044	5,250,957	5,465,235	5,643,626	5,802,944	5,813,706	5,916,175
Adj. R^2	0.279	0.411	0.268	0.294	0.248	0.272	0.284	0.280	0.267
Excluded collateral type		Government bond	Bank bond	Monetary stabilization bond	Special bond	Financial bond	Municipal bond	Corporate bond	Equity and ETF

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. *Maturity type* $\times$ *Date*, *Borrower type* $\times$ *Lender type* $\times$ *Date*, and *Collateral* $\times$ *Date* fixed effects are all included. The standard errors are clustered at the collateral level.

Table A-7: Robustness check: alternative standard error clustering

Dependent variable: Haircut	(1) All maturities	(2) 1-day	(3) All maturities	(4) 1-day	(5) All maturities	(6) 1-day
Spread	-1.361***	-1.369***	-1.361***	-1.369***	-1.361***	-1.369***
	(-72.53)	(-77.41)	(-5.67)	(-5.61)	(-24.24)	(-24.53)
Size	0.000429***	0.000595***	0.000429	0.000595	0.000429	0.000595*
	(3.70)	(5.17)	(0.43)	(0.59)	(1.30)	(1.80)
Maturity type $\times$ Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Borrower type $\times$ Lender type $\times$ Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Collateral $\times$ Date FE	Yes	Yes	Yes	Yes	Yes	Yes
Cluster Var.	Collateral $\times$ Date (970,175)	Collateral $\times$ Date (927,149)	Collateral + (Year $\times$ Month) + (Day $\times$ DoW) (12,259)	Collateral + (Year $\times$ Month) + (Day $\times$ DoW) (12,059)	Collateral $\times$ ((Year $\times$ Month) + (Day $\times$ DoW)) (642,901)	Collateral $\times$ ((Year $\times$ Month) + (Day $\times$ DoW)) (622,147)
N.Obs	6,162,332	5,917,734	6,162,332	5,917,734	6,162,332	5,917,734
Adj. R^2	0.648	0.279	0.647	0.279	0.648	0.279

The sample period is 2015m1-2020m6. Contracts with an interest rate higher than 10% or with a haircut lower than 0% or higher than 200% are excluded. *t*-Statistics are reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% level, respectively. *DoW* is an abbreviation of "the day of the week". The numbers in parentheses in the cluster variable row are the number of clusters.

Appendix B: Omitted proofs

Proof of Proposition 1. We first analyze the terms of IIS collateralized debt contracts. We define the Lagrangian function for the maximization problem (6) as

$$
\begin{aligned}
L = {} & mq - (1-\omega)(1-\sigma)p - \omega\overline{y}a' + \lambda_1\left[-q + (1-\omega)(1-\sigma)p + \omega\overline{y}a'\right] \\
& + \lambda_2\left[-\sigma q + \gamma\right] + \lambda_3\left[ya' - p\right] + \lambda_4\left[a - a'\right] + \lambda_5 q + \lambda_6 p + \lambda_7 a'
\end{aligned}
$$

where λ_i for $i \in \{1, \ldots 7\}$ are the Lagrange multipliers with $\lambda_i \geq 0$ for all i. The first order conditions are

$$\{q\} : m + \lambda_5 = \lambda_1 + \lambda_2\sigma \tag{22}$$

$$\{p\} : \lambda_3 - \lambda_6 = (1-\omega)(1-\sigma)(\lambda_1 - 1) \tag{23}$$

$$\{a'\} : \lambda_4 - \lambda_7 = (\lambda_1 - 1)\omega\overline{y} + \lambda_3 y. \tag{24}$$

First, suppose that $\lambda_2 = 0$, which indicates the case of fully-IIS collateralized debt. Given $\lambda_2 = 0$, we obtain $\lambda_1 = m + \lambda_5 > 1$, $\lambda_3 - \lambda_6 > 0$, and $\lambda_4 - \lambda_7 > 0$ from (22) - (24). Thus, (7), (9), and (10) must bind, which implies $q = \overline{y}a$, $p = ya$, and $a' = a$. Therefore, $V_{IIS} = (m-1)\overline{y}a$. Finally, from $\lambda_2 = 0$, we have $\gamma \geq \sigma q = \sigma\overline{y}a = \gamma^*$.

Second, suppose that $\lambda_2 > 0$, which indicates the case of partially-IIS collateralized debt. Given $\lambda_2 > 0$, (8) must bind, which implies $q = \frac{\gamma}{\sigma} > 0$ and $\lambda_5 = 0$. Therefore, it must be $a' > 0$ and $\lambda_7 = 0$, because from (7) and (9), $a' = 0$ implies $q \leq 0$. Suppose $\lambda_1 = 0$. Then, from (23) and (24), we obtain $a' = 0$, a contradiction. Thus, $\lambda_1 > 0$ must hold whenever $\lambda_2 > 0$. Then, from the binding (7) and (8), we obtain (19). Given $q = \frac{\gamma}{\sigma}$ and the binding (7), the borrower's surplus is given as $V_{IIS} = \frac{(m-1)\gamma}{\sigma}$. For a moment, we assume that $\lambda_3 = 0$. Given $\lambda_3 = 0$, we obtain, from (23) and (24), $\lambda_6 = (1-\omega)(1-\sigma)(1-\lambda_1) \geq 0$ and $\lambda_4 = \omega\overline{y}(\lambda_1 - 1) \geq 0$. This requires $\lambda_1 = 1$ and $\lambda_4 = \lambda_6 = 0$. Therefore, it must be $0 \leq p \leq ya'$ and $0 < a' \leq a$, and p and a' must satisfy (19) but are not determined uniquely. Next, from (9), (10), and (19), we obtain

$$\frac{\gamma}{\sigma} = (1-\omega)(1-\sigma)p + \omega\overline{y}a' \leq \overline{y}a. \tag{25}$$

Thus, the necessary condition for this case is $\gamma \leq \sigma\overline{y}a = \gamma^*$. If $p = ya'$, (19) implies $\frac{\gamma}{\sigma} = \overline{y}a'$. Therefore, $a' \geq \frac{\gamma}{\sigma\overline{y}}$ must hold to have $p \leq ya'$. On the other hand, if $p = 0$, then (19) gives $\frac{\gamma}{\sigma} = \omega\overline{y}a'$. Thus, $a' \leq \frac{\gamma}{\omega\sigma\overline{y}}$ must hold to have $p \geq 0$. Combined together after applying the restriction $a' \leq a$, we obtain $a' \in \left[\frac{\gamma a}{\gamma^*}, \min\left\{a, \frac{\gamma a}{\omega\gamma^*}\right\}\right]$. Moreover, from (25), we obtain $p = 0$ if and only if $a' = \frac{\gamma}{\omega\sigma\overline{y}}$. Therefore, p is bounded below by 0 if $a \geq \frac{\gamma}{\omega\sigma\overline{y}}$, i.e., $\omega\gamma^* \geq \gamma$, while p is bounded below by $\frac{1}{(1-\omega)\sigma(1-\sigma)}\left[\gamma - \omega\gamma^*\right]$ otherwise. Moreover, p is maximized when $p = ya'$, and by substituting $p = ya'$ into (25), we obtain $\frac{\gamma}{\sigma} = (1-\sigma)ya'$, which means $ya' = \frac{\gamma}{\sigma(1-\sigma)}$. Therefore, p is bounded above by $\frac{\gamma}{\sigma(1-\sigma)}$. Combined together, we obtain $p \in \left[\max\left\{0, \frac{\gamma-\omega\gamma^*}{(1-\omega)\sigma(1-\sigma)}\right\}, \frac{\gamma}{\sigma(1-\sigma)}\right]$. Now suppose $\lambda_3 > 0$, which implies $p = ya' > 0$ from (9) so $\lambda_6 = 0$. Then, from (19), we obtain $a' = \frac{\gamma}{\sigma\overline{y}}$, and the necessary condition for this case is again $\gamma \leq \sigma\overline{y}a = \gamma^*$. This is the knife edge case of the case with $\lambda_3 = 0$.

We now investigate the terms of IS collateralized debt contract. In the maximization problem (12), it must be that $q > 0$ and $a' > 0$ to satisfy (14). It is also obvious that (13) must bind; otherwise, the borrower could increase the surplus without violating any constraints. Then, from (13) and (14), we obtain

$$q = (1-\omega)p + \omega ya' - \frac{\gamma}{1-\sigma} \geq \frac{\gamma}{\sigma}. \tag{26}$$

Thus, $\gamma \leq \sigma\overline{y}a = \gamma^*$ must hold; otherwise, constraints (13) - (16) cannot be satisfied simultaneously. Substituting $q = (1-\omega)p + \omega ya' - \frac{\gamma}{1-\sigma}$ into the objective function (12), we obtain

$$V_{IS} = \max_{p,a'} \left\{(1-\sigma)(m-1)\left[(1-\omega)p + \omega ya'\right] - m\gamma\right\}$$

subject to (15) - (17) and (26). Now, it becomes obvious that it must be $a' = a$ and $p = ya$ to maximize the objective function. Then, $q = ya - \frac{\gamma}{1-\sigma}$ and $V_{IS} = (m-1)\overline{y}a - m\gamma$. Because the borrower can always choose not to trade, it must be $V_{IS} = (m-1)\overline{y}a - m\gamma \geq 0$, which requires $\gamma \leq \frac{m-1}{m\sigma}\gamma^*$. Thus, the necessary condition for the existence of an IS loan contract that is not dominated by the no trading option is $\gamma \leq \min\left\{\gamma^*, \frac{m-1}{m\sigma}\gamma^*\right\}$.

We finish the proof by discussing of the optimal collateralized debt. First, it is easy to verify that $V_{IIS} = (m-1)\min\{\overline{y}a, \frac{\gamma}{\sigma}\}$ weakly increases with γ while $V_{IS} = (m-1)\overline{y}a - m\gamma$ decreases with γ. Specifically,

$$\lim_{\gamma \searrow 0} V_{IIS} < \lim_{\gamma \searrow 0} V_{IS} \text{ and } \lim_{\gamma \nearrow \gamma^*} V_{IIS} > \lim_{\gamma \nearrow \gamma^*} V_{IS}.$$

Additionally, when $\gamma = \gamma^{**}$, $V_{IIS} = \frac{(m-1)\gamma^{**}}{\sigma} = (m-1)\frac{\gamma^*}{\sigma} - m\gamma^{**} = V_{IS}$, so we obtain the cutoff level γ^{**} by (18). Note that $\gamma^{**} < \min\left\{\gamma^*, \frac{m-1}{m\sigma}\gamma^*\right\}$. More specifically, the type of IIS collateralized debt must be partially-IIS when $\gamma = \gamma^{**}$ because $\gamma^{**} < \gamma^*$.

Thus, the borrower induces the lender to acquire information (i.e., IS collateralized debt) only if $\gamma < \gamma^{**}$, and does not trigger information acquisition otherwise. Furthermore, when $\gamma \geq \gamma^{**}$, if $\gamma \in [\gamma^{**}, \gamma^*)$, the borrower offers partially-IIS collateralized debt to the lender and offers fully-IIS collateralized debt if $\gamma \geq \gamma^*$. Finally, it can by verified from (18) that γ^* and γ^{**} increase with σ, which completes the proof. ■

Proof of Proposition 2. When information about the collateral quality is not produced, the expected value of collateral assets is $\overline{y}a'$. When collateralized debt is fully-IIS, $q = \overline{y}a$ and $a' = a$. Thus, the haircut is $\theta = \frac{v-q}{v} = 0$. Substituting $p = ya$ and $q = \overline{y}a$ into $r = \frac{p-q}{q}$, we obtain the interest rate as $r = \frac{ya - \overline{y}a}{\overline{y}a} = \frac{\sigma}{1-\sigma}$. Next, when collateralized debt is partially-IIS, $q = \frac{\gamma}{\sigma}$ and $v = \overline{y}a'$. Thus, the haircut is given as $\theta = 1 - \frac{\gamma}{\sigma\overline{y}a'}$. According to Proposition 1, we obtain $\sigma\overline{y}a' \in \left[\gamma, \min\{\gamma^*, \frac{\gamma}{\omega}\}\right]$. Therefore, it must be that $\theta \in \left[0, 1 - \max\left\{\frac{\gamma}{\gamma^*}, \omega\right\}\right]$. Given $q = \frac{\gamma}{\sigma}$, the interest rate is $r = \frac{\sigma p}{\gamma} - 1$. Also, from Proposition 1, we obtain $\sigma p \in \left[\max\left\{0, \frac{\gamma - \omega\sigma\overline{y}a}{(1-\omega)(1-\sigma)}\right\}, \frac{\gamma}{1-\sigma}\right]$, which implies $r \in \left[\max\left\{0, \frac{1-\omega\cdot\frac{\gamma^*}{\gamma}}{(1-\omega)(1-\sigma)}\right\} - 1, \frac{1}{1-\sigma} - 1\right]$. Moreover, because p and a' are not determined uniquely in this case, the interest rate r and haircut θ also are not determined uniquely. More precisely, substituting $r = \frac{\sigma p}{\gamma} - 1$ and $\theta = 1 - \frac{\gamma}{\sigma \overline{y} a'}$ into (19), we obtain (20), which determines the pair of (r, θ) for the partially-IIS loan contract.

When the information is produced, the lender's acceptance of the IS debt reveals the information that the dividend state is good. Hence, the expected value of the collateral ya', and, thus, the interest rate and haircut are given as $r = \frac{\gamma}{\overline{y}a - \gamma}$ and $\theta = \frac{\gamma}{\overline{y}a}$. ■

Proof of Proposition 3. Note that, by Assumption 1, IS contracts do not occur for any $(\omega, \sigma, \gamma) \in \Omega \times \Sigma \times \Gamma$ in equilibrium. Let $\hat{r}_{FIIS}$ ($\hat{\theta}_{FIIS}$) and $\hat{r}_{PIIS}$ ($\hat{\theta}_{PIIS}$) denote the interest rate (haircut) when the type of collateralized debt is fully-IIS and partially-IIS, respectively. From Proposition 2, we obtain

$$\hat{r}_{PIIS} = \frac{1}{2}\left[\max\left\{0, \frac{1 - \omega \cdot \frac{\gamma^*}{\gamma}}{(1-\omega)(1-\sigma)}\right\} + \frac{1}{1-\sigma}\right] - 1 \text{ and } \hat{\theta}_{PIIS} = \frac{1}{2}\left[1 - \max\left\{\frac{\gamma}{\gamma^*}, \omega\right\}\right]. \tag{27}$$

First, we examine the effect of ω on $\hat{r}$ and $\hat{\theta}$. Note that γ^* is not affected by ω, and the type of collateralized debt is fully-IIS if $\gamma \geq \gamma^*$ and partially-IIS otherwise. Thus, the type of collateralized debt remains unchanged by variations in ω. Furthermore, $\hat{r}$ and $\hat{\theta}$ do not change with ω when the type of collateralized debt is fully-IIS as shown in Proposition 2. Next, suppose that the contract type is partially-IIS. When $\omega \geq \frac{\gamma}{\gamma^*}$, $\frac{\partial \hat{r}_{PIIS}}{\partial \omega} = 0$ and $\frac{\partial \hat{\theta}_{PIIS}}{\partial \omega} < 0$. Otherwise, we have $\frac{\partial \hat{\theta}_{PIIS}}{\partial \omega} = 0$ and

$$\frac{\partial \hat{r}_{PIIS}}{\partial \omega} = \frac{1}{2(1-\sigma)} \cdot \frac{\partial}{\partial \omega}\left(\frac{2 - \omega - \omega\frac{\gamma^*}{\gamma}}{1-\omega}\right) = \frac{1}{2(1-\sigma)} \cdot \frac{1 - \frac{\gamma^*}{\gamma}}{(1-\omega)^2} < 0,$$

because $\gamma < \gamma^*$ when the type of collateralized debt is partially-IIS. Thus, $\hat{r}$ and $\hat{\theta}$ never change concurrently in response to variations in ω.

We now study the impact of γ on $\hat{r}$ and $\hat{\theta}$. Because $\hat{\theta}_{FIIS} = 0$, responses in $\hat{\theta}$ occur only when the type of collateralized debt remains partially-IIS, or when the type shifts due to changes in γ. First, suppose that $\gamma < \gamma^*$ so that the contract type is partially-IIS. It can be verified that $\frac{\partial\hat{\theta}_{PIIS}}{\partial\gamma} = \frac{\partial\hat{r}_{PIIS}}{\partial\gamma} = 0$ if $\omega \geq \frac{\gamma}{\gamma^*}$, while $\frac{\partial\hat{\theta}_{PIIS}}{\partial\gamma} < 0$ and $\frac{\partial\hat{r}_{PIIS}}{\partial\gamma} > 0$ if $\omega < \frac{\gamma}{\gamma^*}$. Thus, if $\hat{r}(\omega,\sigma,\gamma)$ and $\hat{\theta}(\omega,\sigma,\gamma)$ respond to a change in γ within partially-IIS collateralized debt, they will change in opposite directions.

We complete the proof by showing that the interest rate and haircut also change in opposite directions when the type of collateralized debt shifts between partially-IIS and fully-IIS. Notice that $\lim_{\gamma\to\gamma^*}\hat{r}_{PIIS} = \frac{1}{2(1-\sigma)} - 1 < \hat{r}_{FIIS}$ and $\lim_{\gamma\to\gamma^*}\hat{\theta}_{PIIS} = \hat{\theta}_{FIIS} = 0$. Thus, considering that $\frac{\partial\hat{\theta}_{PIIS}}{\partial\gamma} < 0$ when $\gamma > \omega\gamma^*$, the interest rate increases and the haircut decreases when the type of collateralized debt shifts from partially-IIS to fully-IIS. ■

Proof of Proposition 4. Define $\hat{r}_{FIIS}$, $\hat{\theta}_{FIIS}$, $\hat{r}_{PIIS}$, and $\hat{\theta}_{PIIS}$ as in the proof of Proposition 3. Let $\sigma^* \equiv \frac{\gamma}{\overline{y}a}$. Then, the type of collateralized debt is fully-IIS if $\sigma \leq \sigma^*$, and partially-IIS otherwise.

We first analyze the effects of σ on interest rates. Note from (27) that $\frac{\partial\hat{r}_{PIIS}}{\partial\sigma} > 0$ if $\omega\gamma^* \geq \gamma$. If $\omega\gamma^* < \gamma$, we obtain $\hat{r}_{PIIS} = \frac{(2-\omega)\gamma-\omega\sigma\overline{y}a}{\gamma(1-\omega)(1-\sigma)}$, so

$$\begin{aligned}\frac{\partial\hat{r}_{PIIS}}{\partial\sigma} &= \frac{(2-\omega)\gamma-\omega\overline{y}a}{\gamma(1-\omega)(1-\sigma)^2}\\ &\geq \frac{\left\{(2-\omega)\frac{m-1}{m(1+\sigma)-1}\sigma-\omega\right\}\overline{y}a}{\gamma(1-\omega)(1-\sigma)^2},\end{aligned} \tag{28}$$

where the first inequality comes from the fact that $\gamma \geq \gamma^{**} = \frac{m-1}{m(1+\sigma)-1}\sigma\overline{y}a$. Then, given that $\omega < \frac{(m-1)2\sigma}{m-1+(2m-1)\sigma}$, we obtain $\frac{\partial\hat{r}_{PIIS}}{\partial\sigma} > 0$ from (28).

Also, from Proposition 2, we obtain that $\frac{\partial\hat{r}_{FIIS}}{\partial\sigma} > 0$, and it can be shown that

$$\hat{r}_{FIIS}(\omega,\sigma^*,\gamma) = \lim_{\sigma\to\sigma^*}\hat{r}_{PIIS}(\omega,\sigma,\gamma) = \frac{\sigma^*}{1-\sigma^*}.$$

Note that for all $\sigma \leq \sigma^*$, the contract type is fully-IIS, while the contract type is partially-IIS otherwise. Therefore, it must be that $\hat{r}_{FIIS} < \hat{r}_{PIIS}$, because $\frac{\partial\hat{r}_{FIIS}}{\partial\sigma} > 0$ and $\frac{\partial\hat{\theta}_{PIIS}}{\partial\sigma} > 0$.

To analyze the effects of σ on haircuts, note that $\hat{\theta}_{FIIS} = 0$. Thus, $\hat{\theta}$ responds to a change of σ only if either the type of collateralized debt changes between fully-IIS and partially-

IIS, or if the type remains partially-IIS with the change in σ. First, since $\hat{\theta}_{PIIS} > 0$, if the contract type changes from fully-IIS to partially-IIS in response to an increase in σ, then $\hat{\theta}$ increases. Next, from (27), we obtain $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} = 0$ if $\omega\gamma^* \geq \gamma$ and $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} > 0$ otherwise.

We now investigate responses of $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ to a change in σ toghether. First, if the contract type changes from fully-IIS to partially-IIS in response to an increase in σ, both $\hat{\theta}$ and $\hat{r}$ increase. Next, note that $\frac{\partial \hat{\theta}_{FIIS}}{\partial \sigma} = 0$ since $\hat{\theta}_{FIIS} = 0$. Next, $\frac{\partial \hat{r}_{PIIS}}{\partial \sigma} > 0$, and $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} = 0$ if $\omega\gamma^* \geq \gamma$ and $\frac{\partial \hat{\theta}_{PIIS}}{\partial \sigma} > 0$ otherwise. Combined together, we obtain that whenever both $\hat{r}(\omega, \sigma, \gamma)$ and $\hat{\theta}(\omega, \sigma, \gamma)$ respond to a change in σ, they do so in the same direction. ■

Appendix C: Non-existence of borrower's information acquisition incentives

In this appendix, we demonstrate that the assumption that the borrower cannot obtain information about her tree's quality in the baseline model is made without loss of generality. Specifically, we show that even if the borrower has the ability to acquire information, she will choose not to do so at any cost.

Suppose that the borrower can acquire information about her tree's quality at a cost of $\gamma_b > 0$ units of disutility in period 0 before making an offer to the lender. If the borrower does not acquire the information, then equilibrium outcomes are equivalent to those in the baseline model. Thus, in what follows, we focus on the case in which the borrower acquires the information.

This information, known only to the borrower, categorizes her as either a good-type borrower (with good-quality collateral) or a bad-type borrower (with bad-quality collateral). Consequently, the bargaining problem features a signaling issue, and we adopt the perfect Bayesian equilibrium as our solution concept.

In this environment, separating equilibrium cannot exist because a bad-type borrower will always mimic a good-type borrower. If a lender identifies the collateral as bad, she will reject the collateralized debt offered by the bad-type borrower. Given this mimicry by the bad-type borrower in the first period, the lender has a pooling belief about the borrower's type in equilibrium.

The good-type borrower's choice of collateralized debt depends on the lender's belief. Let $\overline{V}^g_{IIS}$ be the largest possible surplus for the good-type borrower from IIS collateralized debt in equilibrium, and let $\overline{V}^g_{IS}$ be that from IS collateralized debt. Let $\overline{V}^b_{IIS}$ and $\overline{V}^b_{IS}$ be the bad-type borrower's surpluses from mimicking the good-type borrower when the good-type borrower offers IIS debt contract and IS debt contract, respectively.

After obtaining information, the maximized surplus, $\overline{V}^g_{IIS}$, of the good-type borrower from IIS collateralized debt, is given by

$$\overline{V}^{g}_{IIS} = \max_{q,p,a'} \{mq - (1-\omega)p - \omega y a'\} \tag{29}$$

subject to

$$-q + (1-\omega)(1-\sigma)p + \omega \overline{y} a' \geq 0 \tag{30}$$

$$-\sigma q + \gamma \geq 0 \tag{31}$$

$$ya' - p \geq 0 \tag{32}$$

$$a - a' \geq 0 \tag{33}$$

$$q, p, a' \geq 0 \tag{34}$$

When the good-type borrower offers an IIS debt contract, the bad-type borrower will also do so to mimic her in period 1. Because the bad-type borrower will default in period 1 with certainty, her surplus from mimicking is given as follows:

$$\overline{V}^{b}_{IIS} = mq.$$

Next, after checking the dividend state, the good type borrower may offer IS debt contract that induces the lender produce information. The maximized surplus for the good-type borrower from IS collateralized debt, $\overline{V}^{g}_{IS}$, is given by

$$\overline{V}^{g}_{IS} = \max_{q,p,a'} \{[mq - (1-\omega)p - \omega y a']\} \tag{35}$$

subject to

$$(1-\sigma)\left[-q+(1-\omega)p+\omega y a'\right]-\gamma \geq 0 \tag{36}$$

$$\sigma q-\gamma \geq 0 \tag{37}$$

$$y a'-p \geq 0 \tag{38}$$

$$a-a' \geq 0 \tag{39}$$

$$q, p, a' \geq 0. \tag{40}$$

When the lender also checks the dividend state, the bad-type will be discovered by the lender, and the lender will only trade with the good-type borrower. Thus, the bad-type borrower cannot mimic, and the bad type's surplus is given as $\overline{V}^b_{IS}=0$.

The expected maximized surplus of the borrower ex-ante after information acquisition is given as $\overline{V}=\max\{(1-\sigma)\overline{V}^g_{IIS}+\sigma\overline{V}^b_{IIS},(1-\sigma)\overline{V}^g_{IS}+\sigma\overline{V}^b_{IS}\}$. Then, the borrower decides whether to acquire information about the dividend state or not by comparing V and $\overline{V}-\gamma_b$. The next proposition shows that $V>\overline{V}-\gamma_b$, so the borrower never acquires the private information at a cost.

Proposition 5 *The borrower does not acquire information in equilibrium.*

Proof of Proposition 5. We first investigate the terms of IIS collateralized debt contract. We define the Lagrangian function for the maximization problem (29) as

$$\begin{aligned} L = & \, mq-(1-\omega)p-\omega y a'+\lambda_1\left[-q+(1-\omega)(1-\sigma)p+\omega\overline{y}a'\right] \\ & +\lambda_2\left[-\sigma q+\gamma\right]+\lambda_3\left[y a'-p\right]+\lambda_4\left[a-a'\right]+\lambda_5 q+\lambda_6 p+\lambda_7 a' \end{aligned}$$

where λ_i for $i\in\{1,\ldots 7\}$ are the Lagrange multipliers with $\lambda_i\geq 0$ for all i. The first-order conditions are

$$\{q\} : m + \lambda_5 = \lambda_1 + \lambda_2\sigma \tag{41}$$

$$\{p\} : \lambda_3 - \lambda_6 = (1-\omega)((1-\sigma)\lambda_1 - 1) \tag{42}$$

$$\{a'\} : \lambda_4 - \lambda_7 = ((1-\sigma)\lambda_1 - 1)\omega y + \lambda_3 y. \tag{43}$$

Case 1. $\lambda_2 = 0$ and $m(1-\sigma) > 1$.

Given $\lambda_2 = 0$, we obtain $\lambda_1 = m + \lambda_5$. Also, from $\lambda_1 \geq m$, we have $(1-\sigma)\lambda_1 > 1$. Thus, $\lambda_3 - \lambda_6 > 0$ and $\lambda_4 - \lambda_7 > 0$ from (42) and (43). Thus, (30), (32), and (33) must bind, which implies $p = ya$, $q = \overline{y}a$, and $a' = a$. Therefore, $\overline{V}^g_{IIS} = \left(m - \frac{1}{1-\sigma}\right)\overline{y}a$ and $\overline{V}^b_{IIS} = mq = m\overline{y}a$. Finally, from $\lambda_2 = 0$, it must be that $\gamma \geq \sigma\overline{y}a = \gamma^*$. ■

Case 2. $\lambda_2 = 0$ and $(1-\sigma)m < 1$.

Given $\lambda_2 = 0$, we obtain $\lambda_1 = m + \lambda_5 > 0$, so $q = (1-\omega)(1-\sigma)p + \omega\overline{y}a'$. However, $\overline{V}^g_{IIS} = (m - \frac{1}{1-\sigma})q < 0$. Thus, the borrower would not make a fully-IIS collateralized debt. Specifically, first suppose that $\lambda_5 > 0$. Then, $q = 0$, which means no-trade. Now suppose that $\lambda_5 = 0$. Then $\lambda_1 = m$, which implies $(1-\sigma)\lambda_1 < 1$. Thus, $\lambda_3 - \lambda_6 < 0$ and $\lambda_4 - \lambda_7 < 0$ from (41) - (43). Therefore, (30) must bind and $p = a' = 0$, which in turn implies $q = 0$, i.e., no-trade. ■

Case 3. $\lambda_2 = 0$ and $m(1-\sigma) = 1$.

Given $\lambda_2 = 0$, we obtain $\lambda_1 = m + \lambda_5 > 0$. Thus, $q = (1-\omega)(1-\sigma)p + \omega\overline{y}a'$ and $\overline{V}^g_{IIS} = (m - \frac{1}{1-\sigma})q = 0$. If $\lambda_5 > 0$, then $q = 0$, which means no-trade. Now suppose that $\lambda_5 = 0$. Then, $\lambda_1 = m$, which implies $(1-\sigma)\lambda_1 = 1$. Thus, $\lambda_3 - \lambda_6 = \lambda_4 - \lambda_7 = 0$ from (42) and (43). If $\lambda_3 > 0$ and $\lambda_6 > 0$, then $p = a' = 0$, no-trade. Moreover, both $\lambda_4 > 0$ and $\lambda_7 > 0$ cannot hold together, because both $a' = a$ and $a' = 0$ cannot hold together. Thus $\lambda_4 = \lambda_7 = 0$. Now suppose that $\lambda_3 = \lambda_6 = 0$. Then $p \leq ya'$ and $a' \leq a$. This is the knife edge case of either case 1, 2, or case 4. We consider this case as a part of case 1, i.e., restrict our attention that $p = ya$ and $a' = a$.[25] ■

Case 4. $\lambda_2 > 0$

Given $\lambda_2 > 0$, (31) must bind, which gives $q = \frac{\gamma}{\sigma}$ and $\lambda_5 = 0$. Therefore, it must be $a' > 0$ and thus $\lambda_7 = 0$, because $q \leq 0$ by (30) otherwise. Suppose $\lambda_1 = 0$. Then, from (42) and (43), we obtain $a' = 0$, a contradiction. Thus, $\lambda_1 > 0$ must hold whenever $\lambda_2 > 0$. Then, from the binding (30) and (31), we obtain

$$\frac{\gamma}{\sigma} = (1-\omega)(1-\sigma)p + \omega\overline{y}a'. \tag{44}$$

From $q = \frac{\gamma}{\sigma}$ and the binding (30), we obtain the borrower's surplus as $\overline{V}^g_{IIS} = \frac{\gamma}{\sigma}\left[m - \frac{1}{1-\sigma}\right]$ and $\overline{V}^b_{IIS} = mq = \frac{m\gamma}{\sigma}$. Therefore, the equilibrium is no-trade if $m(1-\sigma) < 1$.

For the rest of case 4, we assume that $m(1-\sigma) \geq 1$. For a moment, we assume that $\lambda_3 = 0$. Given $\lambda_3 = 0$, we obtain, from (42) and (43), that $\lambda_6 = (1-\omega)(1-\sigma)(1-(1-\sigma)\lambda_1) \geq 0$ and $\lambda_4 = ((1-\sigma)\lambda_1 - 1)\omega y \geq 0$. This requires $(1-\sigma)\lambda_1 = 1$ and $\lambda_4 = \lambda_6 = 0$. Therefore, it must be $0 \leq p \leq ya'$ and $0 < a' \leq a$, and p and a' must satisfy condition (44) but are not determined uniquely. Next, from (32), (33), and (44), we obtain $\frac{\gamma}{\sigma} = (1-\omega)(1-\sigma)p + \omega\overline{y}a' \leq \overline{y}a$, and thus the necessary condition for this case is $\gamma \leq \sigma\overline{y}a = \gamma^*$. As in the proof of Proposition 1, we obtain $p \in \left[\max\left\{0, \frac{\gamma - \omega\gamma^*}{(1-\omega)\sigma(1-\sigma)}\right\}, \frac{\gamma}{\sigma(1-\sigma)}\right]$ and $a' \in \left[\frac{\gamma a}{\gamma^*}, \min\left\{a, \frac{\gamma a}{\omega\gamma^*}\right\}\right]$. Now suppose $\lambda_3 > 0$, which implies $p = ya' > 0$ from (32) so $\lambda_6 = 0$. Then, from (44), we obtain $a' = \frac{\gamma}{\sigma\overline{y}} = \frac{\gamma a}{\gamma^*}$, and the necessary condition for this case is again $\gamma \leq \sigma\overline{y}a = \gamma^*$. This is the knife edge case of the case with $\lambda_3 = 0$ above. ■

In summary, if $m(1-\sigma) < 1$, the equilibrium IIS collateralized debt is no-trade. On the other hand, when $m(1-\sigma) \geq 1$, $\overline{V}^g_{IIS} = \left(m - \frac{1}{1-\sigma}\right)\overline{y}a$ and $\overline{V}^b_{IIS} = m\overline{y}a$ if $\gamma \geq \gamma^*$, and $\overline{V}^g_{IIS} = \frac{m\gamma}{\sigma} - \frac{\gamma}{\sigma(1-\sigma)}$ and $\overline{V}^b_{IIS} = \frac{m\gamma}{\sigma}$ otherwise.

We now investigate the terms of IS contracts. In the loan contract problem (35), it must be $q > 0$ and $a' > 0$ to satisfy (37) and (38). It is also obvious that (36) must bind; otherwise,

[25] The value of q may vary in case 3, depending on the values of p and a'. However, due to the parametric condition $m(1-\sigma) = 1$, $\overline{V}^g_{IIS} = 0$ is fixed regardless of (q, p, a'). Thus, restricting $p = ya$ and $a' = a$ for case 3 does not harm the richness of the results.

the borrower could increase the surplus without violating any constraints. Then, from (36) and (37), we obtain

$$q = (1-\omega)p + \omega y a' - \frac{\gamma}{1-\sigma} \geq \frac{\gamma}{\sigma}. \tag{45}$$

Substituting $q = (1-\omega)p + \omega y a' - \frac{\gamma}{1-\sigma}$ into the objective function (35), we obtain

$$\overline{V}^g_{IS} = \max_{p,a'} \left\{ (m-1)\left[(1-\omega)p + \omega y a'\right] - \frac{m\gamma}{1-\sigma} \right\}$$

subject to (38) - (40) and (45). Now, it becomes obvious that it must be $a' = a$ and $p = ya$ to maximize the objective function. Then, $q = ya - \frac{\gamma}{1-\sigma}$ and $\overline{V}^g_{IS} = (m-1)ya - \frac{m\gamma}{1-\sigma}$. From (45), $\gamma \leq \sigma(1-\sigma)ya = \gamma^*$ must hold. Finally, the borrower can always choose not to trade. Thus, it must be $\overline{V}^g_{IS} = (m-1)ya - \frac{m\gamma}{1-\sigma} \geq 0$, which requires $\gamma \leq \frac{(m-1)\gamma^*}{\sigma m}$. Thus, the necessary condition for the existence of an IS loan contract that is not dominated by the no trading option is $\gamma \leq \min\left\{\gamma^*, \frac{(m-1)\gamma^*}{\sigma m}\right\}$. Finally, the lender would not trade with the bad-type borrower, thus, $\overline{V}^b_{IS} = 0$.

We finalize the proof by showing that the borrower has no incentive to acquire the information. It is trivial that the borrow does not acquire the information when $m(1-\sigma) < 1$, so assume that $m(1-\sigma) \geq 1$. The borrower's maximized surplus when the borrower does not acquire the information is $\max\{V_{IIS}, V_{IS}\}$. If the borrower decides to acquire the information, then she first bears the cost $-\gamma_b$. Then, she offers a collateralized debt to maximize her surplus with probability $1-\sigma$ and mimics the good-type borrower with probability σ. Thus, the borrower's ex-ante maximized surplus who acquires the information is

$$\max\left\{-\gamma_b + (1-\sigma)\overline{V}^g_{IIS} + \sigma\overline{V}^b_{IIS}, -\gamma_b + (1-\sigma)\overline{V}^g_{IS} + \sigma\overline{V}^b_{IS}\right\}.$$

Notice that the required conditions for γ for the equilibrium IIS collateralized debt and IS collateralized debt are identical when the borrower acquires the information or not. Thus, it suffices to show that $V_{IIS} > -\gamma_b + (1-\sigma)\overline{V}^g_{IIS} + \sigma\overline{V}^b_{IIS}$ and $V_{IS} > -\gamma_b + (1-\sigma)\overline{V}^g_{IS} + \sigma\overline{V}^b_{IS}$.

We first show that $V_{IIS} > -\gamma_b + (1-\sigma)\overline{V}^g_{IIS} + \sigma\overline{V}^b_{IIS}$. Note that $V_{IIS} = (m-1)\overline{y}a$ if $\gamma \geq \gamma^*$, and $V_{IIS} = \frac{(m-1)\gamma}{\sigma}$ otherwise. Suppose that $\gamma \geq \gamma^*$. Then

$$-\gamma_b + (1-\sigma)\overline{V}^g_{IIS} + \sigma\overline{V}^b_{IIS} = -\gamma_b + m\overline{y}a - \overline{y}a = V_{IIS} - \gamma_b.$$

Now suppose that $\gamma < \gamma^*$. Then

$$-\gamma_b + (1-\sigma)\overline{V}^g_{IIS} + \sigma\overline{V}^b_{IIS} = -\gamma_b + \frac{m\gamma}{\sigma} - \frac{\gamma}{\sigma} = V_{IIS} - \gamma_b.$$

Next, from the above analysis, we obtain

$$-\gamma_b + (1-\sigma)\overline{V}^g_{IS} + \sigma\overline{V}^b_{IS} = -\gamma_b + (m-1)\overline{y}a - m\gamma = V_{IS} - \gamma_b.$$

Thus, $V_{IS} > -\gamma_b + (1-\sigma)\overline{V}^g_{IS} + \sigma\overline{V}^b_{IS}$, which completes the proof. ■