연세대학교 상경대학

경제연구소

Economic Research Institute

Yonsei University

서울시 서대문구 연세로 50

50 Yonsei-ro, Seodaemun-gu, Seoul, Korea

TEL: (+82-2) 2123-4065 FAX: (+82-2) 364-9149

E-mail: yeri4065@yonsei.ac.kr

http://yeri.yonsei.ac.kr/new

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Jihwan Do

Yonsei University

Jeanine Miklós-Thal

University of Rochester

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Jihwan Do †1 and Jeanine Miklós-Thal ‡2

[1]School of Economics, Yonsei University
[2]Simon Business School, University of Rochester; CEPR

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Abstract

The cloud services industry, which is currently dominated by a few large providers, has come under scrutiny from antitrust authorities worldwide. One concern is that "egress fees"—charges for transferring data out of a provider's cloud—could harm competition and welfare by discouraging multi-clouding, whereby a user combines services from several providers. Motivated by this policy concern, we analyze the effects of banning price discrimination against multi-stop shoppers in a market where multi-product firms sell complementary goods to buyers with elastic demands, and multi-stop shoppers impose higher service costs than one-stop shoppers. We find that if buyers are locked into a specific product combination, then a ban on price discrimination against multi-stop shoppers raises social welfare for a wide range of demand functions. If product choices are endogenous and buyers' product preferences are weak, however, then a ban on price discrimination tends to harm social welfare.

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†Email: econ.jihwando@yonsei.ac.kr
‡Email: jeanine.miklos-thal@simon.rochester.edu

1 Introduction

Cloud computing services ("cloud services") are a cornerstone of today's digital economy. Extending far beyond data storage and computing power, these services encompass databases, networking, analytics, software, artificial intelligence (AI) and machine learning tools, and Internet-of-Things services, among others.[1] Cloud service providers enable organizations to access these resources on demand over the Internet, thereby eliminating the need for costly on-premise infrastructure and accelerating the development and deployment of new goods and services. Cloud computing also provides essential infrastructure for training and deploying AI and machine learning models at scale.

The cloud services industry, which originated with the launch of Amazon Web Services (AWS) in 2006 and has since grown to generate global revenues above 600 billion dollars,[2] is currently a concentrated market. A small group of large cloud services providers, each offering a full suite of cloud services, command high combined market shares. According to recent studies by regulators, the two largest providers, AWS and Microsoft Azure, have combined market shares between 50 and 80 percent in key geographic markets.[3]

Several business practices of the major cloud services providers have recently drawn scrutiny from regulators and competition authorities worldwide.[4] One practice of concern is the egress fees charged by the major cloud service providers—fees customers pay when transferring data out of a provider's cloud.[5] Although data transfers impose costs for cloud service providers, such as bandwidth and infrastructure usage costs, regulators estimate that current egress fees exceed providers' incremental costs of processing these transfers.[6]

Two concerns have been raised about the potential adverse competition effects of above-cost data transfer fees.[7] One concern is that such fees could discourage buyers from switching providers. The other concern, which motivates our analysis, is that data transfer fees could

[1] The three largest cloud service providers—Amazon Web Services, Microsoft Azure, and Google Cloud Platform—each offer more than 200 distinct cloud products and services (Gans et al., 2023).

[2] International Data Corporation (IDC) Worldwide Semiannual Public Cloud Services Tracker, https://www.idc.com/getdoc.jsp?containerId=prUS52343224.

[3] See ACM (2022, p.34); Ofcom (2023, paragraph 6.8); KFTC (2022, p.2); JFTC (2022, Figure 2-12).

[4] In the US, the Federal Trade Commission issued a Request for Information into the business practice of Cloud Computing Providers in 2023. In the UK, the Competition and Markets Authority (CMA) is currently undertaking a cloud services market investigation following a market study by Ofcom (2023). Other countries in which the industry is or was under regulatory scrutiny include the Netherlands, France, South Korea, and Japan.

[5] Other concerns include software licensing practices, contractual arrangements such as minimum spending accounts or cloud credits, and technical barriers to interoperability.

[6] See CMA (2025, Appendix Q) for a detailed analysis of providers' costs of data transfers.

[7] See, for instance, CMA (2024).

discourage multi-clouding, whereby a customer uses the services of multiple cloud providers in parallel, as opposed to relying on only a single cloud provider.[8]

More specifically, regulators are concerned that data transfer fees discourage the practice of "integrated multi-clouding," in which customers mix and match services from several cloud providers. To give a concrete example (see Ofcom (2023, Box 5.3)), consider a digital advertising agency that uses compute, storage and other services on Cloud A to deliver customized advertisements, but uses the data warehouse and other analytics and machine-learning services on Cloud B to process the resulting data and assess advertising performance. An integrated multi-cloud architecture like this requires ongoing data transfer from Cloud A to Cloud B, hence the advertising agency must pay data transfer fees on an ongoing basis, in addition to paying for the services it uses. A concern, therefore, is that above-cost data transfer fees may lead to too little multi-clouding from a social welfare perspective, either by leading to inefficiently low consumption levels for multi-clouders or by discouraging the adoption of integrated multi-cloud architectures altogether.

In the European Union, egress fees will be regulated under the European Data Act, which entered into force in January 2024.[9] The Data Act, whose goal is to promote a fair and competitive data market within Europe, contains two key provisions on egress fees. For multi-clouders ("in-parallel use of services"), the regulation specifies that providers "may impose data egress charges, but only for the purpose of passing on egress costs incurred, without exceeding such costs." (Article 34(2)) and states that "This is important [...] for the successful deployment of multi-cloud strategies..." (paragraph (99)). For switchers, the regulation specifies that data egress fees should be abolished (see Article 29, along with the definition of switching charges in Article 2(36)).[10] These rules will apply after a three-year transition period.

Although egress fees will be regulated in the European Union soon and similar regulations are being considered in other jurisdictions, little is known about the likely economic effects of such regulations. This paper seeks to contribute to filling this knowledge gap by analyzing the concern that unregulated data transfer fees harm social welfare and buyers of cloud services by discouraging multi-clouding.

[8]Using customer data sets from AWS, Microsoft, and GCP to analyze the prevalence of multi-clouding, CMA (2025, Appendix I) finds that 35-40 percent of all spend (at these three providers) is by customers that multi-cloud.

[9]Regulation (EU) 2023/2854 of the European Parliament and of the Council of 13 December 2023 on harmonised rules on fair access to and use of data amending Regulation (EU) 2017/2394 and Directive (EU) 2020/1828 (Data Act); available for download at: https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=OJ:L_202302854&qid=1734448380817

[10]Amazon, Microsoft, and Google each announced policies to remove egress fees for switchers following the passage of the Data Act. However, these policies do not apply to multi-clouders.

We consider a duopoly market in which multi-product firms sell complementary services to a mass of buyers. A key feature of our setup is that multi-stop shoppers, who combine the services of several firms, can be costlier to serve than one-stop shoppers, who buy all services from the same firm. In the cloud services industry, these cost differences correspond to the data transfer costs that sellers on both sides incur when serving multi-clouders. Another important feature of our setup is that buyers have elastic demands. The elasticity of demand captures the key aspect of the cloud that users can flexibly scale up or down consumption, and it allows us to analyze the effects of regulation on the intensive margin as well as the extensive margin of consumption, both of which are important to assess the welfare effects of any regulation.

Our analysis addresses two sets of questions. First, do firms price discriminate against multi-stop shoppers in equilibrium when they are allowed to do so? Second, what are the effects of banning price discrimination against multi-stop shoppers on social welfare, firm profits, and consumer surplus? Throughout the analysis, price discrimination against multi-stop shoppers is defined as a price premium for multi-stop shoppers that exceeds the marginal cost premium of serving them. A ban on price discrimination against multi-stop shoppers in the model corresponds to a ban on above-cost data transfer fees in our motivating example of cloud services.

We first consider a model variant in which buyers are exogenously allocated to a specific combination of products, so that prices have no effect on the share of one-stop versus multi-stop shoppers but only affect quantity choices. In the context of cloud services, such a situation might occur if buyers are locked into a specific cloud architecture due to tight integration between that architecture and the buyer's internal systems and processes or due to engineering lock-in—whereby engineers develop skills tailored to a particular cloud architecture.

In this independent-markets model, discrimination against multi-stop shoppers arises in a symmetric equilibrium as long as the cost differential between serving the two shopper types is not too large. The price discrimination against multi-stop shoppers is driven by the familiar double marginalization effect (or 'Cournot effect') that occurs when sellers of complementary goods set price non-cooperatively.[11] For the effects of a ban on such price discrimination, the curvature of the demand function plays an important role. We show that a ban on price discrimination raises aggregate output and is beneficial for social welfare if the demand function (which determines a buyer's quantity given the total price per unit of the complementary goods) is "not too convex."[12] Moreover, under the same condition on demand curvature, a ban

[11]On the pricing of complements, see Cournot (1838), Sonnenschein (1968), and Amir and Gama (2019).

[12]In cases where the curvature of inverse demand varies along the demand curve, an additional condition is that the demand does not becomes more convex too fast as price falls/quantity rises.

on price discrimination against multi-stop shoppers also benefits firms, who are caught in a prisoners' dilemma under unrestricted pricing. The effects of a ban on aggregate consumer surplus are less clear-cut, but we show that under linear demand, a ban on price discrimination raises aggregate consumer surplus if the cost premium of serving multi-stop shoppers is not too large. Overall, our analysis of the independent-markets model suggests that a ban on discrimination against multi-stop shoppers raises overall social welfare as well as aggregate profits and aggregate consumer surplus for a wide range of functional forms of demand.

Next, we allow for demand inter-dependencies between the one-stop and multi-stop shopper markets, analyzing a model of competitive mixed-bundling with elastic demands and linear tariffs. Each buyer's product choices are now based on the buyer's random match utilities for the different products and on prices. Prices thus affect buyer decisions at the extensive margin as well as the intensive margin. We show that if discrimination against multi-stop shoppers arises in equilibrium under independent markets, then it also arises in equilibrium when markets are interdependent. Moreover, if buyers' product preferences are sufficiently weak, then price discrimination against multi-stop shoppers arises under interdependent markets even if the cost premium of serving multi-stop shoppers exceeds the equilibrium price premium paid by multi-stop shoppers under independent markets.

Since discrimination against multi-stop shoppers discourages multi-stop shopping at the extensive margin when markets are interdependent, one might expect that allowing discrimination raises greater competition concerns in the interdependent-markets model than in the baseline independent-markets model. Our analysis reveals that this is not the case. We show that price discrimination against multi-stop shoppers can be welfare and consumer-surplus enhancing in the presence of sufficiently strong demand inter-dependencies even for functional forms of demand (such as linear) that would yield the opposite result when markets are independent. Intuitively, the welfare-enhancing effect of discrimination arises because, by encouraging buyers to switch from multi-stop shopping to one-stop shopping, price discrimination leads to a greater share of buyers consuming quantities closer to the socially optimal level and to marginal cost efficiencies. Moreover, price discrimination can also intensify competition, sometimes even leading to lower prices for all buyers, which benefits infra-marginal shoppers (those who do not switch) and raises their consumption levels closer to socially optimal levels.

The paper is organized as follows. Section 2 discusses connections to existing literature. Section 3 presents the model. Section 4 contains the analysis and results for the case of independent markets, while Section 5 deals with the case of interdependent markets. Conclusions and policy implications are in Section 6. Proofs are relegated to the Appendix.

2 Related literature

Third-degree price discrimination Our analysis of the independent-markets model is related to the literature on third-degree price discrimination, which studies the effects of allowing firms to charge different prices for the same good to buyers in different markets. Within that literature, our analysis relates most closely to the work on price discrimination in symmetric oligopoly, where the same set of symmetrically differentiated sellers compete in multiple independent markets (Holmes, 1989; Adachi and Matsushima, 2014; Armstrong and Vickers, 2001; Miklós-Thal and Shaffer, 2021a,b; Adachi, 2023; Dertwinkel-Kalt and Wey, 2023).[13] What distinguishes our model from this literature is that each firm sells two complementary goods, rather than a single good. This implies that under price discrimination, two-stop shoppers pay a total price above what a monopolist would charge. In contrast, existing research analyzes settings in which conduct is more competitive than monopoly in all markets.[14]

Another distinguishing feature of our model is that service costs differ across the different markets (one-stop shoppers versus two-stop shoppers), while most existing work assumes symmetric costs. Exceptions can be found in recent contributions by Chen et al. (2021) and Adachi (2023), who consider oligopoly settings in which the sellers' marginal costs differ across markets. Chen et al. (2021) consider markets that differ only in costs and show that allowing firms to charge different prices in different markets is often beneficial for welfare, profit, and consumer surplus. Adachi's (2023) model allows for asymmetries in costs, competitive conduct, and market-level demand elasticity, and he derives conditions for positive or negative social welfare effects that involve pass-through rates, conduct parameters, and profit margins. Unlike our paper, both of these studies focus on the effects of what Chen and Schwartz (2015) have dubbed "differential pricing," comparing the discriminatory prices to a uniform price that ignores cost differences. We instead define prices to be discriminatory if the difference in the prices paid by different shopper types differs from the cost difference of serving them, which allows us to gain insights into the effects of banning above-cost data transfer fees in our motivating example. This definition of price discrimination —as absolute price differences that fail to reflect absolute cost differences— is in line with the definitions in Phlips (1983) and in

[13]On third-degree price discrimination by a monopoly seller, see Pigou (1920); Robinson (1933); Schmalensee (1981); Varian (1985); Aguirre (2006, 2008); Cowan (2007); Aguirre et al. (2010).

[14]One exception is Adachi and Matsushima (2014) who, in a linear-demand framework, allow single-product firms to sell complementary goods. They conclude that price discrimination lowers social welfare and consumer surplus if goods are complementary in the strong market (where price rises) but substitutes/independent in the weak market (where price falls). Our finding that discrimination against two-stop shoppers harms social welfare and consumer surplus in the case of linear demand and no service cost difference across markets is consistent with their insights. However, our analysis is not restricted to linear demand and symmetric service costs.

Tirole (1988)'s textbook. Stigler (1987) proposes an alternative definition according to which price discrimination occurs when the ratio of prices differs from the ratio of marginal costs. Clerides (2004) discusses the relationship between these two definitions.

Competitive mixed bundling Our analysis of the model variant in which the demands of one-stop shoppers and two-stop shoppers are inter-dependent makes a contribution to the literature on competitive mixed bundling (Matutes and Regibeau, 1992; Anderson and Leruth, 1993; Reisinger, 2004; Thanassoulis, 2007; Armstrong and Vickers, 2010; Zhou, 2021).

Most models of competitive mixed bundling, with the exception discussed below, assume that buyers have inelastic demands. In our model, buyers have elastic demands. This difference is important for the welfare effects of discrimination. When buyers have inelastic demands, then a bundling discount in excess of the cost difference between serving two-stop versus one-stop shoppers harms welfare because it implies an inefficient pattern of shopping choices. When buyers have elastic demands, however, then the welfare effect of allowing such bundling discounts are a priori ambiguous, and as we show can be positive, because discrimination can raise the consumption of (some or all) buyers closer to efficient levels.

Armstrong and Vickers (2010) provide a comprehensive analysis of competitive mixed bundling that considers both inelastic and elastic demand. A key difference between their analysis and ours lies in the class of tariffs being considered.[15] In our model, firms are assumed to use linear tariffs both when discrimination is allowed (that is, when tariffs can depend on whether a buyer opts for one-stop or two-stop shopping) and when discrimination is banned (that is, when tariffs cannot depend on whether a buyer opts for one-stop or two-stop shopping). In contrast, Armstrong and Vickers (2010) compare *non-linear* tariffs when discrimination is allowed to linear tariffs when discrimination is banned. Not surprisingly, this difference greatly affects the analysis and results. In Armstrong and Vickers (2010), the unrestricted pricing game has an equilibrium in which each firms offers a menu of efficient two-part tariffs with marginal prices equal to marginal costs. The key welfare trade-off that arises is then between excessive marginal prices (under non-discriminatory linear tariffs) versus excessive loyalty (under discriminatory non-linear tariffs). When the firms use linear tariffs in both the discriminatory and the non-discriminatory regime, as in our model, the effects of allowing discrimination are more complex, because discrimination does not necessarily lead

[15] Another difference is that on-stop shoppers are cheaper to serve than one-stop shoppers in our model. In Armstrong and Vickers (2010), all shopper types are equally costly to serve, but buyers may incur exogenous two-stop shopping cost. Adding such a cost on the buyer side to our model would not significantly alter the analysis.

to lower marginal prices for all shopper types.[16]

In short, this paper offers the first analysis of competitive mixed bundling in which buyers have elastic demands and firms use linear tariffs in all regimes.

Economics of the cloud The literature on the economics of cloud computing services is still in its infancy. A recent report by Biglaiser et al. (2024) offers a survey of the industry, the main anti-competitive concerns and policy initiatives in the sector, and key economic concepts relevant for studying the industry.

Some recent research focuses on the policy concern that data transfer fees deter switching between cloud providers. Gans et al. (2023) explore the economic effects and implementation of the ban on egress fees in the European Data Act. Their discussion includes a simple model in which unregulated data transfer fees are equal to the marginal cost of providing data transfers. Banning data transfer costs therefore leads to inefficiently high levels of data transfer, and, as they argue, may trigger increases in other fees. Biglaiser et al. (2025) analyze the effects of egress fees on switching decisions in a dynamic model of competition between single-product firms. They focus on the role of long-term contracts, showing that when such contracts are feasible, egress fees reduce firm profits while improving consumer surplus. Conversely, when long-term contracts are infeasible, allowing egress fees benefits firms and harms consumer surplus. None of these studies analyzes the policy concern that above-cost data transfer fees could harm competition by discouraging multi-clouding, which is the focus of our analysis.

Less closely related to our analysis, a few recent papers analyze pricing by a monopolistic provider of cloud services. Bergemann and Wang (2025) and Bergemann and Deb (2025) analyze committed spend contracts, in which buyers who commit to minimum spending levels over a period of time receive discounts, another business practice in the cloud computing industry that has attracted regulatory scrutiny. Both papers consider a monopolistic seller that offers a portfolio of services to buyers with uncertain future demands. Bergemann and Wang (2025) characterize the optimal dynamic mechanism and show that it can be implemented by a committed spend contract. Bergemann and Deb (2025) show that a committed spend mechanism is robustly optimal when the seller does not know the true distribution of the buyer's demands. Also focusing on a monopolistic cloud services provider, Hummel and Schwarz (2022) analyze the provider's capacity and pricing decisions when serving customers with uncertain demands in multiple regions.

[16] The only other discussion of competitive bundling with elastic demand that we are aware of is a brief analysis in the appendix of the working paper version of Zhou (2017). His analysis considers linear tariffs both when discrimination is allowed and when it is banned, as in our paper. However, his paper deals with *pure* bundling of independent goods, whereas we analyze mixed bundling with perfect complements.

3 Model

We consider a market in which two firms ($i = A, B$) sell two products ($k = 1, 2$) to a mass of buyers of measure one. The two products are perfect complements, with a one-to-one consumption ratio. The four possible product combinations are denoted by $ij \in \{AA, AB, BA, BB\}$, where ij indicates that product 1 is supplied by firm i and 2 by firm j. We will say that a buyer one-stop shops if $ij \in \{AA, BB\}$, and that a buyer two-stop shops if $ij \in \{AB, BA\}$.

Each firm incurs a constant marginal cost of production $c \geq 0$ for each of the two products. Serving two-stop shoppers can create additional marginal costs for firms.[17] We denote the additional per-unit cost incurred by the seller of product 1 by $c_E \geq 0$ (where E stands for egress), and the additional per-unit cost incurred by the seller of product 2 by $c_I \geq 0$ (where I stands for ingress). The sum of the additional marginal costs of serving a two-stop shopper are denoted by $c_D = c_E + c_I$.

The prices that firm i charges to one-stop shoppers are denoted by $\bar{p}_1^i$ and $\bar{p}_2^i$, and the prices it charges to two-stop shoppers by p_1^i and p_2^i.[18] Note that, since the two products are perfect complements, buyer demand and firm profits depend only on the sum $p_{12}^i \equiv \bar{p}_1^i + \bar{p}_2^i$, not on $\bar{p}_1^i$ and $\bar{p}_2^i$ individually. We define prices to be discriminatory if the difference in the prices for the different shopper types differs from the cost difference of serving them, that is, if $p_1^i - \bar{p}_1^i \neq c_E$ or $p_2^i - \bar{p}_2^i \neq c_I$. Moreover, we will say that two-stop shoppers are discriminated against in a symmetric equilibrium (where $p_k^A = p_k^B = p_k$ for all $k \in \{1, 2, 12\}$) if they pay a price premium that exceeds the total cost difference of serving them, that is, if $p_1 + p_2 - p_{12} > c_D$.

We will consider two model variants of buyer behavior. In Section 4, buyers are exogenously allocated to a product combination, hence the demands for the four different product combinations are independent. In Section 5, we allow for demand inter-dependencies across markets. The first model variant can be obtained as a limiting case of the second.

In both model variants, a buyer's quantity choice is given by the demand function

$$q(P) = \arg\max_{q \geq 0} \left(u(q) - Pq\right).$$

where P is the total per-unit price of the product combination that the buyer purchases ($P = p_{12}^i$ if the buyer purchases both products from seller i, $P = p_1^A + p_2^B$ if the buyer purchases combination AB, and so forth). The utility function $u(\cdot)$ is twice differentiable, with $u(0) = 0$

[17]In the cloud services example, such costs arise because integrated multi-clouding requires data to be transferred from the cloud of the seller of product 1 (which can be thought of as 'data storage') to the cloud of the seller of product 2 (which can be thought of 'data analytics'), and such data transfer is costly for providers.

[18]In the cloud services application, the prices charged to two-stop shoppers correspond to the sum of the price of service 1 plus the egress fee, and the sum of the price of service 2 plus the ingress fee.

and $u'(P) > 0 > u''(P)$ for $P \geq 0$. A buyer's indirect utility function is denoted by

$$V(P) = \max_{q \geq 0} \left(u(q) - Pq \right) = u\left(q\left(P\right)\right) - Pq\left(P\right),$$

which is assumed to be continuously differentiable.

Assumption 1. The demand function $q(P)$ satisfies the following regularity conditions:

(1) There exists a choke price $\overline{P} \in (2c + c_D, \infty]$ such that $q(P) > 0$ if and only if $P < \overline{P}$,

(2) $q(P)$ is thrice continuously differentiable with $q'(P) < 0$ for all $P < \overline{P}$ and

$$\lim_{P \to \overline{P}} \left(\frac{q(P)}{-q'(P)} \right) = 0.$$

(3) $2q'(P) + (P - 2c)q''(P) < 0$ for all $P < \overline{P}$.

Assumption 1(3) requires that the profit of a monopoly seller serving a one-stop shopper is strictly concave. The limit condition in Assumption 1(2) is a technical condition that always holds if the slope of $q(P)$ is bounded below zero for prices below the choke price.

The elasticity of the demand function will be denoted by $\varepsilon(P) = \frac{-Pq'(P)}{q(P)}$. We further denote the Lerner index of the bundle of the two products given a total marginal cost C by

$$L(P; C) = \frac{P - C}{P}.$$

The inverse of $q(P)$ will be denoted by $p(q)$, and the curvature of the inverse demand function by

$$\sigma(q) = -q\frac{p''}{p'} = \frac{qq''}{(q')^2}.$$

In the independent-markets model variant, an exogenous share $(1-\gamma) \in (0,1)$ of buyers are one-stop shoppers, half of them captive to AA and the other half captive to BB. The remaining share γ of buyers are two-stop shoppers, half of them captive to AB and half of them captive to BA.

The second model variant, in which markets are allowed to be interdependent, is a discrete-continuous choice framework in which each of the two products is horizontally differentiated across firms. To model product preferences, we adopt a multi-product version of Perloff and Salop (1985)'s random utility framework.[19] A buyer has a match utility $x_k^i \in [0,1]$ for firm i's

[19]See Zhou (2021) for an application of the random utility framework in a competitive mixed bundling model with inelastic demands.

product k, where x_k^i is a random drawn from some probability distribution. A buyer's gross utility from consuming q units of product combination $ij \in \{AA, AB, BA, BB\}$ is now

$$u(q) + t(x_1^i + x_2^j),$$

where $t(x_1^i + x_2^j)$ is the random utility component. The parameter $t > 0$ measures the importance of product preferences relative to the base utility from consumption. A buyer's relative preference for firm B's product k over firm A's product, $z_k \equiv x_k^B - x_k^A$, is assumed to follow a joint cumulative distribution function $H(z_1, z_2)$ with a density $h(z_1, z_2) > 0$ on $[-1, 1]^2$. To simplify the analysis, we focus on cases in which buyers' preferences are symmetric across sellers and across products:

$$h(z_1, z_2) = h(-z_1, -z_2) = h(z_2, z_1).$$

4 Independent markets

4.1 Pricing

Unrestricted pricing Suppose each firm i can price discriminate between shoppers in the three different markets $\{ii, ij, ji\}$ that it serves. In its one-stop shopper market, firm i then optimally sets the price p_{12}^i to solve monopoly profit maximization problem

$$\max_{p_{12}^i} \quad \frac{1-\gamma}{2}\left(p_{12}^i - 2c\right) q\left(p_{12}^i\right).$$

The first-order condition determines the one-stop shopper equilibrium price $p_{12}^i = P_M$:

$$P_M : q(P_M) + (P_M - 2c)\, q'(P_M) = 0, \tag{1}$$

or, written using the Lerner index and the elasticity of demand, $L(P_M; 2c)\varepsilon(P_M) = 1$. By the strict concavity of the monopoly profit function (Assumption 1(3)), $\sigma(q(P_M)) < 2$.

In the two-stop shoppers markets, the seller of product 1 sets its price p_1^i to solve

$$\max_{p_1^i} \quad \frac{\gamma}{2}\left(p_1^i - c - c_E\right) q\left(p_1^i + p_2^j\right).$$

and the seller of the complementary product 2 sets its price p_2^j to solve

$$\max_{p_2^j} \quad \frac{\gamma}{2}\left(p_2^j - c - c_I\right) q\left(p_1^i + p_2^j\right).$$

The first-order conditions for the equilibrium prices (p_1, p_2) paid by two-stop shoppers are thus

$$\begin{aligned} q(p_1 + p_2) + (p_1 - c - c_E)\, q'(p_1 + p_2) &= 0, \\ q(p_1 + p_2) + (p_2 - c - c_I)\, q'(p_1 + p_2) &= 0, \end{aligned}$$

which implies that $p_1 - c_E = p_2 - c_I$. Adding up the two first-order conditions yields the following condition determining the total per-unit price $P_D = p_1 + p_2$ paid by two-stop shoppers:

$$P_D : q(P_D) + (P_D - 2c - c_D)\frac{q'(P_D)}{2} = 0, \tag{2}$$

or, equivalently, $L(P_D; 2c + c_D)\varepsilon(P_D) = 2$. The regularity assumptions in Assumption 1(3) imply that $\sigma(q(P_D)) < 1$,[20] (2) has a unique solution, the prices set by the two firms for two-stop shoppers are strategic substitutes, and the equilibrium is locally stable.

It must be that $P_D > P_M$, because $P_D \leq P_M$ would imply the following contradiction:

$$0 \leq q(P_D) + (P_D - 2c)\,q'(P_D) < 2q(P_D) + (P_D - 2c - c_D)\,q'(P_D) = 0,$$

where the inequalities follow from (1), Assumption 1, and $c_D \geq 0$. Independent suppliers of complements set higher prices than a monopoly supplier of both products would because of the familiar double marginalization effect, also known as the 'Cournot effect.' Hence, under discrimination, two-stop shoppers pay a higher total price than one-stop shoppers even if there is no cost difference between serving the two types of shoppers ($c_D = 0$). If the firms incur additional marginal costs to serve two-stop shoppers ($c_D > 0$), then this further raises the gap between the two prices.

When there is no service cost difference ($c_D = 0$), then the double marginalization effect directly implies that two-stop shoppers are discriminated against in equilibrium, i.e., that $P_D - P_M > c_D$. Whether this continues to be true for $c_D > 0$ depends on the size of the cost difference and the pass-through rate in the two-stop shopper markets, which is equal to

$$\frac{\partial P_D}{\partial c_D} = \frac{1}{3 + (P_D - 2c - c_D)\frac{q''(P_D)}{q'(P_D)}} = \frac{1}{3 - 2\sigma(q(P_D))} < 1, \tag{3}$$

by Assumption 1. Hence, $P_D - P_M < c_D$ for large enough c_D. Nonetheless, since $P_D > P_M$ at $c_D = 0$ and P_D is continuous in c_D, there always exists a $\bar{c}_D > 0$ such that $P_D - P_M > c_D$ for all $c_D < \bar{c}_D$.

In summary, provided the service cost differential is not too large, there is discrimination against two-stop shoppers at the unrestricted prices. This discrimination is due to the double-marginalization effect that arises when competing firms sell complementary goods. The remainder of the analysis in this section will focus on cases in which $P_D - P_M > c_D$.

[20] Using the first-order condition in (2), the concavity condition in Assumption 1(3), evaluated at P_D, can be rewritten as $\frac{P_D - 2c - c_D}{P_D - 2c} > \sigma(q(P_D))$, which implies $1 > \sigma(q(P_D))$.

Restricted pricing We now analyze pricing under restrictions on the firms' ability to set differential prices to one-stop shoppers versus two-stop shoppers. Recall that the prices firm i charges to one-stop shoppers are denoted by $\bar{p}_1^i$ and $\bar{p}_2^i$, summing up to a total price per unit of $p_{12}^i = \bar{p}_1^i + \bar{p}_2^i$. Suppose now that each firm faces the restrictions $p_1^i \leq \bar{p}_1^i + r_1$ and $p_2^i \leq \bar{p}_2^i + r_2$. Each seller i then solves the following constrained maximization problem:

$$\max_{\left(p_{12}^i, p_1^i, p_2^i, \bar{p}_2^i\right)} \left(\frac{1-\gamma}{2} \left(p_{12}^i - 2c\right) q\left(p_{12}^i\right) + \frac{\gamma}{2} \left(p_1^i - c - c_E\right) q\left(p_1^i + p_2^j\right) + \frac{\gamma}{2} \left(p_2^i - c - c_I\right) q\left(p_1^j + p_2^i\right) \right)$$

$$\text{s.t. } p_1^i \leq p_{12}^i - \bar{p}_2^i + r_1, \text{ and } p_2^i \leq \bar{p}_2^i + r_2.$$

The equilibrium prices depend on r_1 and r_2 only through their sum $r = r_1 + r_2$, because any slack in one of the two constraints can be transferred to the other constraint by adjusting $\bar{p}_2^i$. Focusing on cases in which the unrestricted prices discriminate against two-stop shoppers ($P_D - P_M > c_D$), a ban on discrimination thus corresponds to imposing $r = c_D$.

The equilibrium prices depend on r as follows. If $r \geq P_D - P_M$, buyers pay the unrestricted discriminatory prices P_M and P_D derived previously. If $r < P_D - P_M$, then

$$p_{12}^i = P(r) > P_M, \quad p_1^i + p_2^i = P(r) + r < P_D,$$

and $p_1^i - c_E = p_2^i - c_I$ for each i, where $P(r)$ is determined by the first-order condition[21]

$$(1-\gamma)\left(q(P) + (P - 2c)\, q'(P)\right) + \gamma \left(q(P+r) + (P + r - 2c - c_D) \frac{q'(P+r)}{2} \right) = 0. \quad (4)$$

The left-hand side of this first-order condition is a weighted average of each firm's marginal profit in its one-stop shopper market and the sum of its marginal profits in the two-stop shopper markets. Intuitively, the condition ensures that starting from symmetric prices P for one-stop shoppers and $p_1 + p_2 = P + r$ for two-stop shoppers, there is no profitable unilateral deviation to prices $(P + 2\epsilon, p_1 + \epsilon, p_2 + \epsilon)$ that keep the constraint on the price difference binding.

Compared to the unrestricted discriminatory prices, one-stop shoppers pay more but two-stop shoppers pay less when there is a constraint on the degree of differential pricing. Applying the implicit function theorem to (4) yields the rate at which the price for one-stop shoppers changes in the degree of differential pricing that is allowed:

$$P'(r) = \frac{-\gamma \Delta\pi_D}{(1-\gamma)\Delta\pi_M + \gamma\Delta\pi_D}, \quad (5)$$

where

$$\Delta\pi_M \equiv 2q'(P) + (P - 2c)\, q''(P) < 0,$$
$$\Delta\pi_D \equiv \frac{3}{2} q'(P+r) + (P + r - 2c - c_D) \frac{q''(P+r)}{2} < 0.$$

[21] The second-order sufficient condition follows from Assumption 1.

The regularity assumptions in Assumption 1 imply that $P'(r) \in (-1, 0)$. Hence, as the degree of differential pricing r that is allowed rises (within the range in which it is binding), the equilibrium one-stop shopper price falls, while the equilibrium two-stop shopper price rises: $P'(r) + 1 \in (0, 1)$ for all $r \in [0, P_D - P_M)$.

4.2 Effects of price discrimination against two-stop shoppers

We now analyze the effect of banning discrimination. Throughout the analysis, we continue to focus on cases in which $P_D - P_M > c_D$, so that the firms would discriminate against two-stop shoppers in the absence of a ban. Moreover, we restrict attention to demand functions with a constant inverse demand curvature σ in the main text. Such demand functions are commonly used in the IO literature (see, for instance, Bulow and Pfleiderer (1983); Genesove and Mullin (1998)), and the restriction greatly simplifies the exposition. In Appendix B, we provide a detailed analysis of the effects of price discrimination against two-stop shoppers when the curvature of inverse demand can vary along the demand curve, and we also derive conditions on the curvature of direct (as opposed to inverse) demand. Although the analysis and conditions in our formal propositions are more involved in that more general case, the qualitative results from the analysis with a constant inverse demand curvature carry over.

4.2.1 Effects of discrimination on output and social welfare

Following the marginal-effects method introduced by Schmalensee (1981) to analyze the effects of monopolistic third-degree price discrimination, and used by Holmes (1989) and Aguirre et al. (2010) among others, we denote the degree of differential pricing that is allowed by $r \in [0, P_D - P_M]$.[22] A ban on price discrimination corresponds to the restriction $r = c_D$, while unrestricted pricing corresponds to $r \geq P_D - P_M$. The marginal change in social welfare as more differential pricing is allowed is

$$\begin{aligned} W'(r) &= (1-\gamma)(P(r) - 2c)\, q'(P(r))\, P'(r) \\ &\quad + \gamma (P(r) + r - 2c - c_D)\, q'(P(r) + r)\,(P'(r) + 1), \end{aligned} \tag{6}$$

and the effect of allowing price discrimination on social welfare can be found by integrating the marginal welfare effects as the constraint rises from $r = c_D$ to $r = P_D - P_M$:

$$\Delta W = \int_{c_D}^{P_D - P_M} W'(r)\, dr$$

[22] As shown in the previous subsection, the equilibrium prices depend on r_1 and r_2 only through $r = r_1 + r_2$. Hence, r fully captures restrictions on the ability of firms to set differential prices for one-stop versus two-stop shoppers.

Denoting the marginal change in the total quantity consumed as more discrimination is allowed by

$$Q'(r) = (1-\gamma)q'(P(r))P'(r) + \gamma q'(P(r)+r)(P'(r)+1), \tag{7}$$

the marginal welfare effect of more discrimination can be decomposed as follows:

$$W'(r) = \underbrace{(P(r)-2c)\,Q'(r)}_{\text{(value of) marginal output effect}} + \underbrace{\gamma(r-c_D)\,q'(P(r)+r)(P'(r)+1)}_{\text{marginal allocation effect}}. \tag{8}$$

The marginal output effect has the sign of $Q'(r)$. The sign of the marginal allocation effect depends on whether r lies below or above c_D. For $r > c_D$, a marginal increase in the degree of discrimination r worsens allocative efficiency and this effect is strictly negative. Intuitively, for any given total output, allocative efficiency is maximized if output is allocated to the buyers who value it most above marginal costs, which requires the price difference to equal the cost difference. It follows that for any $r \geq c_D$, $Q'(r) \leq 0$ implies that $W'(r) < 0$. Hence, if $Q'(r) \leq 0$ for all $r \in [c_D, P_D - P_M]$, then a ban on discrimination improves social welfare.

Since a negative marginal output effect is a sufficient condition for a negative marginal welfare effect, we evaluate the effect of price discrimination on the total quantity sold. Using (5) and simplifying, we obtain

$$Q'(r) = \underbrace{\left(\frac{\gamma(1-\gamma)}{2}\right)\left(\frac{q'(P)q'(P+r)}{(1-\gamma)\Delta\pi_M + \gamma\Delta\pi_D}\right)}_{<0} \times \left(1 + 2(P-2c)\frac{q''(P)}{q'(P)} - (P+r-2c-c_D)\frac{q''(P+r)}{q'(P+r)}\right)$$

Hence,

$$Q'(r) \overset{sign}{=} -1 + 2(P-2c)\frac{q''(P)}{-q'(P)} - (P+r-2c-c_D)\frac{q''(P+r)}{-q'(P+r)}, \tag{9}$$

$$= -1 + \sigma\left(2L(P;2c)\varepsilon(P) - L(P+r;2c+c_D)\varepsilon(P+r)\right). \tag{10}$$

where $P = P(r)$. The expression in (9) directly implies that discrimination reduces aggregate output if the demand function is linear. Our next proposition shows that, more generally, a sufficient condition for discrimination to have a negative marginal output effect throughout the relevant range—and, thus, for price discrimination to harm social welfare—is that the demand function is not "too convex."

Proposition 1. Price discrimination against two-stop shoppers reduces aggregate output and harms social welfare if

$$\sigma \leq \frac{1}{L(P(c_D);2c)\varepsilon(P(c_D))}, \tag{11}$$

where the upper bound on σ in (11) lies between $1-\gamma+\frac{\gamma}{2}\frac{q'(P(c_D)+c_D)}{q'(P(c_D))} \in (0,1)$ and 1.

The intuition for the role of demand curvature for the effect of discrimination on aggregate output can be understood as follows. Suppose that price discrimination, which makes prices more dispersed across buyers, does not change the average price (i.e., that $(1-\gamma)P(c_D) + \gamma(P(c_D)+c_D) = (1-\gamma)P_M + \gamma P_D$). If demand is concave, price discrimination then leads to a fall in aggregate output; if demand is convex (linear), it leads to a rise (no change) in aggregate output. It follows that price discrimination that leads to a strictly higher average price causes lower aggregate output when demand is concave or linear (and, by continuity, if demand is convex but not too much so).[23]

The impact of price discrimination on the average price, in turn, is driven by the difference in competitive conduct across markets (and demand curvature). When the prices for two-stop shoppers are set non-cooperatively and demand is linear, then the average price is lower under a ban on price discrimination than with discrimination because of the double marginalization effect in the two-stop shopper markets. Intuitively, firm i internalizes the full effect of a price change on the quantity demanded by its one-stop shoppers when changing p^i_{12}, but it internalizes only part of the effect on the quantities demanded by two-stop shoppers when changing p^i_1 and p^i_2. This implies that, given linear demand, a firm's marginal profit in its one-stop shopper markets falls in price at a faster rate than the sum of the marginal profits in the two-stop shopper markets falls in price.[24] Pricing in the presence of a ban on discrimination is therefore "biased" towards P_M, the discriminatory price in the one-stop shopper market, and price discrimination leads to a higher average price and, hence, a lower aggregate output.

Our finding that price discrimination against two-stop shoppers reduces aggregate output is qualitatively consistent with findings in the literature on third-degree price discrimination in symmetric oligopoly. A key takeaway of that literature is that price discrimination is more

[23] This can also be seen by rewriting (7) as

$$Q'(r) = q'(P(r))\left((1-\gamma)P'(r) + \gamma\frac{-q'(P(r)+r)}{-q'(P(r))}(P'(r)+1)\right).$$

If demand is concave, then $\frac{-q'(P(r)+r)}{-q'(P(r))} \geq 1$, and hence

$$Q'(r) \leq q'(P(r))\underbrace{((1-\gamma)P'(r) + \gamma(P'(r)+1))}_{\text{marginal effect on average price}},$$

which is strictly negative if discrimination raises the average price. Similarly, if demand is convex, a marginal reduction in average price implies a positive marginal output effect.

[24] Formally, given linear demand with slope $-\beta < 0$, the marginal profit in the one-stop shoppers markets has slope -2β, while the sum of marginal profits in the two-stop shopper markets has slope $-\frac{3}{2}\beta$. More generally, the marginal profit in the one-stop shopper markets falls at a faster rate than the sum of the marginal profits in the two-stop shopper markets, and more discrimination raises the average price (i.e., $(1-\gamma)P'(r) + \gamma(P'(r)+1) > 0$), if and only if $2q'(P) + (P-2c)q''(P) < \frac{3}{2}q'(P+r) + (P+r-2c-c_D)\frac{q''(P+r)}{2}$.

likely to reduce aggregate output when competition is less intense in the market where price rises than in the market where prices falls.[25] Our finding that price discrimination lowers aggregate output when demand is not "too convex" and the curvature of inverse demand is constant (or, as shown in Appendix B, does not rise too fast as price falls) is in line with this insight, because conduct is "more competitive" in the one-stop shopper markets (monopoly conduct) than in the two-stop shopper markets (above-collusive conduct due to the double marginalization effect).

4.2.2 Effects of discrimination on firm profits

Next, we analyze the effect of price discrimination against two-stop shoppers on the firms' aggregate profits. The marginal effect of more discrimination on the firms' aggregate profits is

$$\begin{aligned}2\Pi'(r) &= (1-\gamma)\,P'(r)\left(q\left(P\left(r\right)\right)+\left(P\left(r\right)-2c\right)q'\left(P\left(r\right)\right)\right)\\ &\quad+\gamma\left(P'\left(r\right)+1\right)\left(q\left(P\left(r\right)+r\right)+\left(P\left(r\right)+r-2c-c_D\right)q'\left(P\left(r\right)+r\right)\right).\end{aligned}$$

The marginal effect in the first line is positive because more discrimination raises the profits from one-stop shoppers by lowering the price towards the monopoly level P_M. The marginal effect in the second line is negative, because more discrimination distorts the price paid by two-stop shoppers further above the monopoly level.[26]

In the proof of the next proposition, we show that the marginal profit effect is negative for all r in the relevant range if demand is not "too convex." The condition in Proposition 1, which ensures that discrimination has a negative effect on aggregate output, is also sufficient to ensure that price discrimination against two-stop shoppers lowers aggregate firm profits. In this case, the gain in profits in the one-stop shopper markets, where quantity rises but price-cost margins are lower, is insufficient to compensate for the loss in profits in the two-stop shopper markets, where quantity falls by a greater amount than the quantity gain in the one-stop shopper markets and price-cost margins are higher.

Proposition 2. Price discrimination against two-stop shoppers reduces firm profits if (11) holds.

[25]Holmes (1989) shows that this holds when demand is linear in all markets, and Miklós-Thal and Shaffer (2021b) show that it holds when the market-level demand functions are concave with a common constant curvature of inverse demand and the conduct parameters (that measure the intensities of competition in the two markets) are constant.

[26]Given $\sigma < 1$, the non-discriminatory price paid by two-stop shoppers, $P(c_D) + c_D$, lies above the price that a monopolist with marginal cost $2c + c_D$ would charge. This is because (i) $P(c_D)$ exceeds the price P_M that a monopolist with cost $2c$ would charge, and (ii) a monopolist's pass-through rate from marginal cost to price is less than 1 when demand is strictly log-concave.

Price discrimination against two-stop shoppers creates a prisoners' dilemma for the firms when it leads to a reduction in aggregate profit. Although each firm unilaterally benefits from practicing price discrimination, both firms are better off in the presence of a ban on price discrimination.

Bulow-Pfleiderer Demand: Consider the family of demand functions $q(P) = \left(\frac{a-P}{b}\right)^{\frac{1}{d}}$, where $a > 2c + c_D$ and $b, d > 0$. This family, which was also used by Bulow and Pfleiderer (1983), encompasses linear demand ($d = 1$), as well as cases of strictly concave ($d > 1$) or strictly convex ($d < 1$) demand. Demand is log-concave with a constant curvature of inverse demand $\sigma = 1-d < 1$. The discriminatory prices are $P_M = \frac{ad+2c}{1+d}$ and $P_D = \frac{2ad+2c+c_D}{1+2d}$, [27] and the unrestricted equilibrium features discrimination against two-stop shoppers if $c_D < \frac{a-2c}{2(1+d)}$.

For concave demand ($d \geq 1$), we know from Propositions 1 and 2 that discrimination against two-stop shoppers reduces aggregate output, social welfare, and firm profits. For strictly convex demand ($d < 1$), the upper bound on the inverse demand curvature in condition (11) is at least

$$1 - \gamma + \frac{\gamma}{2}\frac{q'(P(c_D) + c_D)}{q'(P(c_D))} \geq 1 - \gamma + \frac{\gamma}{2}.$$

It follows that for $\frac{\gamma}{2} \leq d < 1$, our propositions again imply that discrimination against two-stop shoppers reduces aggregate output, social welfare, and firm profits. In summary, Propositions 1 and 2 imply that discrimination has negative output, welfare, and profit effects for any $d \geq \frac{\gamma}{2}$.

For $0 < d < \frac{\gamma}{2}$ and $c_D = 0$, we can obtain closed-form expressions for the equilibrium prices and effects of discrimination. In this case, straightforward (but tedious) calculations show that price discrimination continues to reduce aggregate output, social welfare, and total profits. For $0 < d < \frac{\gamma}{2}$ and $0 < c_D < \frac{a-2c}{2(1+d)}$, we are not able to obtain closed-form expressions, but extensive numerical simulations suggest that the effects of price discrimination against two-stop shoppers on aggregate output, social welfare, and profits remain negative.

4.2.3 Effects of discrimination on consumer surplus

Finally, we consider the effects of discrimination against two-stop shoppers on aggregate consumer surplus. For a given $r \geq c_D$, aggregate consumer surplus is

$$CS(r) = (1 - \gamma)V(P(r)) + \gamma V(P(r) + r).$$

[27] Assumption 1 (1) and (2) hold for all $d > 0$, but Assumption 1 (3) holds only when $d \geq 1$. Nonetheless, it can be checked that each profit function in the discriminatory pricing regime has a single interior peak, ensuring the existence of unique monopoly and duopoly equilibrium prices, respectively.

The marginal effect of discrimination on aggregate consumer surplus is therefore

$$\begin{aligned} CS'(r) &= (1-\gamma)V'\left(P(r)\right)P'(r) + \gamma V'\left(P(r)+r\right)\left(P'(r)+1\right) \\ &= -(1-\gamma)q\left(P(r)\right)P'(r) - \gamma q\left(P(r)+r\right)\left(P'(r)+1\right). \end{aligned} \tag{12}$$

Price discrimination against two-stop shoppers affects consumer surplus through two channels. First, discrimination raises price dispersion. Since consumer surplus is convex in price, this means that keeping the average price constant, discrimination would raise consumer surplus (see also Waugh (1944) and Chen et al. (2021)). Second, discrimination changes the average price. If discrimination reduces the average price, then this second effect goes in the same direction as the first effect and buyers benefit from price discrimination. If discrimination raises the average price, however, which, as discussed earlier, happens for a wide range of demand functions, the two effects go in opposite directions and the effect of discrimination on aggregate consumer surplus is a priori ambiguous.[28]

The next proposition shows that consumer surplus is quasi-convex in the degree of discrimination if demand is weakly concave: $CS(r)$ is either decreasing everywhere, or first decreasing and then increasing, or increasing everywhere. Moreover, one of the first two cases applies, that is, consumer surplus is initially decreasing, if the condition for a negative output effect from Proposition 1 holds and the cost premium of serving two-stop shoppers is not too large. However, unlike in our earlier analyses of welfare and profits, this condition is not sufficient to conclude that price discrimination has a negative effect overall.

Proposition 3. The following holds:

(i) $CS(r)$ is quasi-convex for $r \in [c_D, P_D - P_M]$ if $\sigma \leq 0$.

(ii) If condition (11) holds, then there exists $\bar{c}_D > 0$ such that $CS'(c_D) \leq 0$ for $c_D < \bar{c}_D$.

(iii) $CS'(P_D - P_M) \leq 0$ if and only if $\frac{P_M - 2c}{2-\sigma} \leq \frac{P_D - 2c - c_D}{3 - 2\sigma}$.

Under linear demand, Proposition 3 implies that aggregate consumer surplus first falls (provided c_D is small enough) and then rises as the degree of price discrimination rises. Intuitively, the marginal consumer surplus effect becomes positive as the degree of discrimination rises because the difference in the quantities purchased by one-stop shopper versus two-stop shoppers

[28]This can be seen formally by noting that (12) implies

$$CS'(r) \geq -q(P)\underbrace{\left((1-\gamma)P'(r) + \gamma(P'(r)+1)\right)}_{\text{marginal change in average price}}.$$

This condition implies that $CS'(r) \geq 0$ when the marginal change in average price is negative, but leaves the sign of $CS'(r)$ undetermined when the marginal change in average price is strictly positive.

rises with the degree of discrimination. The extent to which one-stop shoppers benefit from a further marginal price reduction is therefore larger, and the extent to which two-stop shoppers are harmed by a further marginal price increase smaller, at high degrees of discrimination. It is worth noting that information about the price-cost margins and the pass-through rates (equal to $\frac{1}{2-\sigma}$ and $\frac{1}{3-2\sigma}$ in one-stop and two-stop shopper markets, respectively) at the discriminatory pricing equilibrium is sufficient to evaluate whether the condition in Proposition 3(iii) holds.

To further explore the effects of price discrimination against two-stop shoppers on aggregate consumer surplus, we return to our previous example.

Bulow-Pfleiderer Demand: Consider the example from the previous subsection in which $q(P) = \left(\frac{a-P}{b}\right)^{\frac{1}{d}}$, where $a > 2c + c_D$ and $b, d > 0$. As shown before, condition (11) holds if $d > \frac{\gamma}{2}$. Hence, $CS'(c_D) \leq 0$ for c_D close enough to zero if $d > \frac{\gamma}{2}$. Moreover, the necessary and sufficient condition for $CS'(P_D - P_M) \leq 0$ in Proposition 3 becomes

$$\frac{a-2c}{(1+d)^2} \leq 2\left(\frac{a-2c-c_D}{(1+2d)^2}\right),$$

which is equivalent to $d \leq \frac{1}{\sqrt{2}}\left(\frac{\sqrt{(a-2c)(a-2c-c_D)}-2c_D}{a-2c+c_D}\right) \leq \frac{1}{\sqrt{2}}$, with equality of the latter for $c_D = 0$. Hence, Proposition 3 implies that discrimination reduces aggregate consumer surplus if c_D is close enough to zero and $d \in (\frac{\gamma}{2}, \frac{1}{\sqrt{2}})$. If $d > \frac{1}{\sqrt{2}}$ and c_D small, then $CS(r)$ is U-shaped in r, having initially negative and then positive slope as r rises from c_D to $P_D - P_M$.

For $c_D = 0$, we are able to obtain closed-form solutions for prices and directly compute the the consumer surplus effect of discrimination against two-stop shoppers:

$$CS(0) = \frac{bd}{1+d}\left(-\frac{(\gamma-2)(a-2c)}{b(2d-\gamma+2)}\right)^{\frac{1}{d}+1},$$

and

$$CS(P_D - P_M) = (1-\gamma)\left(\frac{bd}{1+d}\right)\left(\frac{a-2c}{bd+b}\right)^{\frac{1}{d}+1} + \gamma\left(\frac{bd}{1+d}\right)\left(\frac{a-2c}{2bd+b}\right)^{\frac{1}{d}+1}.$$

Straightforward (but tedious) calculations reveal that discrimination reduces consumer surplus, that is, $CS(0) > CS(P_D - P_M)$, for all $d > 0$.

Figures 1 and 2 illustrate these insights. Figure 1 illustrates that for $c_D = 0$, price discrimination against two-stop shoppers harms not only social welfare and firm profits for all values of d, but it also reduces consumer surplus. Figure 2 illustrates that for $c_D > 0$, the effect of discrimination on consumer surplus can be positive, while the effect on social welfare remains negative.

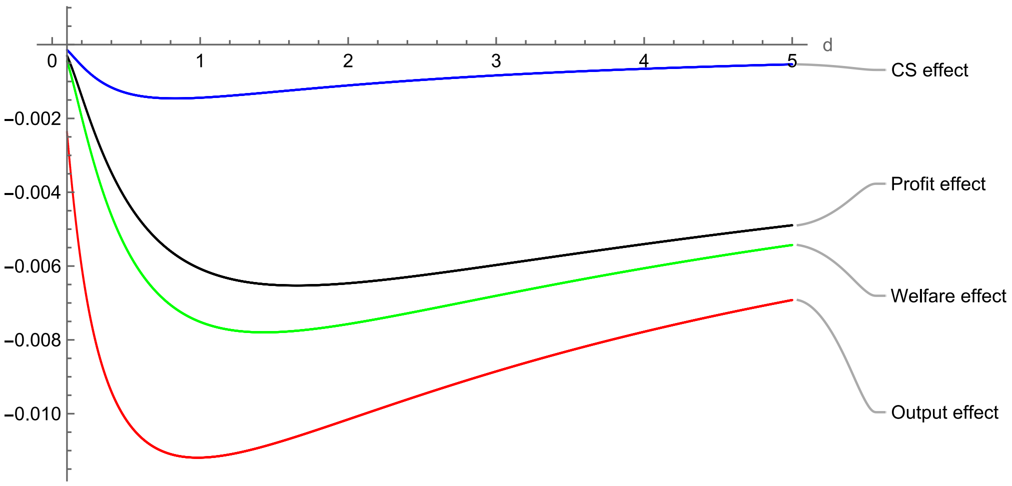

Figure 1: Effects of discrimination against two-stop shoppers when $q(P) = (1-P)^{\frac{1}{d}}$, $c = 0.1$, $c_D = 0$, and $\gamma = \frac{1}{2}$.

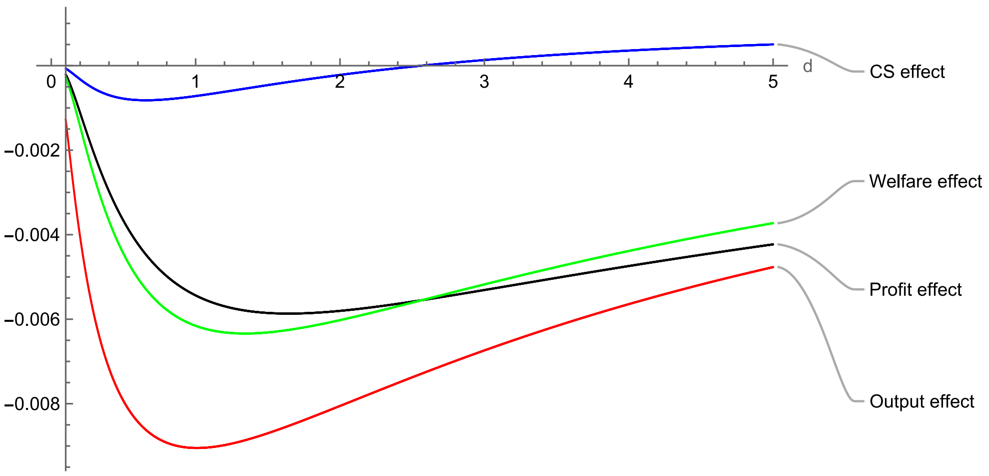

Figure 2: Effects of discrimination against two-stop shoppers when $q(P) = (1-P)^{\frac{1}{d}}$, $c = 0.1$, $c_D = 0.01$, and $\gamma = \frac{1}{2}$.

5 Interdependent markets

We now allow the demands for different product combinations to be interdependent. Each buyer chooses which products to purchase and how much to buy. The net utility of a one-stop shopper buying from firm i is $V(p_{12}^i) + t\,(x_1^i + x_2^i)$. The net utility of a two-stop shopper of product combination ij is $V(p_1^i + p_2^j) + t\left(x_1^i + x_2^j\right)$. As mentioned earlier, a buyer's relative preference for firm B's product k over firm A's product, $z_k = x_k^B - x_k^A$, follows a joint cumulative distribution function $H\left(z_1, z_2\right)$ with a density $h\left(z_1, z_2\right)$ on $[-1, 1]^2$, which is symmetric across sellers and across products, i.e., $h(z_1, z_2) = h(-z_1, -z_2) = h(z_2, z_1)$.

It should be noted that independent markets arise as a limiting case of this more general setting. Keeping the preference distribution fixed, the markets for different product combinations become less interdependent as t rises. In the limit, as $t \to \infty$, each buyer's decision as to which product combination to buy is determined solely by the buyer's relative match values (hence, each buyer's product choice is determined solely by which of the four quadrants in the (z_1, z_2)-space the buyer is located in), while prices determine quantity choices. Hence, the demands in the markets for different product combinations are independent from one another in the limit.[29]

5.1 Prices

Unrestricted pricing As in the independent-markets case, we first consider unrestricted pricing. The analysis focuses on a symmetric equilibria with prices

$$p_{12}^A = p_{12}^B = p_{12},$$

$$p_1^A + p_2^B = p_1^B + p_2^A = p_{12} + \delta,$$

where δ denotes the "bundling" discount received by one-stop shoppers. Two-stop shoppers are discriminated against in equilibrium if and only if $\delta > c_D$.

For $\delta > 0$, the proportion of two-stop shoppers among buyers is

$$\Phi\left(\delta, p_{12}\right) = 2\int_{-1}^{\frac{V(p_{12}+\delta)-V(p_{12})}{t}} \int_{\frac{V(p_{12})-V(p_{12}+\delta)}{t}}^{1} h(z_1, z_2) dz_2 dz_1.$$

[29]Alternatively, our previous independent-markets setting with shares $1-\gamma$ and γ of one-stop shoppers and two-stop shoppers also obtains if $t > V(0)$ and the distribution $h(z_1, z_2)$ is degenerate, assigning mass $\frac{\gamma}{2}$ to each of the points $(-1, -1)$ and $(1, 1)$ and mass $\frac{1-\gamma}{2}$ to each of the points $(-1, 1)$ and $(1, -1)$. Each buyer again considers only their preferred product combination in this case, hence the markets for the different product combinations have independent demands.

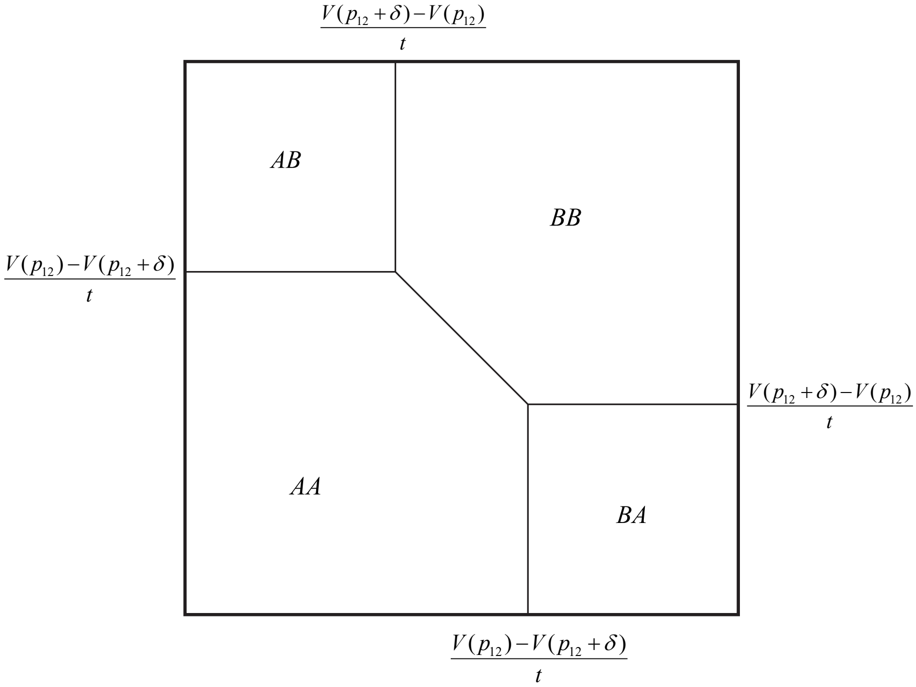

Figure 3: The pattern of shopping choices when $\delta > 0$.

Figure 3 illustrates the segmentation of buyers for $\delta > 0$ and $t > V(p_{12}) - V(p_{12}+\delta)$, where the latter ensures a strictly positive share of two-stop shoppers. We will call an equilibrium with symmetric prices and strictly positive shares of two-stop and one-stop shoppers a symmetric mixed-bundling equilibrium.

Assuming $\delta > 0$, prices in a symmetric mixed-bundling equilibrium must satisfy the following first-order conditions. First, it must be unprofitable for an individual firm to change one or both of its single-product prices. Starting at symmetric prices, a marginal change in p_1^i, keeping all other prices unchanged, is unprofitable for firm i if

$$\begin{aligned} 0 =& \frac{-q(p_{12}+\delta)\eta}{t} \left(2\left(p_1 - c - c_E\right) q\left(p_{12}+\delta\right) - (p_{12} - 2c)q(p_{12})\right) \\ &+ \frac{\Phi}{2}\left(q\left(p_{12}+\delta\right) + \left(p_1 - c - c_E\right) q'\left(p_{12}+\delta\right)\right), \end{aligned} \tag{13}$$

where η is the line integral along each of the horizontal and vertical segments in Figure 3:[30]

$$\begin{aligned}\eta &\equiv \int_{\frac{V(p_{12})-V(p_{12}+\delta)}{t}}^{1} h\left(\frac{V(p_{12}+\delta)-V(p_{12})}{t}, z_2\right) dz_2 \\ &= \int_{-1}^{\frac{V(p_{12}+\delta)-V(p_{12})}{t}} h\left(z_1, \frac{V(p_{12})-V(p_{12}+\delta)}{t}\right) dz_1 > 0,\end{aligned}$$

The first line in (13) captures the effects of buyers switching from ij to ii and from ij to jj when p_1^i increases. The second line captures the impact on profits from inframarginal buyers of ij. Similarly, starting at symmetric prices, a marginal change in p_2^i is unprofitable for i if

$$\begin{aligned}0 =& \frac{-q(p_{12}+\delta)\eta}{t}\left(2\left(p_2-c-c_I\right)q\left(p_{12}+\delta\right)-(p_{12}-2c)q(p_{12})\right) \\ &+\frac{\Phi}{2}\left(q\left(p_{12}+\delta\right)+\left(p_2-c-c_I\right)q'\left(p_{12}+\delta\right)\right).\end{aligned} \tag{14}$$

Conditions (13) and (14) imply that $p_1 - c_E = p_2 - c_I$ in equilibrium, as in the case of independent markets.

Adding up these two first-order conditions yields the following condition on the equilibrium prices p_{12} and $p_{12}+\delta$ paid by one-stop shoppers and two-stop shoppers:

$$\begin{aligned}0 =& \frac{\partial \Phi}{\partial \delta}\left(\left(p_{12}+\delta-2c-c_D\right)q\left(p_{12}+\delta\right)-(p_{12}-2c)q(p_{12})\right) \\ &+\Phi\left(2q\left(p_{12}+\delta\right)+\left(p_{12}+\delta-2c-c_D\right)q'\left(p_{12}+\delta\right)\right),\end{aligned} \tag{15}$$

where

$$\frac{\partial \Phi}{\partial \delta} = -\frac{4q(p_{12}+\delta)\eta}{t} < 0.$$

Intuitively, condition (15) rules out deviations in which firm i changes p_k^i from p_k to $\widetilde{p}_k$ for each $k = 1, 2$ such that $p_1 + \widetilde{p}_2 = \widetilde{p}_1 + p_2$. This deviation does not alter the prices for one-stop shoppers, but it changes the prices of combinations AB and BA by the same amount. The right-hand side of (15) is the deviation's overall impact on the deviating seller's profit, where the second line captures the impact on profits from inframarginal two-stop shoppers, and the first line captures the marginal profit impact of buyers switching between two-stop shopping and one-stop shopping. Figure 4 depicts these effects in the case of price increases. If condition

[30] The equality follows from the symmetry of h. To see this, observe that

$$\begin{aligned}&\int_{\frac{V(p_{12})-V(p_{12}+\delta)}{t}}^{1} h\left(\frac{V(p_{12}+\delta)-V(p_{12})}{t}, z_2\right) dz_2 = \int_{\frac{V(p_{12})-V(p_{12}+\delta)}{t}}^{1} h\left(z_2, \frac{V(p_{12}+\delta)-V(p_{12})}{t}\right) dz_2 \\ &= \int_{\frac{V(p_{12})-V(p_{12}+\delta)}{t}}^{1} h\left(-z_2, \frac{V(p_{12})-V(p_{12}+\delta)}{t}\right) dz_2 = \int_{-1}^{\frac{V(p_{12}+\delta)-V(p_{12})}{t}} h\left(z_1, \frac{V(p_{12})-V(p_{12}+\delta)}{t}\right) dz_1,\end{aligned}$$

where the last equality is due to a change of variables.

(15) holds and $p_1 - c_E = p_2 - c_I$, then there are no profitable marginal deviations to different single-product prices.

Second, neither firm must have an incentive to marginally change its one-stop shopper price. Starting at symmetric prices, a marginal change in p_{12}^i, keeping all other prices unchanged, is unprofitable for firm i if

$$\begin{aligned} 0 = & \left(\frac{1-\Phi}{2}\right)(q(p_{12}) + (p_{12} - 2c)q'(p_{12})) \\ & + \frac{\eta q(p_{12})}{t}\left((p_{12} + \delta - 2c - c_D)q(p_{12} + \delta) - 2(p_{12} - 2c)q(p_{12})\right) \\ & - \frac{\beta q(p_{12})}{t}\left(p_{12} - 2c\right) q\left(p_{12}\right), \end{aligned} \tag{16}$$

where β is the line integral for the diagonal segment in Figure 3:

$$\beta \equiv \int_{\frac{V(p_{12}+\delta)-V(p_{12})}{t}}^{\frac{V(p_{12})-V(p_{12}+\delta)}{t}} h\left(-z_2, z_2\right) dz_2 = \int_{\frac{V(p_{12}+\delta)-V(p_{12})}{t}}^{\frac{V(p_{12})-V(p_{12}+\delta)}{t}} h\left(z_1, -z_1\right) dz_1 > 0.$$

Seller i's deviation of slightly increasing p_{12} has three effects on i's profits: first, it changes i's profits from its one-stop shoppers; second, it induces some one-stop shoppers of seller i to switch to two-stop shopping; finally, it induces some one-stop shoppers of seller i to switch to one-stop shopping at the rival seller $j \neq i$. These effects are captured, in order, by each line of the right-hand side of (16), and illustrated in Figure 5.

In the limiting case of independent markets (as $t \to \infty$), buyers do not switch between product combinations in response to price changes, hence the first line in (15) and the second and third lines in (16) are zero; one-stop shoppers thus pay P_M and two-stop shoppers pay P_D in the limit. When the markets are interdependent, however, (15) and (16) imply that each firm can profitably deviate from the independent-markets unrestricted equilibrium prices:

Lemma 1. Starting from the independent-markets discriminatory pricing equilibrium, each firm has a strict unilateral incentive to slightly decrease the price offered to one-stop shoppers and to slightly raise the prices offered to two-stop shoppers.

The intuition for why these deviations are profitable is as follows. At the independent-markets equilibrium prices P_M and P_D for one-stop and two-stop shoppers, respectively, one-stop shoppers are more profitable for firms than two-stop shoppers: the prices they pay are not distorted away from the monopoly level, and they are cheaper to serve. A unilateral deviation to a slightly lower one-stop price is therefore profitable: it has zero first-order effect on the profits earned per (infra-marginal) one-stop shopper, but it induces some buyers to switch towards one-stop shopping with the deviator. Similarly, a unilateral deviation to slightly higher

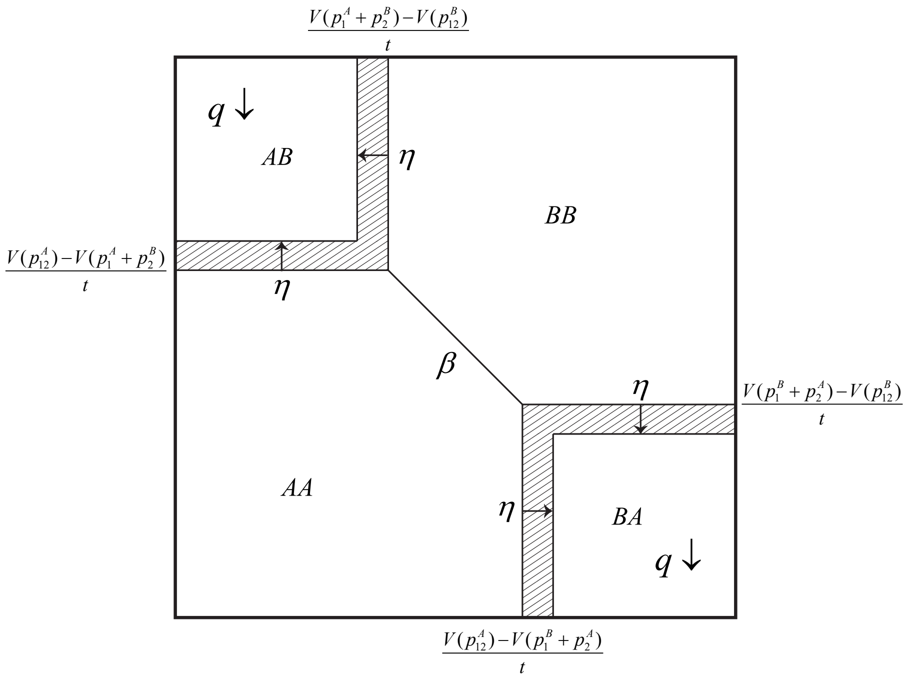

Figure 4: Firm A increases p_1 and p_2 by the same amount, keeping p_{12} unchanged.

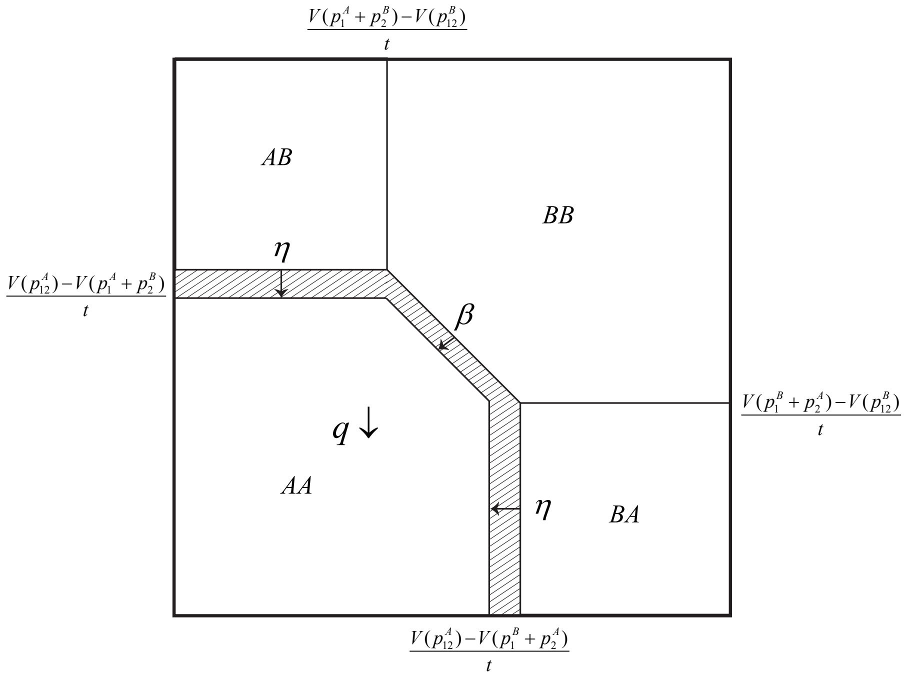

Figure 5: Firm A increases p_{12}, keeping p_1 and p_2 unchanged.

two-stop shopper prices is profitable: the deviation has zero first-order effect on the deviator's profits from (infra-marginal) two-stop shoppers, and it raises the share of one-stop shoppers among the deviator's buyers (without changing the number of buyers for each product).

Based on this reasoning, one might expect that $p_{12} < P_M$ and $p_{12} + \delta > P_D$ in equilibrium, but only the former turns out to be generally true, as our next proposition, which characterizes key features of the unrestricted prices in a symmetric mixed-bundling equilibrium, shows.

Proposition 4. In a symmetric mixed-bundling equilibrium,

(i) $p_{12} < P_M$,

(ii) $\delta = p_1 + p_2 - p_{12} > \min\{c_D, P_D - P_M\}$,

(iii) if $t \leq V(P_M)$, then $p_1 + p_2 \leq P_D$ and $\delta = p_1 + p_2 - p_{12} > c_D$,

(iv) $p_1 - c_E = p_2 - c_I$.

The first takeaway from Proposition 4 is that the competition for one-stop shoppers that arises once markets are interdependent drives the one-stop shopper equilibrium price below the monopoly price P_M. The intuition behind this result can be easily understood for $\delta > 0$. If $p_{12} \geq P_M$ and $\delta > 0$, all prices exceed the monopoly price P_M. Clearly, each seller then has an incentive to reduce p_{12}, keeping the single-product prices p_1 and p_2 unchanged, because doing so increases both the proportion of one-stop shoppers (who are more profitable) and the profit earned per one-stop shopper.

The second takeaway from Proposition 4 is that if discrimination against two-stop shoppers arises in equilibrium when markets are independent, it will also arise in equilibrium when markets are interdependent. Moreover, for small enough t, discrimination against two-stop shoppers arises in the latter case even if it wouldn't arise with independent markets:

Corollary 1. There is discrimination against two-shoppers in a symmetric mixed-bundling equilibrium if either (i) $P_D - P_M > c_D$, or (ii) t is small enough.

To understand the intuition behind the discrimination against two-stop shoppers, suppose that $0 < \delta \leq c_D$. One-stop shoppers are then more profitable than two-stop shoppers, because

$$(p_{12} - 2c + \underbrace{\delta - c_D}_{\leq 0})\underbrace{q(p_{12} + \delta)}_{<q(p_{12})} < (p_{12} - 2c)q(p_{12}).$$

Hence, keeping the total number of buyers for each of its two products constant, a firm prefers a higher share of one-stop shoppers among its buyers. If $p_1 + p_2 < P_D$, then this implies that slightly increasing both of its two-stop shopper prices is a profitable deviation for each firm, because it (i) raises the share of one-stop shoppers among the deviator's buyers (without changing its total number of buyers per product) and (ii) raises the deviator's profits from its two-stop shoppers. Since $\delta \leq P_D - P_M$ implies $p_1 + p_2 < P_D$ (given that $P_{12} < P_M$ in equilibrium), it follows that $\delta > \min\{c_D, P_D - P_M\}$ in equilibrium, otherwise each firm could profitably deviate to higher two-stop shopper prices. By a similar reasoning, each firm has a profitable discrete deviation to higher two-stop shopper prices if t is small and $0 < \delta \leq c_D$. In particular, for t low enough so that all buyers prefer one-stop shopping at price p_{12} over not buying (formally, two-stop shopping with zero quantity), each firm could profitably raise its two-stop shopper prices to infinity without any change to its one-stop shopper price. Such a deviation would cause all shoppers to opt for one-stop shopping, with each firm serving half of the buyers. Since one-stop shoppers are more profitable than two-stop shoppers when $0 < \delta \leq c_D$, this deviation would be profitable. It follows that for low enough t, discrimination against two-stop shoppers arises in a symmetric mixed-bundling equilibrium even if $P_D - P_M < c_D$.

Figure 6 illustrates the unrestricted equilibrium prices under linear demand and a uniform distribution of relative brand preferences. We can see that while $p_{12} < P_M$ for all values of t, the total price $p_1 + p_2$ paid by two-stop shoppers lies below P_D for low values of t (in line with Proposition 4(iii)), but exceeds P_D at higher values of t. As $t \to \infty$, the prices converge to the unrestricted independent-market equilibrium prices P_M and P_D.[31]

Restricted pricing Suppose that there is discrimination against two-stop shoppers at the unrestricted equilibrium prices ($\delta > c_D$). As in the case of independent markets, if each firm now faces the restrictions $p_1^i \leq \bar{p}_1^i + r_1$ and $p_2^i \leq \bar{p}_2^i + r_2$, the equilibrium prices depend on r_1 and r_2 only through their sum $r = r_1 + r_2$. Moreover, for any $r < \delta$, the constraint on the price difference is binding in equilibrium, i.e., $p_{12} = p_1 + p_2 - r$. (See Appendix B.2 for detailed derivations.)

We can characterize the first-order equilibrium condition for $r \in [c_D, \delta]$ by ruling out unilateral deviations to different prices that keep the constraint on the price difference binding. Starting from symmetric price levels (p_{12}, p_1, p_2) with $p_1 + p_2 - p_{12} = r$, suppose firm i changes its prices to $(\widetilde{p}_{12}, \widetilde{p}_1, \widetilde{p}_2)$, where $\widetilde{p}_{12} = p_{12} + 2\epsilon$ and $\widetilde{p}_k = p_k + \epsilon$ for $k = 1, 2$, so that $\widetilde{p}_1 + \widetilde{p}_2 - \widetilde{p}_{12} = p_1 + p_2 - p_{12} = r$. For such a deviation to be unprofitable for small ϵ, the

[31]The results in Proposition 4 are robust to the introduction of an exogenous shopping cost $m \in [0, t)$ for two-stop shoppers. Given fixed prices, such a cost reduces the share of two-stop shoppers: for instance, the indifference condition between AA and AB, now requires $z_2 = \frac{V(p_{12}) - V(P_{12}+\delta)+m}{t} > \frac{V(p_{12}) - V(P_{12}+\delta)}{t}$. As a result, the two-stop shopping cost expands the boundary where buyers are indifferent between AA and BB, which further intensifies competition and the result $p_{12} < P_M$ remains valid. The reasoning behind the above-cost bundling discount continues to hold as well.

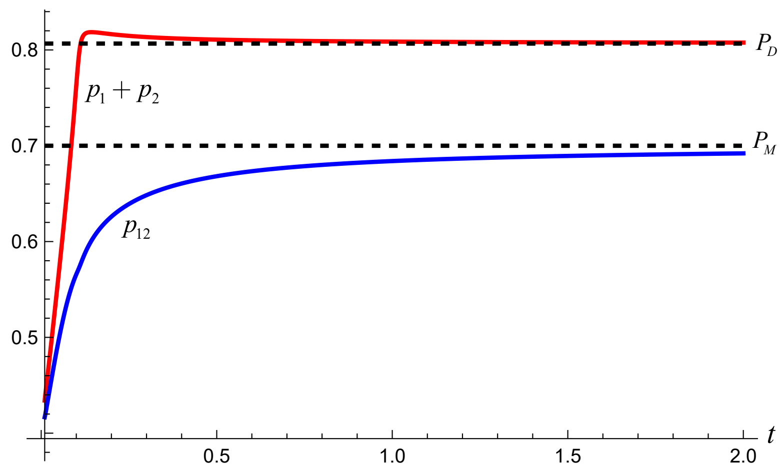

Figure 6: Unrestricted equilibrium prices for $q(P) = 1 - P, c = 0.2, c_D = 0.02$, uniform $h(z_1, z_2)$.

equilibrium one-stop shopper price $p_{12} = \widehat{P}(r)$ must satisfy

$$
\begin{aligned}
&(1-\Phi)\left(q\left(\widehat{P}\right)+\left(\widehat{P}-2c\right)q'\left(\widehat{P}\right)\right)+\Phi\left(q\left(\widehat{P}+r\right)+\left(\widehat{P}+r-2c-c_D\right)\frac{q'\left(\widehat{P}+r\right)}{2}\right)\\
&-\frac{\eta\left(2q\left(\widehat{P}\right)-q\left(\widehat{P}+r\right)\right)}{t}\left(2\left(\widehat{P}-2c\right)q\left(\widehat{P}\right)-\left(\widehat{P}+r-2c-c_D\right)q\left(\widehat{P}+r\right)\right)\\
&-\frac{\eta q\left(\widehat{P}+r\right)}{t}\left(\widehat{P}+r-2c-c_D\right)q\left(\widehat{P}+r\right)-\frac{2\beta q\left(\widehat{P}\right)}{t}\left(\widehat{P}-2c\right)q\left(\widehat{P}\right)\\
&=0.
\end{aligned}
\tag{17}
$$

The first line in this first-order condition captures profit effects at the intensive margin of consumption. For $\Phi = \gamma$, this line equals zero at the constrained equilibrium price $P(r)$ that solves (4) in the independent-markets case. The second and third lines capture the additional effects that arise due to buyers' switching between different product choices. Figure 7 illustrates these additional effects for $\epsilon > 0$. The second line captures switching between one-stop shopping with the deviator and two-stop shopping. The third line captures switching between two-stop shopping and one-stop shopping with the competitor and switching from one-stop shopping with the deviator to one-stop shopping with the competitor.

The condition in (17) is complex, and comparative statics analyses of the restricted prices do not yield general results. In contrast to the independent-markets case, it is not necessarily true that one price falls and the other rises with r. Instead, it is possible that both prices fall as the degree of discrimination r rises. This is shown in our next example, which considers a special limiting case of the distribution of brand preferences that generates intense competition

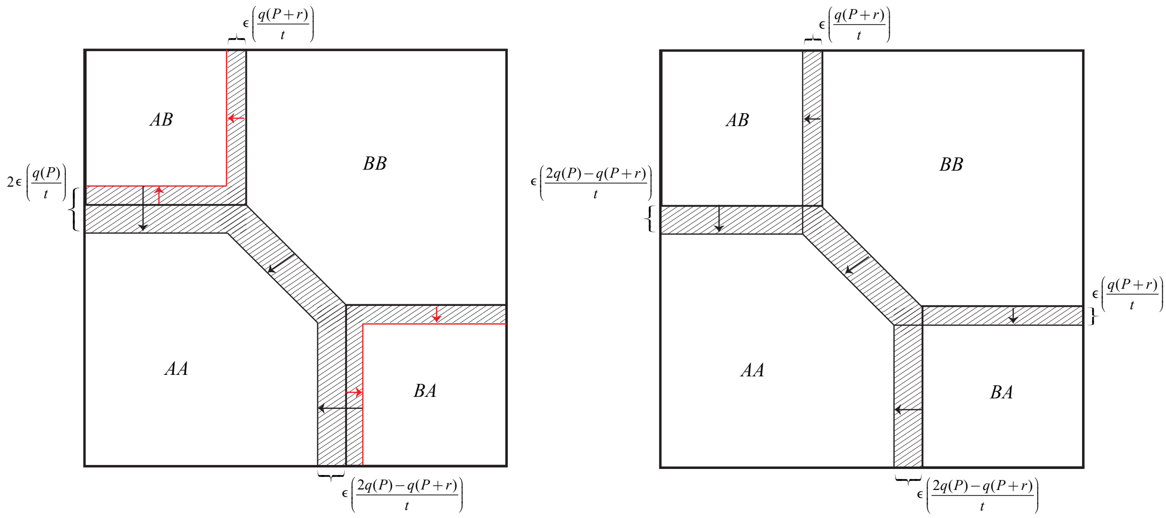

Figure 7: The left panel depicts the effects of the changes in p_1^A, p_2^A (red) and p_{12}^A (black). The right panel depicts the net effects on product choices.

for one-stop shoppers when price discrimination is allowed.

Linear Uniform Diagonal Example: Suppose buyers' relative brand preferences (z_1, z_2) are uniformly distributed on the diagonal line segment between $(-1, 1)$ and $(1, -1)$, so that $z_1 + z_2 = 0$ for each buyer (i.e., buyers' relative brand preferences exhibit perfect negative correlation). The cumulative distribution function of relative brand preferences is then

$$H(z_1, z_2) = \begin{cases} 0 & \text{for } z_1 + z_2 \leq 0\,, \\ \frac{z_1+z_2}{2} & \text{for } z_1 + z_2 > 0. \end{cases}$$

Given this distribution of the relative brand preferences, $x_1^A + x_2^A = x_1^B + x_2^B$ for all buyers, hence each buyer is indifferent between product combinations AA and BB when prices are symmetric. We further assume indifferent buyers are allocated symmetrically across their preferred options. Moreover, the demand function is linear and $c_D = 0$, so that a ban on price discrimination is equivalent to uniform pricing ($r = 0$).

Proposition 5. In the *Linear Uniform Diagonal Example*, for sufficiently large t,

(i) there exists a unique symmetric equilibrium under unrestricted pricing, with equilibrium prices that satisfy $p_{12} = 2c$ and $p_1 + p_2 \in (2c, P_D)$,

(ii) there exists a unique symmetric equilibrium under uniform pricing, with equilibrium prices that satisfy $p_{12} = p_1 + p_2 \in (2c, P_D)$,

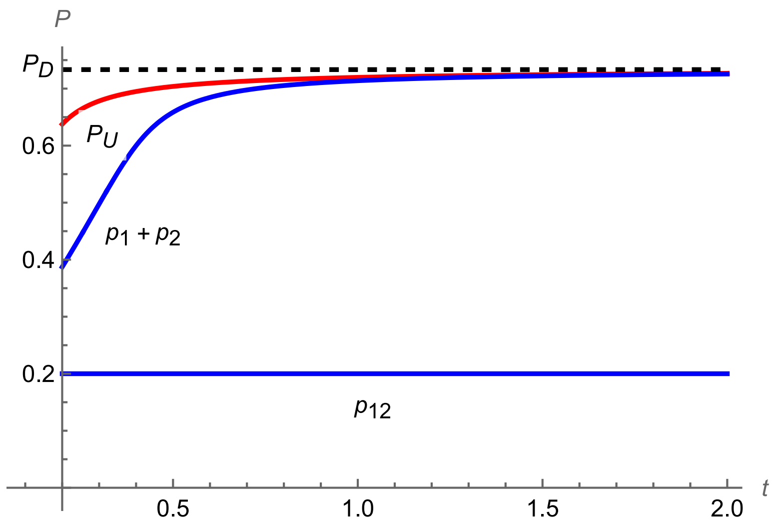

Figure 8: Equilibrium prices in the *Linear Uniform Diagonal Example* for $q(P) = 1 - P$ and $c = 0.1$. The blue lines depict the equilibrium prices under discriminatory pricing. The red line depicts the equilibrium price under non-discriminatory (uniform) pricing.

(iii) *all* buyers pay a lower price in the discriminatory-pricing equilibrium than in the uniform-pricing equilibrium.

When all buyers are located on the diagonal line along which $z_1 + z_2 = 0$, a standard Bertrand-game undercutting argument implies that $p_{12} = 2c$ under discriminatory pricing. In the discriminatory-pricing equilibrium, buyers close enough to $(z_1, z_2) = (0, 0)$ one-stop shop and pay prices at marginal cost; buyer closer to the extremes of the diagonal line segment two-stop shop and pay prices strictly above marginal costs.

When prices are restricted to be uniform, then all buyers are two-stop shoppers: buyers with $z_1 < 0$ buy product combination AB, while buyers with $z_1 > 0$ buy BA. The firms earn strictly positive margins in this equilibrium, because each seller would have a profitable unilateral deviation to higher prices under uniform pricing at marginal costs.

As shown in the proof of Proposition 5, all buyers, including two-stop shoppers, pay a strictly lower price in the discriminatory-pricing equilibrium than in the uniform-pricing equilibrium for high enough t. Figure 8 offers a graphical illustration. Intuitively, discrimination intensifies competition for one-stop shoppers, driving the price they pay down to marginal costs. The low price for one-stop shoppers, in turn, drives down the equilibrium price paid by two-stop shoppers as well, so that ultimately all shoppers gain from price discrimination.

That allowing firms to discriminate can intensify competition, and potentially lead to lower prices for all buyers, has been shown previously in models of competitive mixed bundling with inelastic demands (Armstrong and Vickers (2010), Zhou (2021)). The same economic effects are at play here. As explained by Armstrong and Vickers (2010), discrimination expands the margin of competition for "doubly profitable" one-stop shoppers, which intensifies competition. Unlike in models with inelastic demands, however, this competition-intensifying effect of discrimination alters not just what but also how much buyers consume in our setting. This matters for the welfare effects of price discrimination, which we turn to next.

5.2 Effects of discrimination against two-stop shoppers

Denote by $\Phi(r) = \Phi\left(r, \widehat{P}(r)\right)$ the equilibrium proportion of two-stop shoppers given a limit on discrimination $r \in [c_D, \delta]$. Moreover, as in the independent-markets case, let us denote the marginal change in the aggregate quantity consumed by *infra-marginal* shoppers (who do not switch their product choices) as more discrimination is allowed by

$$\widehat{Q}'(r) = (1-\Phi(r))\, q'\left(\widehat{P}(r)\right)\widehat{P}'(r) + \Phi(r) q'\left(\widehat{P}(r)+r\right)\left(\widehat{P}'(r)+1\right).$$

The marginal welfare effect can then be decomposed as follows:[32]

$$\begin{aligned}\widehat{W}'(r) = & \underbrace{\left(\widehat{P}(r)-2c\right)\widehat{Q}'(r) + \Phi(r)\,(r-c_D)\,q'\left(\widehat{P}(r)+r\right)\left(\widehat{P}'(r)+1\right)}_{\text{marginal effects due to consumption changes of infra-marginal shoppers}} \\ & + \underbrace{\left(-\Phi'(r)\right)\left(c_D q\left(\widehat{P}(r)+r\right) + \int_{q\left(\widehat{P}(r)+r\right)}^{q\left(\widehat{P}(r)\right)} (p(q)-2c)\,dq\right)}_{\text{marginal effects of switching (ignoring match values)}} \\ & + \underbrace{\Phi'(r)\left(V\left(\widehat{P}(r)\right) - V\left(\widehat{P}(r)+r\right)\right)}_{\text{marginal match-quality effect}}.\end{aligned} \tag{18}$$

The first two terms in (18) are analogous to the two effects in the independent-markets case (see (8)). If more discrimination causes all prices to fall due to the competition-intensifying effect of discrimination, then both of these terms are positive for any $r \geq c_D$. If more discrimination lowers the price paid by one-stop shoppers but raises the price paid by two-stop shoppers, then the second term is negative, while the first term will continue to be positive provided the average price (given the share $\Phi(r)$ of two-stop shoppers) falls and demand is concave or linear (or, more generally, "not too convex").

The second and third line of (18) capture the additional marginal welfare effects that arise because of buyer switching between different product combinations. The second line captures

[32] Details on the derivation of the marginal match-quality effect can be found in Appendix B.2.

the marginal effects of buyers switching from two-stop shopping to one-stop shopping, ignoring the buyers' random match utilities. It is strictly positive if and only if more discrimination lowers the share of two-stop shoppers:

$$\Phi'(r) = \frac{4\eta\left(r, \widehat{P}(r)\right)\left(\left(q\left(\widehat{P}(r)\right) - q\left(\widehat{P}(r)+r\right)\right)\widehat{P}'(r) - q\left(\widehat{P}(r)+r\right)\right)}{t} < 0, \qquad (19)$$

A sufficient condition for $\Phi'(r) < 0$ is $\widehat{P}'(r) < 0$, that is, the price paid by one-stop shoppers is decreasing in r.[33] Allowing more discrimination then induces a positive mass of two-stop shoppers to switch to one-stop shopping, and this improves social welfare because it raises the consumption levels of the switchers closer to the socially optimal level, and it leads to marginal-cost savings. The "marginal match-quality effect" in the last line of (18) captures the effect of buyer switching between different product combinations on the aggregate surplus from buyer match values. This effect is strictly negative if $\Phi'(r) < 0$, because buyers who switch from two-stop shopping to one-stop shopping to pay less obtain lower match values after the switch.

In the special case of unit demand, the total marginal welfare effect in (18) simplifies to[34]

$$\widehat{W}'(r) = -\Phi'(r)\, c_D + \Phi'(r)\, r < 0.$$

Discrimination always harms social welfare in this case because it leads to an inefficient pattern of consumption; see also Armstrong and Vickers (2010). When demand is elastic, this is no longer necessarily true. Allowing price discrimination can raise social welfare in this case, including for functional forms of demand (such as linear) where discrimination would be harmful under independent markets. As discussed, buyer switching from two-stop shopping to one-stop shopping raises the consumption levels of switchers closer to the socially efficient levels, a welfare-enhancing effect. Moreover, because of the competition-intensifying effect of price discrimination discussed earlier, price discrimination can also lead to more efficient consumption levels for some (or even all) infra-marginal shoppers, which is another welfare-enhancing effect.

Figures 9 and 10 show the effects of price discrimination under linear demand when relative product preferences are uniformly distributed on the square $[-1, 1]^2$. Unlike in the *Linear Uniform Diagonal Example*, where all shoppers pay less under discrimination, which directly

[33]Although we were unable to prove that $\widehat{P}'(r) < 0$ is generally true, we could not find a counter-example in which $\widehat{P}'(r) \geq 0$.

[34]To see this, note that under unit demand, $V(P) - V(P+r) = r$ while the first line and the second term in the second line of (18) are all zero. Moreover, $\Phi'(r) = -\frac{4\eta}{t} < 0$.

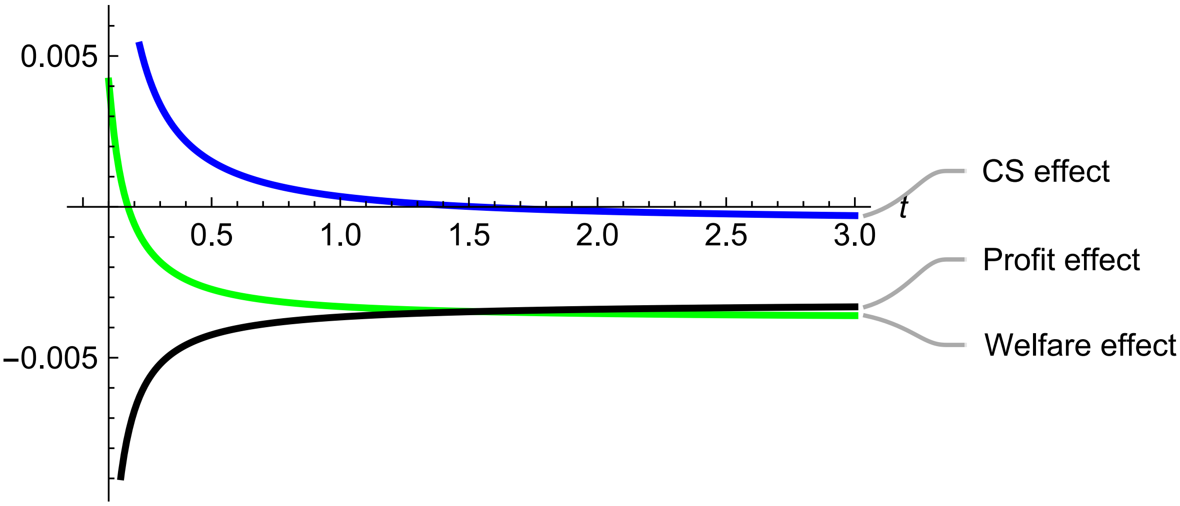

Figure 9: Effects of discrimination against two-stop shoppers when $x_k^B = 1 - x_k^A \sim U[0,1]$, $q(P) = 1 - P$, $c = 0.2$, and $c_D = 0$.

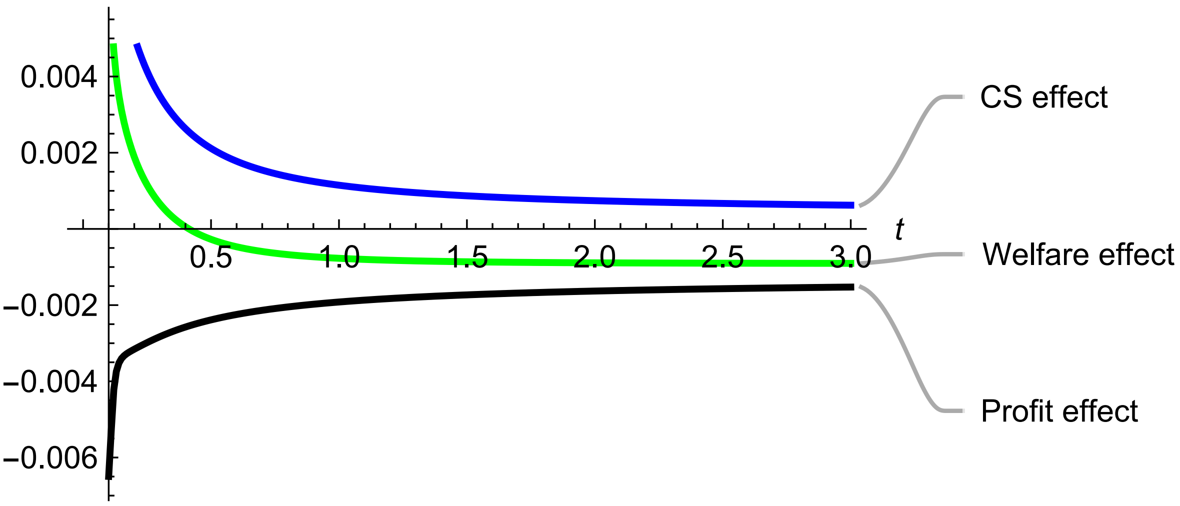

Figure 10: Effects of discrimination against two-stop shoppers when $x_k^B = 1 - x_k^A \sim U[0,1]$, $q(P) = 1 - P$, $c = 0.2$, and $c_D = 0.1$.

implies a positive consumer surplus effect, allowing price discrimination leads to a lower price for one-stop shoppers and a higher price for two-stop shoppers in these examples.

The figures reveal that for weak product preferences (low t), discrimination raises both social welfare and consumer surplus. Intuitively, when t is low, price discrimination induces more buyers to switch from two-stop to one-stop shopping, which benefits social welfare because it raises the consumption levels of switchers and leads to marginal cost efficiencies. Moreover, the competition-intensifying effect of price discrimination, which benefits consumer surplus and social welfare but harms profits, is stronger when buyers have weaker horizontal product preferences. These positive effects on social welfare outweigh the negative effects for low t. The cost efficiency effect is stronger the larger c_D, which provides an intuitive explanation for why the range of t for which discrimination raises social welfare is larger when $c_D = 0.1$ (Figure 10) than when $c_D = 0$ (Figure 9).

For $t \to \infty$, the equilibrium prices and the effects of discrimination are as in the independent-markets model variant. Consistent with our formal results for linear demand and independent markets, Figures 9 and 10 show that for large t, price discrimination against two-stop shoppers harms social welfare and profits; moreover, consistent with the linear-demand example in Section 4, price discrimination against two-stop shoppers also harms consumer surplus for $c_D = 0$.

6 Conclusion

Motivated by recent competition concerns in the cloud services industry, we have analyzed the effects of price discrimination against multi-stop shoppers in a duopoly market where multi-product firms sell complementary goods to final buyers with elastic demands and multi-stop shoppers are costlier to serve than one-stop shoppers. Our analysis shows that both the curvature of the buyers' demand function and the strength of product preferences play important roles for evaluating the effects of price discrimination. If the markets for different product combinations are independent, price discrimination against two-stop shoppers was shown to harm social welfare provided demand is not "too convex." In the case of interdependent markets, however, price discrimination was shown to have additional welfare-improving effects, which imply that price discrimination against two-stop shoppers can raise social welfare even under linear demand.

The implications of our analysis for policies regarding data transfer fees in the cloud can be summarized as follows. If data transfer fees primarily discourage multi-clouding at the intensive margin but lead to little switching from multi-clouding to single-clouding, our analysis suggests than a ban on above-cost data transfer fees is likely to raise social welfare. If, instead, buyers switch readily between cloud architectures in response to price changes, our analysis suggests that a ban on above-cost data transfer fees may well harm social welfare. Overall, our analysis therefore suggests that the strength of buyers' product preferences and degree of buyer lock-in are important factors for the welfare effects of a ban.

Our analysis is, of course, subject to several limitations. One limitation for drawing policy implications from our analysis is that we have assumed marginal data transfer costs to be known and constant. In practice, these costs may be hard to measure for regulators, and they may vary over time and by region, which may make it difficult to implement a regulation that ties data transfer fees to costs. Another limitation is that our analysis, which focuses on the pricing strategies of incumbent multi-product firms, does not consider potential entry or growth by new competitors. Thus, our analysis cannot capture the concern that egress fees might harm future competition by making it harder for competitors, especially those that offer only a subset of services, to enter the industry or expand.[35] Third, we do not consider any potential effects of regulating data transfer fees on firms' infrastructure investment incentives.

[35] Related concerns are addressed in the literature on bundling and entry deterrence; see Whinston (1990); Nalebuff (2004); Choi and Stefanadis (2001, 2006).

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Appendix A: Proofs

Proof of Proposition 1

We first show that $R(r) \equiv 2L(P(r);2c)\varepsilon(P(r)) - L(P(r)+r;2c+c_D)\varepsilon(P(r)+r)$ is decreasing and non-negative for all $r \in [c_D, P_D - P_M]$. Recall that $\sigma < 1$ (demand is strictly log-concave), otherwise Assumption 1(3) would be violated at $P = P_D$. This implies that $\varepsilon' > 0$ (Marshall's second law of demand is satisfied), and hence

$$R'(r) = 2\underbrace{P'(r)}_{<0}\left(\underbrace{\frac{\partial L(P;2c)}{\partial P}\varepsilon(P)}_{>0} + \underbrace{L(P;2c)}_{>0}\underbrace{\varepsilon'(P)}_{>0}\right)$$

$$-\underbrace{(P'(r)+1)}_{>0}\left(\underbrace{\frac{\partial L(P+r;2c+c_D)}{\partial(P+r)}\varepsilon(P+r)}_{>0} + \underbrace{L(P+r;2c+c_D)}_{>0}\underbrace{\varepsilon'(P+r)}_{>0}\right) < 0,$$

where the positivity of the Lerner indices follows follows from the first-order condition in (4). Moreover, $R(P_D - P_M) = 0$ by the first-order conditions (1) and (2). It follows that $R(r) \geq 0$ for all $r \in [c_D, P_D - P_M]$.

If demand is concave ($\sigma \leq 0$), then $R(r) \geq 0$ directly implies that the right-hand side of (10) is strictly negative and thus $Q'(r) < 0$ for all $r \in [c_D, P_D - P_M]$.

If $\sigma \in (0,1)$, then $R(r) \geq 0 > R'(r)$ implies that $Q'(r) < 0$ for all $r \in (c_D, P_D - P_M]$ if $Q'(c_D) \leq 0$. Using (10), $Q'(c_D) \leq 0$ if at $P = P(c_D)$

$$\underbrace{\sigma(P-2c)}_{>0}\left(2\frac{-q'(P)}{q(P)} - \frac{-q'(P+c_D)}{q(P+c_D)}\right) \leq 1. \tag{20}$$

We have that $\frac{-q'(P)}{q(P)} \leq \frac{-q'(P+c_D)}{q(P+c_D)}$, with a strict inequality for $c_D > 0$, because $\sigma < 1$. Therefore, a sufficient condition for (20) is that at $P = P(c_D)$,

$$\sigma(P-2c)\frac{-q'(P)}{q(P)} \leq 1, \tag{21}$$

or, equivalently, $\sigma \leq \frac{1}{L(P(c_D);2c)\varepsilon(P(c_D))}$, the condition in the proposition.

It remains to show that $\frac{1}{L(P(c_D);2c)\varepsilon(P(c_D))} \geq 1 - \gamma + \frac{\gamma}{2}\frac{q'(P(c_D)+c_D)}{q'(P(c_D))}$. By the first-order condition in (4), we have that for $P = P(c_D)$,

$$P - 2c = \frac{(1-\gamma)q(P) + \gamma q(P+c_D)}{(1-\gamma)(-q'(P)) + \frac{\gamma}{2}(-q'(P+c_D))} \leq \frac{q(P)}{(1-\gamma)(-q'(P)) + \frac{\gamma}{2}(-q'(P+c_D))}.$$

Therefore, at $P = P(c_D)$,

$$(P-2c)\frac{-q'(P)}{q(P)} \leq \frac{1}{1-\gamma+\frac{\gamma}{2}\frac{q'(P+c_D)}{q'(P)}},$$

which is equivalent to $\frac{1}{L(P(c_D);2c)\varepsilon(P(c_D))} \geq 1-\gamma+\frac{\gamma}{2}\frac{q'(P(c_D)+c_D)}{q'(P(c_D))}$. Since $L(P(c_D);2c)\varepsilon(P(c_D)) > 1$, it must be that $1-\gamma+\frac{\gamma}{2}\frac{q'(P(c_D)+c_D)}{q'(P(c_D))} < 1$. $\square$

Proof of Proposition 2

Using (5) and simplifying, we obtain

$$2\Pi'(r) = \underbrace{\frac{\gamma\,(1-\gamma)\,\Delta\pi_M\Delta\pi_D}{(1-\gamma)\,\Delta\pi_M+\gamma\Delta\pi_D}}_{<0}\left(\frac{q(P+r)+(P+r-2c-c_D)q'(P+r)}{\Delta\pi_D}-\frac{q(P)+(P-2c)q'(P)}{\Delta\pi_M}\right).$$

Using $\Delta\pi_M = q'(P)\,(2-L(P;2c)\varepsilon(P)\sigma)$ and $\Delta\pi_D = \frac{q'(P+r)}{2}\,(3-L(P+r;2c+c_D)\varepsilon(P+r)\sigma)$, it follows that

$$2\Pi'(r) \overset{sign}{=} \widetilde{z}_M(P)-\widetilde{z}_D(P+r), \tag{22}$$

where

$$\widetilde{z}_M(P) \equiv \frac{\frac{q(P)}{q'(P)}+P-2c}{2-L\,(P;2c)\,\varepsilon\,(P)\,\sigma},\ \widetilde{z}_D(P+r) \equiv \frac{\frac{q(P+r)}{q'(P+r)}+P+r-2c-c_D}{\frac{3}{2}-\frac{1}{2}L(P+r;2c+c_D)\varepsilon(P+r)\sigma}.$$

Three observations then imply the result. First, the numerators and denominators in $\widetilde{z}_M(P)$ and $\widetilde{z}_D(P)$ are all positive. Positivity of the numerators follows from $P \geq P_M$ and the discussion in footnote 26. Positivity of the denominators is implied by Assumption 1(3).

Second, from the expression for the marginal output effect in (10), it is easy to see that $Q'(r) < 0$ if and only if $2-L(P;2c)\varepsilon(P)\sigma > \frac{3}{2}-\frac{1}{2}L(P+r;2c+c_D)\varepsilon(P+r)\sigma$. Hence, condition (11), which is a sufficient condition for $Q'(r) < 0$ for all $r \in [c_D, P_D-P_M]$ (as shown in the proof of Proposition 1), is also a sufficient condition for the denominator of $\widetilde{z}_M(P)$ to exceed that of $\widetilde{z}_D(P+r)$.

Third, the numerator of $\widetilde{z}_M(P)$ is smaller than that of $\widetilde{z}_D(P)$, because $P-2c \leq P+r-2c-c_D$ for all $r \geq c_D$, and $q(P)/q'(P)$ is increasing in P if $\sigma < 1$ (which must hold, otherwise Assumption 1(3) would be violated at $P = P_D$).

It follows that if (11) holds, then $\widetilde{z}_M(P) \leq \widetilde{z}_D(P+r)$ and thus $2\Pi'(r) \leq 0$ for all $r \in [c_D, P_D-P_M]$. $\square$

Proof of Proposition 3

Using (5) and simplifying, we obtain

$$CS'(r) = \underbrace{\frac{\gamma(1-\gamma)\Delta\pi_M\Delta\pi_D}{(1-\gamma)\Delta\pi_M+\gamma\Delta\pi_D}}_{<0}\left(\hat{z}_D(P+r)-\hat{z}_M(P)\right),$$

where

$$\hat{z}_M(P) \equiv \frac{\frac{q(P)}{-q'(P)}}{2 - L(P;2c)\varepsilon(P)\sigma}, \; \hat{z}_D(P+r) \equiv \frac{\frac{q(P+r)}{-q'(P+r)}}{\frac{3}{2} - \frac{1}{2}L(P+r;2c+c_D)\varepsilon(P+r)\sigma}.$$

Hence,

$$CS'(r) \overset{sign}{=} \hat{z}_M(P) - \hat{z}_D(P+r). \tag{23}$$

Part (i): We have that

$$\hat{z}'_M(P) \overset{sign}{=} -\underbrace{\frac{\partial(q(P)/q'(P)}{\partial P}}_{>0 \text{ if } \sigma<1} \underbrace{(2 - L(P;2c)\varepsilon(P)\sigma)}_{>0} + \underbrace{\frac{q(P)}{-q'(P)}}_{>0} \underbrace{\frac{\partial(L(P;2c)\varepsilon(P))}{\partial P}}_{>0 \text{ if } \sigma<1} \sigma.$$

Hence, $\hat{z}'_M(P) < 0$ if $\sigma \leq 0$. Similarly,

$$\hat{z}'_D(P) \overset{sign}{=} -\underbrace{\frac{\partial(q(P)/q'(P)}{\partial P}}_{>0 \text{ if } \sigma<1} \underbrace{\left(\frac{3}{2} - \frac{1}{2}L(P;2c+c_D)\varepsilon(P)\sigma\right)}_{>0} + \underbrace{\frac{q(P)}{-q'(P)}}_{>0} \frac{1}{2} \underbrace{\frac{\partial(L(P;2c+c_D)\varepsilon(P))}{\partial P}}_{>0 \text{ if } \sigma<1} \sigma$$

is strictly negative if $\sigma \leq 0$. Since $P' < 0 < P' + 1$, it follows that the right-hand side of (23) is increasing in r if $\sigma \leq 0$. Thus, $CS(r)$ is quasi-convex for $r \in [c_D, P_D - P_M]$.

Part (ii): For $r = 0$, the two terms in (23) have the same numerator. Therefore, $CS'(0) \leq 0$ if and only if

$$2 - L(P;2c)\varepsilon(P)\sigma \geq \frac{3}{2} - \frac{1}{2}L(P;2c+c_D)\varepsilon(P)\sigma \iff Q'(0) \leq 0.$$

As shown in the proof of Proposition 1, $Q'(c_D) \leq 0$ if condition (11) holds. We can conclude that if $c_D = 0$ and (11) holds, then $CS'(c_D) \leq 0$. Moreover, it follows from the proof of Proposition 1 that if $c_D > 0$ and (11) holds, then $Q'(c_D) < 0$. Therefore, by continuity, $CS'(c_D) \leq 0$ for c_D close enough to zero if (11) holds.

Part (iii): At $r = P_D - P_M$, the right-hand side of (23) is equal to

$$\frac{\frac{q(P_M)}{-q'(P_M)}}{2-\sigma} - \frac{2\frac{q(P_D)}{-q'(P_D)}}{3-2\sigma} = \frac{P_M - 2c}{2-\sigma} - \frac{P_D - 2c - c_D}{3-2\sigma}.$$

The statement in the proposition follows directly. □

Proof of Proposition 4

Part (iv) follows directly from (13) and (14). The proof of parts (i) to (iii) proceeds by proving a series of claims.

Claim 1. In a symmetric mixed-bundling equilibrium, $p_{12} < P_M$.

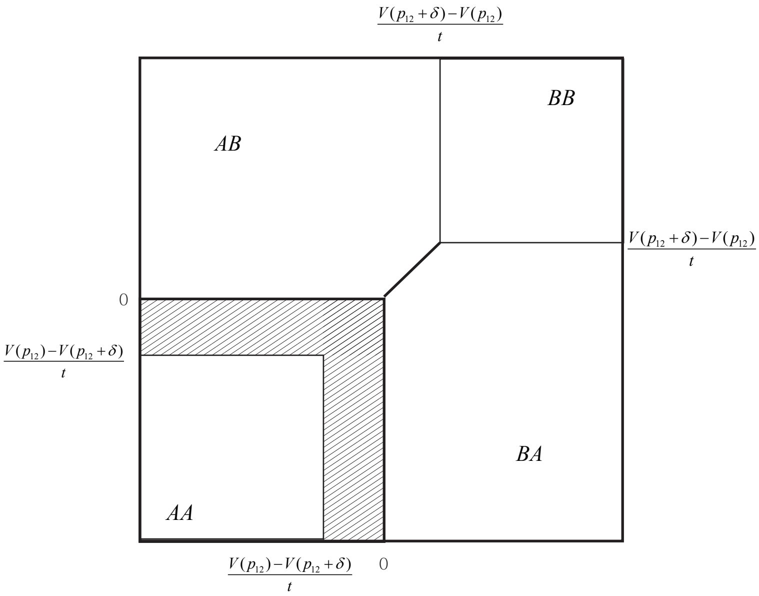

Figure 11: Effects of reducing p_{12}^A from p_{12} to $p_{12}+\delta$ (when $\delta < 0$) while keeping $p_1^A = p_1$ and $p_2^A = p_2$ unchanged.

Proof. First, suppose $p_{12} \geq P_M$ and $\delta \geq 0$. Then, the marginal effect on firm i's profit of increasing p_{12}^i is

$$\begin{aligned}
&\left(\frac{1-\Phi}{2}\right)(q(p_{12}) + q'(p_{12})(p_{12}-2c)) \\
&+\frac{\eta q(p_{12})}{t}(q(p_{12}+\delta)(p_{12}+\delta-2c-c_D) - 2q(p_{12})(p_{12}-2c)) \\
&-\frac{\beta q(p_{12})}{t}(p_{12}-2c)\,q\,(p_{12}),
\end{aligned}$$

which is equal to the right-hand side of (16). Note that the first term is weakly negative because $q(p_{12}) + q'(p_{12})(p_{12}-2c) \leq 0$ for $p_{12} \geq P_M$. The second term is strictly negative because

$$\begin{aligned}
2q\,(p_{12})\,(p_{12}-2c) > q\,(p_{12})\,(p_{12}-2c) &\geq q\,(p_{12}+\delta)\,(p_{12}+\delta-2c) \\
&\geq q\,(p_{12}+\delta)\,(p_{12}+\delta-2c-c_D)
\end{aligned}$$

when $p_{12} \geq P_M > 2c$ and $\delta \geq 0$. Finally, the third term is also strictly negative because $p_{12} \geq P_M$ implies $p_{12} > 2c$. Hence, we can conclude that $p_{12} < P_M$ if $\delta \geq 0$.

Second, suppose $p_{12} \geq P_M$ and $\delta < 0$. The proportion of two-stop shoppers given $\delta < 0$ is

$$\Phi^-(\delta, p_{12}) = 1 - 2H\left(\frac{V(p_{12}) - V(p_{12}+\delta)}{t}, \frac{V(p_{12}) - V(p_{12}+\delta)}{t}\right).$$

There are two subcases to consider: (i) $p_{12}+\delta \geq P_M$, and (ii) $p_{12}+\delta < P_M$. In case (i), it is a profitable deviation for each seller to reduce p_{12}^i from p_{12} to $p_{12}+\delta$ because such a deviation not only increases the profits from one-stop shoppers but also induces some two-stop shoppers to switch to one-stop shopping (Figure 11), which further allows saving data transfer costs. More precisely, the proportion of two-stop shoppers who would switch to the combination AA when seller A deviates is $H(0,0) - \frac{1-\Phi^-}{2}$. Thus, the total impact of the deviation on A's profit is

$$\begin{aligned}
&\left(\frac{1-\Phi^-}{2}\right)\left(q(p_{12}+\delta)(p_{12}+\delta-2c) - q(p_{12})(p_{12}-2c)\right)\\
&+\frac{1}{2}\left(H(0,0) - \frac{1-\Phi^-}{2}\right)\left(q(p_{12}+\delta)(p_{12}+\delta-2c) - q(p_{12}+\delta)(p_1-c-c_E)\right)\\
&+\frac{1}{2}\left(H(0,0) - \frac{1-\Phi^-}{2}\right)\left(q(p_{12}+\delta)(p_{12}+\delta-2c) - q(p_{12}+\delta)(p_2-c-c_I)\right)
\end{aligned}$$

where the first line captures the change of profits from the buyers who would have chosen AA even without the deviation, and the second line (resp. third line) captures the change of profits from the buyers who would switch from combination AB (resp. BA) to AA following the deviation.

Clearly, the first effect is strictly positive since $p_{12} > p_{12}+\delta \geq P_M$. In addition, the sum of the second and third effects, which equals to

$$\frac{1}{2}\left(H(0,0) - \frac{1-\Phi^-}{2}\right)\left(2q(p_{12}+\delta)(p_{12}+\delta-2c) - q(p_{12}+\delta)(p_{12}+\delta-2c-c_D)\right),$$

is also strictly positive because

$$2q(p_{12}+\delta)(p_{12}+\delta-2c) > q(p_{12}+\delta)(p_{12}+\delta-2c) > q(p_{12}+\delta)(p_{12}+\delta-2c-c_D).$$

Therefore, the deviation of reducing p_{12}^i from p_{12} to $p_{12}+\delta$ strictly increases i's profit, which implies that case (i) is not possible in equilibrium. A similar argument applies to case (ii), in which case each seller finds it profitable to reduce p_{12}^i from p_{12} to P_M. □

Claim 2. In a symmetric mixed-bundling equilibrium, $\delta > 0$.

Proof. Suppose $\delta \leq 0$. Then, the proportion of two-stop shopper is $\Phi^-(\delta, p_{12})$, as defined in Claim 1, and following first-order condition must hold for equilibrium prices:[36]

$$\begin{aligned}
0 =& \frac{\partial \Phi^-}{\partial \delta}\left((p_{12}+\delta-2c-c_D)\,q(p_{12}+\delta) - (p_{12}-2c)\,q(p_{12})\right) \qquad (24)\\
&+ \Phi^-\left(2q(p_{12}+\delta) + (p_{12}+\delta-2c-c_D)\,q'(p_{12}+\delta)\right).
\end{aligned}$$

[36]The interpretation of this condition is similar to that of (15).

However, $p_{12} < P_M$ implies that (24) cannot hold because

$$\begin{aligned}&\left(p_{12}+\delta-2c-c_D\right)q\left(p_{12}+\delta\right)-\left(p_{12}-2c\right)q\left(p_{12}\right)\\<&\left(p_{12}+\delta-2c\right)q\left(p_{12}+\delta\right)-\left(p_{12}-2c\right)q\left(p_{12}\right)\leq 0,\end{aligned}$$

and $p_{12}+\delta \leq P_M < P_D$, where the latter implies

$$2q\left(p_{12}+\delta\right)+\left(p_{12}+\delta-2c-c_D\right)q'\left(p_{12}+\delta\right)>0.$$

Therefore, each seller finds it profitable to increase the single-product prices p_1 and p_2 by the same amount, keeping p_{12} unchanged. This is a contradiction, so we conclude that $\delta > 0$ in symmetric bundling equilibrium. □

Claim 3. In a symmetric mixed-bundling equilibrium, $\delta > \min\{c_D, P_D - P_M\}$.

Proof. By way of contradiction, suppose $\delta \leq \min\{c_D, P_D - P_M\}$. Then, $p_{12} \geq 2c$ because otherwise, each seller obtains strictly negative profits in equilibrium. Furthermore, since $p_{12} < P_M$, it follows that

$$p_{12}+\delta < P_D,$$

which implies that

$$2q\left(p_{12}+\delta\right)+\left(p_{12}+\delta-2c-c_D\right)q'\left(p_{12}+\delta\right)>0.$$

Then, the right-hand side of (15),

$$\begin{aligned}&\frac{\partial\Phi}{\partial\delta}\left(\left(p_{12}+\delta-2c-c_D\right)q\left(p_{12}+\delta\right)-\left(p_{12}-2c\right)q(p_{12})\right)\\&+\Phi\left(2q\left(p_{12}+\delta\right)+\left(p_{12}+\delta-2c-c_D\right)q'\left(p_{12}+\delta\right)\right),\end{aligned}$$

is strictly positive because

$$\begin{aligned}&\left(p_{12}+\delta-2c-c_D\right)q\left(p_{12}+\delta\right)-\left(p_{12}-2c\right)q(p_{12})\\=&(p_{12}-2c)\left(q(p_{12}+\delta)-q(p_{12})\right)+(\delta-c_D)q\left(p_{12}+\delta\right)\leq 0,\end{aligned}$$

where the inequality follows from $0<\delta\leq c_D$ and $p_{12}-2c\geq 0$. □

Claim 4. If $t \leq V(P_M)$, then $\delta > c_D$ and $p_1 + p_2 \leq P_D$ in a symmetric mixed-bundling equilibrium.

Proof. Starting from symmetric equilibrium prices (p_{12}, p_1, p_2), consider a unilateral deviation to prices (p_{12}, ∞, ∞). If $t < V(p_{12})$, then the deviation induces all buyers to opt for one-stop

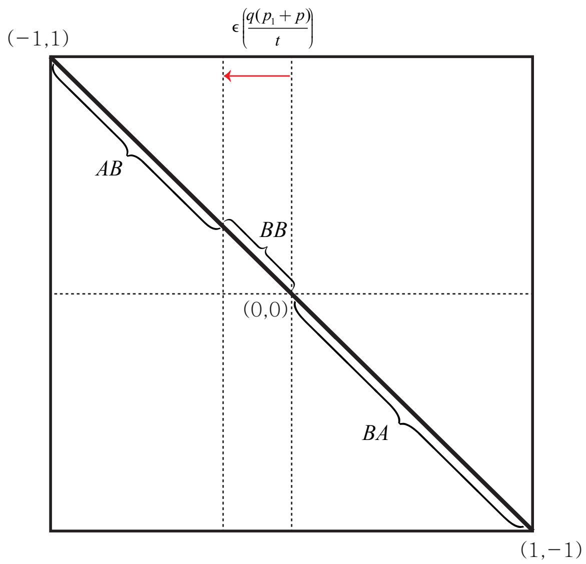

Figure 12: The effect of increasing p_1^A by ϵ on demand segments.

shopping,[37] with half of buyers purchasing both goods from the deviating firm. It follows that for the deviation to be unprofitable, it must be that

$$(p_{12} - 2c + \delta - c_D)\, q(p_{12} + \delta) \geq (p_{12} - 2c)\, q(p_{12}).$$

Given $\delta > 0$, this implies $\delta > c_D$. Moreover, it also implies $p_1 + p_2 \leq P_D$, otherwise the first-order condition in (15) would be violated.

Finally, since $p_{12} < P_M$, $V(p_{12}) > V(P_M)$. Hence, $t \leq V(P_M)$ is a sufficient condition for $t < V(p_{12})$. □

Proof of Proposition 5

First, consider the uniform pricing regime, with $p_1 = p_2 = p$ denoting single product prices and $P = 2p$. Without a deviation, the buyers with $z_1 < 0$ (resp. $z_1 > 0$) are two-stop shoppers purchasing combination AB (resp. BA).

Now, consider firm A's deviation of increasing p_1^A from $p_1 = p$ to $p + \epsilon$. This deviation will affect the profit earned from each buyer buying combination AB and also induce some of two-stop shoppers buying AB to switch to one-stop shopping at seller B. As illustrated in Figure 12, the mass of these switching buyers is

$$\int_{-\frac{\epsilon q(p_1+p)}{t}}^{0} \left(\frac{1}{2}\right) dx = \frac{\epsilon q\,(p_1 + p)}{2t}.$$

[37]If $t > V(p_{12})$, some of the two-stop shoppers before the deviation prefer to buy nothing (and obtain the match values from two-stop shopping) after the deviation rather than switch to one-stop shopping.

Combining these two effects, the marginal change in profits as $\epsilon \to 0$ is obtained as follows:

$$-\left(\frac{q\left(p_1+p\right)}{2t}\right)\left(p_1-c\right)q\left(p_1+p\right)+\frac{1}{2}\left(q\left(p_1+p\right)+q'\left(p_1+p\right)\left(p_1-c\right)\right). \tag{25}$$

Then, the first-order condition for the equilibrium P is given by $\Delta_u(P)=0$, where

$$\Delta_u\left(P\right)\equiv-\left(\frac{q\left(P\right)}{t}\right)\left(P-2c\right)q\left(P\right)+\left(2q\left(P\right)+q'\left(P\right)\left(P-2c\right)\right).$$

Claim 1. $p_1=p_2=\frac{P^u}{2}$ is the unique uniform-pricing equilibrium for sufficiently large t.

Proof. We first show that there exists a unique solution to this first-order condition. Note that

$$\Delta_u(2c)=2q(2c)>0>\Delta_u(P_D)=-\frac{q(P_D)}{t}(P_D-2c)q(P_D).$$

By continuity, there exists $P^u\in(2c,P_D)$ such that $\Delta_u(P^u)=0$. For uniqueness, observe that

$$\frac{d\Delta_u}{dP}=-3-\left(-\frac{1}{t}\left(P-2c\right)q\left(P\right)+\left(\frac{q\left(P\right)}{t}\right)\left(q\left(P\right)-\left(P-2c\right)\right)\right).$$

Since the terms in the bracket can be made arbitrarily close to zero uniformly for sufficiently large t, $\Delta_u(P)$ is strictly decreasing for sufficiently large t, so P^u is unique.

Next, from the derivative of (25) with respect to p_1, the global optimality follows if

$$-1-\left(\frac{q(p_1+p^u)}{2t}\right)\left(q\left(p_1+p^u\right)-\left(p_1-c\right)\right)+\frac{1}{2t}\left(p_1-c\right)q\left(p_1+p^u\right)<0$$

for all p_1. Since the second and third terms can be made uniformly convergent to zero as t grows, this second-order condition also holds, completing the proof. □

We next consider discriminatory pricing, where each firm charges the bundle price p_{12} to its one-stop shoppers and the single-product price p to two-stop shoppers. First, note that $p_{12}=2c$ in equilibrium: if $p_{12}<2c$, it is profitable for each firm to strictly increase p_{12}, while fixing $p_1=p_2=p$; if $p_{12}>2c$, then a slight undercutting p_{12}, keeping $p_1=p_2=p$ unchanged, is a profitable deviation. Finally, given the rival firm choosing $p_{12}=2c$, each firm has no incentive to increase p_{12} since its only effect is that the rival firm would attract all one-stop shoppers.

It remains to characterize the single-product price p. Suppose firm A chooses p_1 and p_2, while firm B is choosing $(2c,p,p)$. The proportions of each two-stop shoppers are then

$$\Phi_{AB}=\int_{-1}^{\frac{V(p_1+p)-V(2c)}{t}}\left(\frac{1}{2}\right)dx=\frac{1}{2}\left(1-\frac{V(2c)-V(p_1+p)}{t}\right),$$

and

$$\Phi_{BA}=\int_{\frac{V(2c)-V(p_2+p)}{t}}^{1}\left(\frac{1}{2}\right)dx=\frac{1}{2}\left(1-\frac{V(2c)-V(p_2+p)}{t}\right).$$

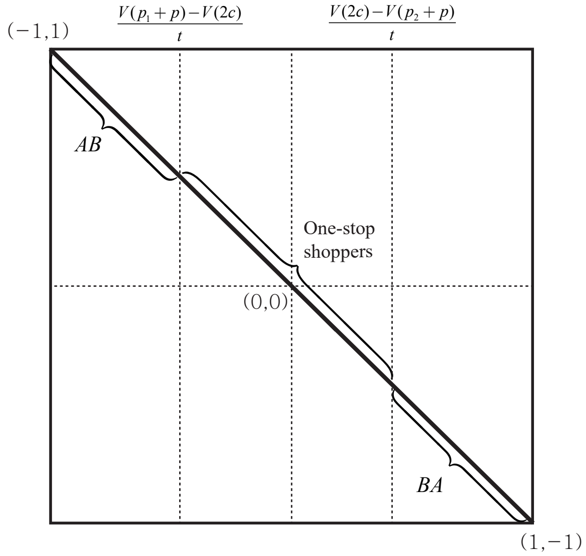

Figure 13: Demand segments when firm A chooses p_1, p_2 and $p_{12} = 2c$, while firm B chooses $p_1 = p_2 = p$ and $p_{12} = 2c$.

The remaining buyers are one-stop shoppers, and each firm serves half of them. The resulting demand segments are depicted in Figure 13. Then, firm A's profit from $(2c, p_1, p_2)$ is

$$\Phi_{AB} \cdot (p_1 - c)q(p_1 + p) + \Phi_{BA} \cdot (p_2 - c)q(p_2 + p).$$

Now, consider a slight increase of p_1, whose marginal profit effect is given by

$$\left(-\frac{q(p_1 + p)}{2t}\right)(p_1 - c)\, q\,(p_1 + p) + \Phi_{AB} \cdot (q(p_1 + p) + (p_1 - c)q'(p_1 + p)), \tag{26}$$

Then, the first order condition for the equilibrium price P is given by $\Delta_d(P) = 0$, where

$$\Delta_d\,(P) \equiv \left(-\frac{q(P)}{2t}\right)(P - 2c)\, q\,(P) + \frac{1}{2}\left(1 - \frac{V(2c) - V(P)}{t}\right)(2q(P) + (P - 2c)q'(P)).$$

Claim 2. $(p_{12}, p_1, p_2) = \left(2c, \frac{P^d}{2}, \frac{P^d}{2}\right)$ is the unique discriminatory-pricing equilibrium for sufficiently large t.

Proof. The existence of a unique solution to the first-order condition follows by a similar argument as Claim 1. To show the optimality, we take the derivative of (26) with respect to p_1, which yields

$$\begin{aligned}&\frac{1}{2t}\,(p_1 - c)\, q\left(p_1 + p^d\right) - \frac{q\left(p_1 + p^d\right)}{2t}\left(q\left(p_1 + p^d\right) - (p_1 - c)\right)\\ -&\left(1 - \frac{V(2c) - V(p_1^d + p)}{t}\right) - \frac{q\left(p_1 + p^d\right)}{2t}\left(q\left(p_1 + p^d\right) - (p_1 - c)\right).\end{aligned}$$

Clearly, this second derivative is negative for all p_1 when t is sufficiently large. Thus, each firm has no incentive to change its price for two-stop shoppers, while keeping the price for one-stop shoppers unchanged.

The proof is completed by noting that there is no profitable deviation that changes prices for one-stop shoppers and two-stop shoppers simultaneously. To see this, consider firm A's deviation to $(\tilde{p}_{12}, \tilde{p}_1, \tilde{p}_2)$. If $\tilde{p}_{12} > 2c$, then the same deviation payoff can be achieved by another deviation, $(2c, \tilde{p}_1, \tilde{p}_2)$. Similarly, if $\tilde{p}_{12} < 2c$ and $\tilde{p}_1, \tilde{p}_2 \geq c$, then $(2c, \tilde{p}_1, \tilde{p}_2)$ yields even higher deviation profit. Lastly, if $\tilde{p}_{12} < 2c$ and $\tilde{p}_1 \geq c > \tilde{p}_2$, then $(2c, \tilde{p}_1, c)$ yields higher deviation profits. □

Finally, we prove that price discrimination lowers all buyers' prices in equilibrium.

Claim 3. For sufficiently large t, all buyers pay a lower price in the discriminatory-pricing equilibrium than in the uniform-pricing equilibrium.

Proof. By the previous results, we know that Δ_u and Δ_d are strictly decreasing for sufficiently large t. Thus, it follows that $P^u > P^d$ if $\Delta_d(P^u) < 0$. Note that

$$\Delta_u(P^u) = 0 \iff 2q(P^u) + q'(P^u)(P^u - 2c) = \frac{q(P^u)}{t}(P^u - 2c)\, q(P^u),$$

and so

$$\Delta_d(P^u) = (P^u - 2c)\, q(P^u) \left[-\frac{q(P^u)}{2t} + \frac{q(P^u)}{2t}\left(1 - \frac{V(2c) - V(P^u)}{t}\right) \right].$$

Note that

$$-\frac{q(P^u)}{2t} + \frac{q(P^u)}{2t}\left(1 - \frac{V(2c) - V(P^u)}{t}\right) = -\frac{q(P^u)}{2t}\left(\frac{V(2c) - V(P^u)}{t}\right) < 0.$$

Therefore, we have $\Delta_d(P^u) < 0$ for sufficiently large t, which completes the proof. □

Appendix B: Additional Results

B.1 Independent markets: effects of discrimination

In this appendix, we analyze the effects of price discrimination against two-stop shoppers in the independent-markets model without imposing that the curvature of inverse demand is constant. In addition, the analysis also contains results expressed in terms of the curvature of the direct demand function, given by $\alpha(P) = \frac{-Pq''}{q'}$.

B.1.1 Effects of discrimination on output and social welfare

As shown in the main text,

$$\begin{aligned} Q'(r) &\overset{sign}{=} -1 + 2(P-2c)\frac{q''(P)}{-q'(P)} - (P+r-2c-c_D)\frac{q''(P+r)}{-q'(P+r)} \\ &= -1 + 2L(P;2c)\sigma(q(P))\varepsilon(P) - L(P+r;2c+c_D)\sigma(q(P+r))\varepsilon(P+r) \\ &= -1 + 2L(P;2c)\alpha(P) - L(P+r;2c+c_D)\alpha(P+r), \end{aligned} \tag{B.1}$$

where $P = P(r)$. Our first proposition shows that discrimination reduces aggregate output if the demand function is not "too convex" and the curvature of demand does not rise too fast as price falls within the relevant range.

Proposition B.1. Price discrimination against two-stop shoppers reduces aggregate output sold and harms social welfare if

(i) demand is not too convex (formally, $\alpha(P(r)) \leq \frac{1}{2}$ for all $r \in [c_D, P_D - P_M]$ and $\sigma < 1$ throughout the relevant range), and

(ii) the curvature of demand does not rise too fast as price falls (formally, $\alpha(P(r)) \leq \alpha(P(r)+r) + \frac{1}{2}$ for all $r \in [c_D, P_D - P_M]$).

Proof. From (B.1), the marginal output effect $Q'(r)$ is weakly negative if

$$2L(P;2c)\alpha(P) - L(P+r;2c+c_D)\alpha(P+r) \leq 1 \tag{B.2}$$

There are two cases to consider: (a) $\alpha(P+r) \geq 0$, and (b) $\alpha(P+r) < 0$. Note that in case (a), if $\alpha(P) \leq \frac{1}{2}$, then the condition in part (ii) of the proposition is satisfied. And in case (b), if the condition in part (ii) of the proposition holds, then $\alpha(P) \leq \frac{1}{2}$.

In case (a),

$$2L(P;2c)\alpha(P) - L(P+r;2c+c_D)\alpha(P+r) \leq 2L(P;2c)\alpha(P),$$

hence (B.2) holds if $\alpha(P) \leq \frac{1}{2}$.

In case (b), first note that $\sigma < 1$ implies $\varepsilon' > 0$ and thus $\varepsilon(P_M) < \varepsilon(P_D)$. Since $2L(P_M; 2c)\varepsilon(P_M) = 2$ and $L(P_D; 2c + c_D)\varepsilon(P_D) = 2$, this in turn implies that $2L(P_M; 2c) > L(P_D; 2c + c_D)$. Given that $L(P; C)$ is increasing in P and $P_M \leq P(r) \leq P(r) + r \leq P_D$, it follows that

$$2L(P(r); 2c) \geq 2L(P_M; 2c) > L(P_D; 2c + c_D) \geq L(P(r) + r; 2c + c_D).$$

Hence, for $\alpha(P + r) < 0$,

$$2L(P; 2c)\alpha(P) - L(P + r; 2c + c_D)\alpha(P + r) < 2L(P; 2c) \left(\alpha(P) - \alpha(P + r)\right).$$

Therefore, $Q'(r) \leq 0$ if $\alpha(P) - \alpha(P + r) \leq \frac{1}{2L(P;2c)}$, which is implied by the condition in part (ii) of the proposition. □

Our next proposition contains a different set of sufficient conditions under which discrimination against two-stop shoppers harms social welfare. Building on an approach introduced by Aguirre et al. (2010) in their analysis of the impact of monopolistic third-degree price discrimination on social welfare, this proposition relies on the following two conditions on the demand function.

Definition 1. The **increasing product condition (IPC)** is satisfied if $w(P; C) \equiv L(P; C)\alpha(P)$ is non-decreasing in P for $P \in [P_M, P_D]$ and $C \in \{2c, 2c + c_D\}$.

Definition 2. The **increasing ratio condition (IRC)** is satisfied if

$$z_M(P) \equiv \frac{P - 2c}{2 - L(P; 2c)\alpha(P)} \text{ and } z_D(P) \equiv \frac{2(P - 2c - c_D)}{3 - L(P; 2c + c_D)\alpha(P)}.$$

are non-decreasing in P for $P \in [P_M, P_D]$.

The IPC holds if (but not only if) the demand function is convex and the curvature of direct demand non-decreasing (i.e., if α is non-negative and weakly increasing). It also holds if demand is convex and log-concave ($\sigma \in [0, 1]$) and the curvature of inverse demand $\sigma(q)$ is non-increasing in quantity (and thus non-decreasing as price rises).[38]

[38]To see this formally, note that

$$\frac{d(L(P; k)\alpha(P))}{dP} = L'\alpha + L\alpha' = (L'\varepsilon + L\varepsilon')\sigma + L\varepsilon\frac{d\sigma(q(P))}{dP}.$$

Since $L' > 0$, the first formulation implies that this derivative is non-negative if $\alpha \geq 0$ and $\alpha' \geq 0$. For the sign of the second formulation, note that if $\sigma \leq 1$, then Marshall's second law of demand is satisfied and $\varepsilon' \geq 0$. Hence, the first term is non-negative if $\sigma \in [0, 1]$. The last term is non-negative if $\frac{d\sigma(q(P))}{dP} = \sigma' q' \geq 0$, which requires $\sigma' \leq 0$.

This IRC condition, which is the condition that was introduced for the monopoly case by Aguirre et al. (2010), holds for a large set of demand functions $q(P)$. Beyond linear demand, it holds for any convex demand function with non-decreasing curvature of direct demand, and for any concave demand with constant curvature of direct demand. It also holds for any log-concave demand function with a positive and non-decreasing curvature of inverse demand. (See Appendix B in Aguirre et al. (2010) for a detailed discussion of sufficient conditions under which $z_M(P)$ is weakly increasing. The same conditions also guarantee that $z_D(P)$ is weakly increasing.)

Importantly, IRC is a weaker condition than IPC. The IPC condition implies IRC, but IRC is satisfied by an even larger set of demand functions. For instance, while IPC does not hold for demand function with constant negative curvature α, the IRC does.

Proposition B.2. If

$$2\alpha(P) - \frac{P}{P+c_D}\alpha(P+c_D) \leq \frac{1}{L(P;2c)} \tag{B.3}$$

$$\Longleftrightarrow$$

$$2\sigma(q(P))\varepsilon(P) - \frac{P}{P+c_D}\sigma(q(P+c_D))\varepsilon(P+c_D) \leq \frac{1}{L(P;2c)} \tag{B.4}$$

at $P = P(c_D)$,

(i) and the IPC holds, then price discrimination against two-stop shoppers reduces aggregate output sold and harms welfare.

(ii) and the IRC holds, then price discrimination against two-stop shoppers harms social welfare.

Proof. Part (i): The IPC implies that the right-hand-side of (B.1), which determines the sign of $Q'(r)$, is decreasing in r, because

$$2\underbrace{w'(P(r);2c)P'(r)}_{\leq 0 \text{ given IPC}} - \underbrace{w'(P(r)+r;2c+c_D)(P'(r)+1)}_{\geq 0 \text{ given IPC}} \leq 0.$$

Hence, given the IPC, if $Q'(c_D) \leq 0$, then also $Q'(r) \leq 0$ for all $r \in (c_D, P_D - P_M]$ and thus $\Delta Q = \int_{c_D}^{P_D-P_M} Q'(r)dr \leq 0$.

Using (B.1), $Q'(c_D) \leq 0$ if and only if at $P = P(c_D)$,

$$2L(P; 2c)\alpha(P) - L(P + c_D; 2c + c_D)\alpha(P + c_D) \leq 1$$

$$\iff$$

$$2L(P; 2c)\alpha(P) - L(P; 2c)\frac{P}{P + c_D}\alpha(P + c_D) \leq 1$$

$$\iff$$

$$2L(P; 2c)\sigma(q(P))\varepsilon(P) - L(P; 2c)\frac{P}{P + c_D}\sigma(q(P + c_D))\varepsilon(P + c_D) \leq 1,$$

which are equivalent to the conditions in the proposition.

Part (ii): Using (6) and the first-order condition that determines $P(r)$, we obtain

$$W'(r) = \underbrace{-\frac{\gamma(1-\gamma)\Delta\pi_M\Delta\pi_D}{(1-\gamma)\Delta\pi_M + \gamma\Delta\pi_D}}_{>0}\left(\frac{P - 2c}{2 - L(P; 2c)\alpha(P)} - \frac{2(P + r - 2c - c_D)}{3 - L(P + r; 2c + c_D)\alpha(P + r)}\right).$$

Hence, $W'(r)$ has the sign of $z_M(P(r)) - z_D(P(r) + r)$. The IRC implies that $z_M(P(r)) - z_D(P(r)+r)$ is decreasing in r, because $z'_M(P(r))P'(r) \leq 0$ and $z'_D(P(r)+r)(P'(r)+1) \geq 0$ given the IRC. It follows that if $W'(c_D) \leq 0$ and the IRC holds, then $W'(r) \leq 0$ for all $r \in [c_D, P_D - P_M]$. By (8), $W'(c_D) = (P(c_D) - 2c)Q'(c_D)$. Hence, $W'(c_D) \leq 0$ if and only if $Q'(c_D) \leq 0$, which, as shown in part (i) of the proof, is equivalent to condition (B.3). □

The proof of Proposition B.2 relies on three main insights. First, conditions (B.3) and (B.4) are each equivalent to $Q'(c_D) \leq 0$, which in turn is equivalent to $W'(c_D) \leq 0$, because the marginal allocation effect of price discrimination is zero at $r = c_D$ (see the decomposition of the marginal welfare effect in (8)). Second, the IPC implies that if $Q'(c_D) \leq 0$, then also $Q'(r) \leq 0$ for all $r > c_D$. Third, the IRC implies that if $W'(c_D) \leq 0$, then also $W'(r) \leq 0$ for all $r > c_D$. Hence, if (B.3) or (B.4) and the IPC hold, then price discrimination has a negative output effect, and if (B.3) or (B.4) and the IRC hold, then price has discrimination has a negative welfare effect.

Condition (B.3) can be simplified in special cases. For $c_D = 0$, it simplifies to an upper bound on the convexity of demand, which is less stringent than the upper bound identified in Proposition B.1:

$$\alpha(P(0)) \leq \underbrace{\frac{1}{L(P(0); 2c)}}_{\geq 1}.$$

Similarly, in the special case of a constant α, the condition simplifies to:

$$\alpha \leq \frac{P(c_D)(P(c_D) + c_D)}{(P(c_D) - 2c)(P(c_D) + 2c_D)}.$$

This upper bound on the degree of convexity strictly exceeds 1 if $2c > \frac{P(c_D)}{P(c_D)+2c_D} c_D$, a sufficient condition for which is $2c > c_D$. If α is not constant and $c_D > 0$, then, all else equal, (B.3) is easier to satisfy when the degree of convexity α increases in price than when it falls in price, echoing condition (ii) in Proposition B.1.

Taken together, Propositions B.1 and B.2 imply that discrimination against two-stop shoppers harms social welfare if demand is not "too convex," and the convexity of the demand function does not rise too rapidly as price falls.

B.1.2 Effects of discrimination on profits

Next, we analyze the effect of discrimination against two-stop shoppers on the firms' profits. To this end, note that

$$\begin{aligned} 2\Pi'(r) &= (1-\gamma) P'(r) \left(q(P(r)) + (P(r) - 2c) q'(P(r)) \right) \\ &\quad + \gamma \left(P'(r) + 1 \right) \left(q(P(r) + r) + (P(r) + r - 2c - c_D) q'(P(r) + r) \right) \\ &= \underbrace{\frac{\gamma(1-\gamma) \Delta\pi_M \Delta\pi_D}{(1-\gamma)\Delta\pi_M + \gamma\Delta\pi_D}}_{<0} \left(2\widetilde{z}_D(P(r) + r) - \widetilde{z}_M(P(r)) \right) \end{aligned}$$

where

$$\widetilde{z}_D(P) \equiv \frac{P - 2c - c_D - \frac{q(P)}{-q'(P)}}{3 - L(P; 2c + c_D)\alpha(P)} = \left(\frac{P - 2c - c_D + \frac{q(P)}{q'(P)}}{P - 2c - c_D} \right) z_D(P),$$

and

$$\widetilde{z}_M(P) \equiv \frac{P - 2c - \frac{q(P)}{-q'(P)}}{2 - L(P; 2c)\alpha(P)} = \left(\frac{P - 2c + \frac{q(P)}{q'(P)}}{P - 2c} \right) z_M(P).$$

Our next proposition shows that price discrimination can reduce equilibrium profits by intensifying price competition. For a large class of demand functions, price discrimination creates a prisoners' dilemma for the firms: although each firm unilaterally benefits from practicing price discrimination, both firms are better off when in the presence of a ban on price discrimination.

Proposition B.3. Given IRC, $\sigma \leq 1$, and (B.3), price discrimination against two-stop shoppers lowers firm profits.

Proof. Suppose $q(\cdot)$ is log-concave, and so $\frac{q(P)}{q'(P)}$ is non-decreasing in P. We first show that IRC implies non-decreasing $\widetilde{z}_D(P)$ and $\widetilde{z}_M(P)$ in the relevant ranges of prices. To this end, observe that

$$P - 2c + \frac{q(P)}{q'(P)} \geq 0 \iff P \geq P_M$$

and, for all $P \geq P(c_D) + c_D$,

$$\begin{aligned}
P - 2c - c_D + \frac{q(P)}{q'(P)} &\geq P(c_D) + c_D - 2c - c_D + \frac{q\left(P(c_D) + c_D\right)}{q'\left(P(c_D) + c_D\right)} \\
&= P(c_D) - 2c + \frac{q\left(P(c_D) + c_D\right)}{q'\left(P(c_D) + c_D\right)} \\
&\geq P(c_D) - 2c + \frac{q\left(P(c_D)\right)}{q'\left(P(c_D)\right)} \\
&\geq 0.
\end{aligned}$$

Also,

$$\begin{aligned}
\frac{\partial}{\partial P}\left(\frac{P - 2c - c_D + \frac{q(P)}{q'(P)}}{P - 2c - c_D}\right) &= \frac{\partial}{\partial P}\left(\frac{\frac{q(P)}{q'(P)}}{P - 2c - c_D}\right) \\
&\overset{sign}{=} \left(P - 2c - c_D\right) \frac{\partial}{\partial P}\left(\frac{q(P)}{q'(P)}\right) - \frac{q(P)}{q'(P)} \\
&\geq 0,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial}{\partial P}\left(\frac{P - 2c + \frac{q(P)}{q'(P)}}{P - 2c}\right) = \frac{\partial}{\partial P}\left(\frac{\frac{q(P)}{q'(P)}}{P - 2c}\right) &\overset{sign}{=} \left(P - 2c\right) \frac{\partial}{\partial P}\left(\frac{q(P)}{q'(P)}\right) - \frac{q(P)}{q'(P)} \\
&\geq 0.
\end{aligned}$$

Since the first terms in $\widetilde{z}_D(P)$ and $\widetilde{z}_M(P)$ are non-decreasing and positive, we conclude that $\widetilde{z}_D(P)$ is non-decreasing in $P \geq P(c_D) + c_D$ and $\widetilde{z}_M(P)$ is non-decreasing in $P \geq P_M$, respectively.

Given this, we next show that $\Pi(r)$ is quasi-concave in $r \geq c_D$. Observe that

$$\begin{aligned}
2\Pi''(r) = {} & \frac{\gamma\left(1-\gamma\right)\Delta\pi_M \Delta\pi_D}{\left(1-\gamma\right)\Delta\pi_M + \gamma\Delta\pi_D}\left(2\widetilde{z}'_D\left(P\left(r\right)+r\right)\left(P'\left(r\right)+1\right) - \widetilde{z}'_M\left(P\left(r\right)\right)P'\left(r\right)\right) \\
& + \left(2\widetilde{z}_D\left(P\left(r\right)+r\right) - \widetilde{z}_M\left(P\left(r\right)\right)\right)\frac{d}{dr}\left(\frac{\gamma\left(1-\gamma\right)\Delta\pi_M \Delta\pi_D}{\left(1-\gamma\right)\Delta\pi_M + \gamma\Delta\pi_D}\right).
\end{aligned}$$

Clearly, the first term is non-positive for all $r \geq c_D$, and the second term is zero at $r = \hat{r}$ if $\Pi'(\hat{r}) = 0$. Hence, $\Pi(r)$ is quasi-concave in $r \geq c_D$, implying that if $\Pi'\left(c_D\right) \leq 0$, then $\Pi'(r) \leq 0$ for all $r \geq c_D$.

Finally, it can be checked that the condition (B.3) ensures $\Pi'(c_D) \leq 0$. To see this, note that $\Pi'(c_D)$ is equivalent to

$$\begin{aligned}
& 2\widetilde{z}_D\left(P\left(c_D\right) + c_D\right) \geq \widetilde{z}_M\left(P\left(c_D\right)\right) \\
\Longleftrightarrow\ & \frac{P\left(c_D\right) - 2c + \frac{q\left(P(c_D)+c_D\right)}{q'\left(P(c_D)+c_D\right)}}{\frac{3}{2} + \frac{1}{2}\left(P\left(c_D\right) - 2c\right)\frac{q''\left(P(c_D)+c_D\right)}{q'\left(P(c_D)+c_D\right)}} \geq \frac{P\left(c_D\right) - 2c + \frac{q\left(P(c_D)\right)}{q'\left(P(c_D)\right)}}{2 + \left(P\left(c_D\right) - 2c\right)\frac{q''\left(P(c_D)\right)}{q'\left(P(c_D)\right)}}.
\end{aligned}$$

This inequality holds if

$$\frac{3}{2}+\frac{1}{2}\left(P\left(c_D\right)-2c\right)\frac{q''\left(P\left(c_D\right)+c_D\right)}{q'\left(P\left(c_D\right)+c_D\right)}\leq 2+\left(P\left(c_D\right)-2c\right)\frac{q''\left(P\left(r\right)\right)}{q'\left(P\left(r\right)\right)},$$

which is equivalent to the condition (B.3), and

$$P\left(c_D\right)-2c+\frac{q\left(P\left(c_D\right)+c_D\right)}{q'\left(P\left(c_D\right)+c_D\right)}\geq P\left(c_D\right)-2c+\frac{q\left(P\left(c_D\right)\right)}{q'\left(P\left(c_D\right)\right)}.$$

The second condition holds if

$$\frac{q(P)}{-q'(P)}\geq\frac{q(P+c_D)}{-q'(P+c_D)},$$

which is implied by log-concavity of $q(\cdot)$. $\square$

Proposition B.3 shows that if the sufficient conditions for a negative welfare effect in Proposition B.2 hold and demand is log-concave (i.e., demand is again not "too convex"), then discrimination against two-stop shoppers harms not only social welfare but it also hurts firm profits. The conditions for a negative profit effect in Proposition B.3 are satisfied by concave demand functions with constant curvature of direct demand (including linear demand).

B.1.3 Effects of discrimination on consumer surplus

As shown in the main text,

$$\begin{aligned}CS'(r) = &-\underbrace{\left(\frac{\gamma(1-\gamma)\Delta\pi_M\Delta\pi_D}{(1-\gamma)\Delta\pi_M+\gamma\Delta\pi_D}\right)}_{>0}\\ &\times\left(\frac{\frac{q(P+r)}{q'(P+r)}}{\frac{3}{2}-\frac{1}{2}L(P+r;2c+c_D)\alpha(P+r)}-\frac{\frac{q(P)}{q'(P)}}{2-L(P;2c)\alpha(P)}\right).\end{aligned}$$

Hence,

$$CS'(r)\overset{sign}{=}\frac{\frac{q(P+r)}{q'(P+r)}}{\frac{3}{2}-\frac{1}{2}L(P+r;2c+c_D)\alpha(P+r)}-\frac{\frac{q(P)}{q'(P)}}{2-L(P;2c)\alpha(P)}. \tag{B.5}$$

Proposition B.4. The following holds:

(i) If (B.3) holds with strict inequality (i.e., if $Q'(c_D)<0$), then there exists $\bar{c}_D>0$ such that $CS'(c_D)\leq 0$ for $c_D<\bar{c}_D$.

(ii) $CS'(P_D-P_M)\leq 0$ if and only if $\frac{P_M-2c}{2-\sigma(q(P_M))}\leq\frac{P_D-2c-c_D}{3-2\sigma(q(P_D))}$.

Proof. Part (i): For $r = 0$, the two terms in (B.5) have the same numerator. Therefore, $CS'(0) \leq 0$ if and only if

$$2 - L(P; 2c)\alpha(P) \geq \frac{3}{2} - \frac{1}{2}L(P; 2c + c_D)\alpha(P) \iff Q'(0) \leq 0.$$

As shown in the proof of Proposition B.2, $Q'(c_D) < 0$ if condition (B.3) holds with strict inequality. We can conclude that if $c_D = 0$ and (B.3) holds, then $CS'(c_D) < 0$. Therefore, by continuity, $CS'(c_D) \leq 0$ for c_D close enough to zero if (B.3) holds with strict inequality.

Part (ii): The proof of Proposition 3 (iii) in the main text applies here as well. □

B.2 Interdependent markets

B.2.1 Derivation of restricted pricing condition

We denote the prices firm i charges to one-stop shoppers by $\bar{p}_1^i$ and $\bar{p}_2^i$, and the prices it charges to two-stop shoppers by p_1^i and p_2^i. The restrictions are given as $p_1^i \leq \bar{p}_1^i + r_1$ and $p_2^i \leq \bar{p}_2^i + r_2$, with $r \equiv r_1 + r_2 \geq 0$. To ensure the restriction to be effective, we assume $r \leq \delta$, where δ is the equilibrium bundling discount in the absence of restrictions in Theorem 4.

Denote by $S_{ij}\left(\mathbf{p}^A, \mathbf{p}^B\right)$ the share of buyers choosing combination ij in a price profile $\left(\mathbf{p}^A, \mathbf{p}^B\right)$. Then, each seller i solves the following constrained maximization problem:

$$\begin{aligned} \max_{\left(p_k^i, \bar{p}_k^i\right)_{k=1,2}} & \Big[S_{ii}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(\bar{p}_1^i + \bar{p}_2^i - 2c\right) q\left(\bar{p}_1^i + \bar{p}_2^i\right) + S_{ij}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(p_1^i - c - c_E\right) q\left(p_1^i + p_2^j\right) \\ & + S_{ji}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(p_2^i - c - c_I\right) q\left(p_1^j + p_2^i\right) \Big] \\ \text{s.t.} \quad & p_1^i \leq \bar{p}_1^i + r_1 \text{ and } p_2^i \leq \bar{p}_2^i + r_2. \end{aligned}$$

As in the analysis for independent markets, it is indeed sufficient to consider the following reduced problem:[39]

$$\begin{aligned} \max_{\left(p_{12}^i, p_1^i, p_2^i\right)} & \Big[S_{ii}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(p_{12}^i - 2c\right) q\left(p_{12}^i\right) + S_{ij}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(p_1^i - c - c_E\right) q\left(p_1^i + p_2^j\right) \\ & + S_{ji}\left(\mathbf{p}^A, \mathbf{p}^B\right)\left(p_2^i - c - c_I\right) q\left(p_1^j + p_2^i\right) \Big] \\ \text{s.t.} \quad & p_1^i + p_2^i - p_{12}^i \leq r. \end{aligned}$$

[39] If $\left(p_{12}^i, p_1^i, p_2^i\right)$ is the solution to the reduced problem, one can find a solution to the original problem by setting $\bar{p}_2^i = p_2^i - r_2$ and $\bar{p}_1^i = p_{12}^i - p_2^i + r_2$.

It can be checked that the constraint must be binding in symmetric equilibrium, $p_{12} = p_1 + p_2 - r$, and we denote by $\widehat{P}(r)$ and $\widehat{P}(r) + r$ the total prices paid by one-stop and two-stop shoppers in equilibrium, respectively.

To characterize the first-order equilibrium condition, let us consider the case in which, starting from some symmetric price levels (p_{12}, p_1, p_2) with $p_1 + p_2 - p_{12} = r$, firm i increases its prices to $(\widetilde{p}_{12}, \widetilde{p}_1, \widetilde{p}_2)$, where $\widetilde{p}_{12} = p_{12} + 2\epsilon$ and $\widetilde{p}_k = p_k + \epsilon$ for $k = 1, 2$. Notice that this price change does not affect the difference between total prices paid by two-stop shoppers and one-stop shoppers, $\widetilde{p}_1 + \widetilde{p}_2 - \widetilde{p}_{12} = p_1 + p_2 - p_{12} = r$. For sufficiently small ϵ, its overall marginal profit impact equals to[40]

$$\begin{aligned}\frac{1}{2} \times \Bigg[& \Phi(r, p_{12}) \left(2q(p_1 + p_2) + (p_1 + p_2 - 2c - c_D) q'(p_1 + p_2)\right) \\ & + 2(1 - \Phi(r, p_{12})) \left(q(p_{12}) + (p_{12} - 2c) q'(p_{12})\right) \\ & + \frac{\partial \Phi(r, p_{12})}{\partial \delta} \left((p_1 + p_2 - 2c - c_D) q(p_1 + p_2) - (p_{12} - 2c) q(p_{12})\right) \\ & + \frac{4\eta(r, p_{12}) q(p_{12})}{t} (p_1 + p_2 - 2c - c_D) q(p_1 + p_2) \\ & - \frac{4q(p_{12})}{t} \left(2\eta(r, p_{12}) + \beta(r, p_{12})\right) (p_{12} - 2c) q(p_{12}) \Bigg].\end{aligned}$$

In equilibrium, such deviations must be unprofitable, yielding that the above is equal to zero at $P = \widehat{P}(r)$. In particular, since $\frac{\partial \Phi(r,P)}{\partial \delta} = -\frac{4\eta(r,P)q(P+r)}{t}$, this condition is given by

$$\begin{aligned}0 = {} & \Phi(r, P) \left(q(P + r) + (P + r - 2c - c_D) \frac{q'(P + r)}{2}\right) \\ & + (1 - \Phi(r, P)) \left(q(P) + (P - 2c) q'(P)\right) \\ & + \frac{2\eta(r, P)}{t} \left(q(P) - q(P + r)\right) \left((P + r - 2c - c_D) q(P + r) - (P - 2c) q(P)\right) \\ & - \frac{2q(P)}{t} \left(\eta(r, P) + \beta(r, P)\right) (P - 2c) q(P)\end{aligned}$$

at $P = \widehat{P}(r)$. By rearranging this, one can check that this is indeed equivalent to (17).

B.2.1 Derivation of marginal match-quality effect

To derive the marginal match-quality effect, let us consider a marginal increase in r by $\epsilon > 0$. If this causes a buyer to switch from AB to AA, then the change of match value to this buyer

[40]This follows from the symmetry of H, which implies that $\frac{\partial S_{ij}}{\partial p_1^i} + \frac{\partial S_{ji}}{\partial p_2^i} = \frac{\partial \Phi(r,p_{12})}{\partial \delta}$; $2\left(\frac{\partial S_{ii}}{\partial p_1^i} + \frac{\partial S_{ii}}{\partial p_2^i}\right) = -\frac{\partial \Phi(r,p_{12})}{\partial \delta}$; $\frac{\partial S_{ii}}{\partial p_{12}^i} = -\frac{q(p_{12})}{t}\left(2\eta(r, p_{12}) + \beta(r, p_{12})\right)$, and $\frac{\partial S_{ij}}{\partial p_{12}^i} = \frac{\eta(r,p_{12})q(p_{12})}{t}$.

is

$$t(x_1^A + x_2^A) - t(x_1^A + x_2^B) = t(x_2^A - x_2^B) = -tz_2.$$

Letting $MV_{AB}(r)$ denote the aggregate match value of such buyers given r, we thus have

$$\frac{MV_{AB}\left(r+\epsilon\right) - MV_{AB}\left(r\right)}{\epsilon}$$
$$\approx \frac{1}{\epsilon} \int_{z_2 = \frac{V\left(\widehat{P}(r)\right) - V\left(\widehat{P}(r)+r\right)}{t}}^{\frac{V\left(\widehat{P}(r+\epsilon)\right) - V\left(\widehat{P}(r+\epsilon)+r+\epsilon\right)}{t}} \int_{z_1=-1}^{\frac{V\left(\widehat{P}(r)+r\right) - V\left(\widehat{P}(r)\right)}{t}} \left(-tz_2\right) h\left(z_1, z_2\right) dz_1 dz_2.$$

Using the Leibniz rule and taking $\epsilon \to 0$ yields

$$MV'_{AB}\left(r\right) = \frac{\Phi'\left(r\right)}{4}\left(V\left(\widehat{P}\left(r\right)\right) - V\left(\widehat{P}\left(r\right) + r\right)\right).$$

Given the symmetry of h, the total marginal change in match values thus amounts to

$$MV'\left(r\right) = 4MV'_{AB}\left(r\right) = \Phi'\left(r\right)\left(V\left(\widehat{P}\left(r\right)\right) - V\left(\widehat{P}\left(r\right) + r\right)\right).$$

The marginal match-quality effects $MV'(r) < 0$ if and only if $\Phi'(r) < 0$. This is intuitive because differential pricing leads some buyers to opt for product combinations they would not choose in the uniform pricing benchmark, where random match values are the only determinants of buyers' shopping decisions.