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Abstract

We revisit Cournot competition with asymmetric demand information by introducing a common input supplier. We characterize a unique equilibrium where information spills over through screening and signaling in vertical contracting. The equilibrium outcomes either coincide with those under complete information or involve quantity distortions. Compared to the independent-supplier case, the presence of the common supplier enhances both consumer and producer surplus under mild downstream competition. Under intense competition, producer surplus can decline, although consumer surplus may still increase. Our findings reveal informational efficiency gains of upstream mergers and the possibility of a welfare improvement even when direct efficiency gains are absent.

JEL classification: D82, D86, L13.
Keywords: Cournot competition; Asymmetric information; Common agency; Information transmission; Vertical contracting; Screening; Signaling.

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1 Introduction

Private information and demand uncertainty play critical roles in shaping market outcomes, especially in markets with imperfect competition. To understand the interactions between information and competition, existing literature on oligopoly models has largely focused on downstream competition for final consumers, treating input costs as exogenously given. Implicit in this approach is the assumption that necessary inputs are either supplied by independent suppliers or produced in-house. Although the vertical contracting literature examines the endogenous determination of input costs, it typically overlooks ex-ante asymmetric information regarding payoff-relevant variables, such as market demand.

However, this simplified depiction of competition does not align with many industries in which inputs are obtained from common suppliers. For example, the semiconductor market is highly concentrated, with buyers—typically large firms producing electronic devices like smartphones or computers—negotiating contractual terms with suppliers based on their demand estimates. A similar example can be found in the automotive industry, where car manufacturers negotiate with common suppliers for components like engines or tires, and their demand forecasts can influence the pricing and terms of these contracts, impacting the competitive dynamics between the firms. Retail giants like Target and Walmart also offer many overlapping products and engage in wholesale negotiations with shared suppliers

When such contracting processes are considered, several questions arise. First, can private demand information, decentralized among competing firms, spill over to rivals? For example, if a smartphone manufacturer places a large order with a shared chip supplier, the competitor might infer high demand by observing the supplier's response—such as offering less favorable terms or charging a higher price—and adjust its own strategy accordingly, perhaps by increasing production or altering pricing. Second, if such spillovers occur, what are their implications for consumers, and do downstream firms have an incentive to select the same supplier? Overall, it remains unclear how the presence of a common upstream supplier impacts market performance. These questions motivate us to revisit the standard model of Cournot competition with asymmetric demand information by introducing a common strategic input supplier.

The model developed and analyzed in this paper is a common agency game where two competing downstream firms act as informed principals, and the upstream supplier is the common agent. Specifically, each downstream firm receives a binary private demand signal and bilaterally contracts with the upstream supplier by making a take-it-or-leave-it offer. Although the upstream supplier lacks private information ex-ante, it may learn the downstream firms' signals by observing the offered contracts. In turn, each downstream firm may attempt to infer its rival's demand information through an offer designed to screen what the supplier has learned.

Our main findings are twofold. First, we characterize a unique equilibrium in which information endogenously spills over through signaling and screening, as described above. Depending on the parameters, this equilibrium either coincides with the Cournot-Nash equilibrium under complete information or features systemic quantity distortions. The former case arises, for instance, when downstream competition is not intense, causing each firm to lack strong incentives to manipulate its rival's demand forecasts to reduce its supply. However, as competition intensifies, the incentives to distort the rival's belief grow stronger, leading to quantity distortions driven by information transmission. Notably, these distortions in individual firms' quantities are not always downward, although aggregate distortions remain negative.

Second, we examine the welfare implications of a common upstream supplier, which depend crucially on parameters such as the intensity of downstream competition. When downstream competition is mild, the presence of a common upstream supplier enhances both consumer and producer surplus compared to the independent-supplier benchmark. This effect is particularly pronounced when downstream firms operate as local monopolists, as information spillover provides each decision-maker, monopolist, with better information, enabling more accurately tailored decisions based on actual demand states. In contrast, under intense competition, consumer welfare may still increase, but producer surplus can decline. This occurs because information spillover causes downstream firms to sell larger quantities under high-demand estimates, while the reverse effect under low-demand estimates is *smaller* due to the limited equilibrium quantities and prices in such scenarios. In particular, the increase in average output due to information leakage can outweigh the negative distortions in aggregate quantities, which ultimately leads to a decline in profits when the firms contract with a common supplier.

Our findings suggest that in modern markets, where input suppliers are frequently shared and overlapped, information asymmetry may have a smaller impact than traditional theories on oligopoly markets predict. Under certain conditions, vertical contracting and information spillovers effectively eliminate information asymmetry, resulting in competition akin to complete information scenarios. Moreover, our welfare analysis highlights the potential informational efficiency gains from upstream mergers, by showing that even when direct efficiency gain is absent or even negative, consumer surplus can improve when transitioning from independent suppliers to a common upstream supplier due to informational effect.

2 Related Literature

This paper contributes to the extensive literature on oligopoly markets with private information. A central theme in this literature is the value of information and the incentives for horizontal competitors to share it. Specifically, our model is closely related to studies of environments with common demand shocks, such as Clarke (1983), Vives (1984, 1990), Li (1985), Gal-Or (1985), Raith (1996), and Myatt and Wallace (2015). The novel feature of this paper is the introduction of vertical layers into horizontal markets, thereby endogenizing input costs in a standard Cournot model. We show how vertical contracting between each competing firm and their common input supplier can lead to information spillovers throughout the entire vertical chain and impact social welfare.

Our paper is also related to the common agency literature (Bernheim and Whinston, 1986), which has various applications, including antitrust, regulation, tacit collusion, multilateral lobbying, and vertical contracting. In particular, we consider privately informed principals, as in Galperti (2015), who studies a general mechanism design framework where each principal has private information.[1] He provides a delegation principle type result that helps characterize equilibrium allocations. In specific settings, such as lobbying and public goods provision, Lima, Moreira and Verdier (2017), Lima and Moreira (2014), and Martimort and Moreira (2010) also examine common agency games with privately informed principals. While our paper employs similar techniques, its main focus and motivation lie in endogenizing input costs in imperfectly competitive markets, which fundamentally distinguishes the current work from these other studies.[2]

The literature on vertical contracting typically examines settings where a supplier offers contracts to multiple competing downstream buyers. A central question in this literature is the "opportunism problem," which arises due to the unobservability of contracts to competing downstream buyers (McAfee and Schwartz, 1994) or as a consequence of asynchronous recontracting in dynamic environments (Do and Miklós-Thal, 2023, 2024). In contrast, our paper considers the opposite setting, where downstream buyers have bargaining power and make offers to their common upstream supplier. In this regard, our work is closely related to Marx and Shaffer (2007) and Miklos-Thal, Rey and Vergé (2011), who analyze models in which two competing retailers simultaneously make take-it-or-leave-it offers to a common upstream manufacturer. Marx and Shaffer (2007) show that a retailer can use upfront payments to exclude its rival in equilibrium, whereas Miklos-Thal, Rey and Vergé (2011) derive the opposite result, finding that exclusion arises only when the contract cannot be contingent on exclusivity. However, unlike our work, these papers abstract away from asymmetric information regarding payoff-relevant variables such as market demand.

Perhaps the closest work to the current paper is Do and Riquelme (2024), where we show that information exchange can occur through a common upstream supplier. However, the baseline model in Do and Riquelme (2024) assumes that each downstream firm is a local monopolist, thus abstracting from the competitive externalities between downstream firms that

[1]For the mechanism design literature on an informed principal, see also Myerson (1983), Maskin and Tirole (1990, 1992), and Mylovanov and Tröger (2012). Unlike these works, we consider multiple privately informed principals who strategically choose mechanisms to signal information to a common agent, whose private information is endogenous.

[2]In a different context, Lima et al. (2024) also compare exclusive agency and common agency. However, Lima et al. (2024) allow direct information disclosure, whereas in our setup, information can only be transmitted indirectly through the common input supplier.

play a central role in the present work. Lima (2021) also studies information transmission in vertical contracting with a common retailer but assumes that private information is verifiable, held by upstream firms, and that upstream firms possess all bargaining power. Moreover, he focuses on comparing fixed-fee contracts and resale price maintenance rather than examining the effects of information spillovers in vertical contracting.

Finally, our paper also contributes to the literature on upstream mergers and their effects on social welfare. Existing studies have highlighted various efficiency gains from upstream mergers such as changes in production technologies, incentives, or market structures. For instance, Antelo and Bru (2006) argue that some benefits of mergers may emerge in the long run through market entry, while Milliou and Pavlou (2013) show that efficiency gains from increased R&D investments or lower wholesale prices can enhance consumer welfare when downstream competition is not too intense. However, the existing literature typically overlooks the impact of a merger on firms' informational structures when evaluating its welfare effects. We complement these works by identifying informational efficiency gains from upstream mergers, which may lead to welfare improvements even when direct efficiency gains are absent or even negative.

The paper is organized as follows: Section 3 introduces our model and presents three benchmark analyses. In Section 4, we derive the main results. Section 5 conducts a welfare analysis and explores the implications for merger policy. The Appendix contains formal proofs of our results.

3 Model

We consider a setting with one upstream firm U and two downstream firms D_1 and D_2. The upstream firm supplies input to D_i, which is transformed into a final product and sold by D_i at zero distribution cost. The upstream firm's marginal production cost is constant at $c \geq 0$.

Market demand is subject to unobservable shock. However, each downstream firm D_i receives a private signal $s_i \in \{h, l\}$ about the underlying demand state, which is drawn from i.i.d. distribution: for each $i = 1, 2$,

$$\rho \equiv \Pr(s_i = h) \in (0, 1).$$

We will refer to s_i as D_i's type. For a pair of signals $s = (s_1, s_2)$, the demands for final products are assumed to arise from a representative consumer who chooses $q = (q_1, q_2)$ to maximize

$$w(q; s) - p_1 q_1 - p_2 q_2,$$

where p_i and q_i are respectively the price and the consumption of D_i's product;

$$w(q; s) = \theta_{s_1, s_2}(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2 + 2\beta q_1 q_2\right)$$

is his utility function where $\beta \in [0, 1]$ measures the degree of substitutability between products; $\theta_{s_1 s_2}$ is the "demand shifter" parameter satisfying

$$\theta_{hh} > \theta_{hl} = \theta_{lh} > \theta_{ll} > c.$$

Note that the consumer's maximization problem yields the following inverse demand for D_i's product:[3]

$$P\left(q_i, q_{-i}; s_i, s_{-i}\right) = \theta_{s_1 s_2} - q_i - \beta q_{-i}.$$

After each downstream firm D_i learns its type, they simultaneously offer a price schedule $f_i\left(q_i\right)$ to the upstream supplier U. Given the offered schedules $f = (f_1, f_2)$, the upstream firm then decides whether to accept each contract or not; if U accepts f_i, it also decides the amount of input to supply to D_i, while if it rejects, no transaction occurs between U and D_i. To be precise, let $g_i \in \{1, 0\}$ denote U's acceptance decision on D_i's offer, and $q_i \geq 0$ denote the amount of input supply to D_i (conditional on acceptance). The contracting outcome between U and D_i is then summarized by $z_i = (f_i, q_i, g_i)$, and the aggregate profit of U is

$$u\left(z_1, z_2\right) = \sum_{i=1,2} g_i \cdot \left(f_i\left(q_i\right) - c q_i\right),$$

and the profit of D_i is

$$\pi(z_i, z_{-i}; s_i, s_{-i}) = g_i \cdot \left(P(q_i, g_{-i} \cdot q_{-i}; s_i, s_{-i}) q_i - f_i\left(q_i\right)\right).$$

Finally, the bilateral surplus of a pair U and D_i is

$$v(z_i, z_{-i}; s_i, s_{-i}) = g_i \cdot \left(P\left(q_i, g_{-i} \cdot q_{-i}; s_i, s_{-i}\right) q_i - c q_i\right).$$

The timing of the game is summarized as follows:

1. Each D_i privately observes s_i.

2. Each D_i simultaneously offers $f_i\left(\cdot\right)$ to U.

3. Given $f = (f_1, f_2)$, U decides whether to accept or reject each f_i.

4. If U accepts f_i, it chooses q_i and receives $f_i(q_i)$ from D_i.

Before analyzing this game, we first examine several benchmark cases that are useful to evaluate the impacts of a common upstream supplier and information spillover through vertical contracting.

[3]This formulation dates back to Dixit (1979), Vives (1984), and Häckner (2000).

Benchmark 1 (Complete information) Suppose that demand signals are publicly observable. Given that the downstream firms have bargaining power and there are no upstream externalities, each downstream firm can induce any target quantity q and fully extract the resulting bilateral surplus by setting

$$f_i(q_i) = \begin{cases} cq & \text{if } q_i = q, \\ -\infty & \text{otherwise.} \end{cases}$$

As a result, the equilibrium quantity of D_i, denoted by $q_i^*(s_i, s_{-i})$, is given by

$$q_i^*(s_i, s_{-i}) = \arg\max_{q_i} \left(P\left(q_i, q_{-i}^*(s_{-i}, s_i); s_i, s_{-i}\right) q_i - cq_i \right),$$

for each $i = 1, 2$, which leads to

$$q_i^*(s_i, s_{-i}) = q_{s_1 s_2}^* \equiv \frac{\theta_{s_1 s_2} - c}{2 + \beta},$$

for each $i = 1, 2$ and $s_1, s_2 \in \{h, l\}$. This quantity profile corresponds to the unique Cournot-Nash equilibrium when the firms have complete information about the demand signals. From the best-response functions, it can be shown that the ex-ante expected profit of each downstream firm equals

$$\Pi^* = \rho E_{\tilde{s}}\left[(q_{h\tilde{s}}^*)^2\right] + (1 - \rho) E_{\tilde{s}}\left[(q_{l\tilde{s}}^*)^2\right]. \quad \diamond$$

Benchmark 2 (Independent suppliers) Next, suppose that each downstream firm privately observes a demand signal but can purchase input from its independent supplier whose marginal production cost is c. Since there is no additional information that a downstream firm gains from interacting with its supplier, the contracting outcome is as if each downstream firm produces input in-house at the marginal cost of c.[4] A Bayesian Cournot-Nash equilibrium $(\bar{q}_1(s_1), \bar{q}_2(s_2))$ then must satisfy, for each $i = 1, 2$ and $s_1, s_2 \in \{h, l\}$,

$$\bar{q}_i(s_i) \in \arg\max_{q_i} E_{\tilde{s}}\left[P\left(q_i, \bar{q}_{-i}(\tilde{s}); s_i, \tilde{s}\right) q_i - cq_i\right],$$

where $\tilde{s}$ denotes the random variable of the rival downstream firm's demand signal. In our model specification, the unique equilibrium is then given by

$$\bar{q}_i(l) = \bar{q}_l \equiv \frac{E_{\tilde{s}}[\theta_{l\tilde{s}}] - c}{2 + \beta} - \frac{\beta\rho\left(E_{\tilde{s}}[\theta_{h\tilde{s}}] - E_{\tilde{s}}[\theta_{l\tilde{s}}]\right)}{2(2 + \beta)},$$

$$\bar{q}_i(h) = \bar{q}_h \equiv \frac{E_{\tilde{s}}[\theta_{h\tilde{s}}] - c}{2 + \beta} + \frac{\beta(1 - \rho)\left(E_{\tilde{s}}[\theta_{h\tilde{s}}] - E_{\tilde{s}}[\theta_{l\tilde{s}}]\right)}{2(2 + \beta)},$$

for each $i = 1, 2$. As we may expect, $\bar{q}_h > \bar{q}_l$, and the ex-ante expected profit of D_i is

$$\bar{\Pi} = \rho(\bar{q}_h)^2 + (1 - \rho)(\bar{q}_l)^2.$$

[4]Alternatively, one can assume that the input is supplied by a competitive input market at unit price c.

Notice that, for given $s \in \{h, l\}$, the complete information quantity $q^*_{s\tilde{s}}$ as a random variable is a mean-preserving-spread of the first term in $\bar{q}_s$: that is, $\frac{E_{\tilde{s}}[\theta_{s\tilde{s}}]-c}{2+\beta} = \rho q^*_{sh} + (1-\rho)\, q^*_{sl}$. When β is sufficiently small, the second term of $\bar{q}_s$ is close to zero, and the convexity of the profit functions thus implies that the downstream firms obtain strictly higher profits when demand signals are public. If β is large enough, on the other hand, the downstream firms may obtain strictly lower profits when demand signals are public, i.e., $\bar{\Pi} > \Pi^*$. As shown later, the impacts of a common upstream supplier and information spillover will also depend crucially on the degree of competitive externalities, in a similar manner. $\diamond$

Benchmark 3 (Independent suppliers with communication) As a final benchmark, suppose that the downstream firms still purchase input from independent suppliers but can exchange public cheap talk messages. Even with such a direct communication device, however, it can be shown that informative cheap talk communication does not occur in equilibrium for any $\beta > 0$. To see this, let us assume that each downstream firm sends message $\hat{h}$ (resp. $\hat{l}$) after receiving signal h (resp. l). On the path of plays, information asymmetry disappears, and the downstream firms' quantities are given by those in Benchmark 1. For any $\beta > 0$, however, the profit of D_i will increase as the rival downstream firm sells less, and each downstream firm with signal h will find it profitable to send $\hat{l}$ and sells q_i such that

$$q_i = \arg\max_{q_i} \left(P\left(q_i, q^*_{\hat{s}l}; h, \hat{s}\right) q_i - c q_i \right),$$

where $\hat{s} \in \{\hat{h}, \hat{l}\}$ is the message sent by the rival downstream firm.[5] This type of double-deviation prevents information exchange through cheap talk messages, and the equilibrium quantities remain the same as those in Benchmark 2. $\diamond$

We conclude the description of our model with the following assumption, which ensures interior solutions.

Assumption 1. $\theta_{hh} - \theta_{hl} \geq \theta_{hl} - \theta_{ll} \geq \theta_{ll} - c$ and $\frac{\theta_{ll}-c}{\theta_{hl}-c} \geq \frac{2-\beta}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right)$.

Intuitively, the assumption requires that the expected demand size, conditional on receiving the high signals, is sufficiently large to ensure that selling positive quantities is always profitable. Notably, it holds for a broad range of parameter values. For instance, if $\theta_{hl} - \theta_{ll} = 2 > \theta_{ll} - c = 1$, it is satisfied for all $0 \leq \beta \leq 1$ so long as $\theta_{hh} - \theta_{hl} \geq 6$. For the case of homogeneous products, $\beta = 1$, the assumption holds even when the expected demand size increases uniformly as the signal becomes more favorable, that is, $\theta_{hh} - \theta_{hl} = \theta_{hl} - \theta_{ll} = \theta_{ll} - c$.

[5] Goltsman and Pavlov (2014) derive a similar impossibility result in horizontal Cournot markets where each firm's production cost is private information and independently drawn from a continuous distribution.

4 Vertical Contracting and Information Spillover

Now, we analyze the model introduced in Section 3. This is a common agency game where two competing downstream firms are the principals, who contract with a common upstream input supplier. Each downstream firm is privately informed about a demand signal, which is unobservable to either its rival downstream firm or the upstream firm. While the upstream firm has no a priori private information, it can learn the rival downstream firm's demand signal when the rival downstream firm's contract offer is separating.

4.1 Equilibrium Notion

Throughout the analysis, we will focus on *fully revealing equilibrium* in which both signaling and screening occur: that is, different types of a downstream firm make different offers, and different types of the upstream firm choose to supply different amounts of quantity. From the perspective of each downstream firm, the vertical contracting game then becomes a principal-agent problem where both the principal and the agent have private information. Specifically, the agent's private information refers to the signal learned from contracting with the rival downstream firm. Following Martimort and Moreira (2010) and Lima and Moreira (2014), we first consider the best response of each downstream firm in this informed principal problem and then find their fixed point, which is the equilibrium notion we consider.

To be more precise, let us consider the problem of an informed downstream firm that holds the following *conjecture* about the rival downstream firm's menu offer:

$$\widehat{m} = \left(\widehat{m}^h, \widehat{m}^l\right) = \left(\left(\left(\widehat{q}_h^h, \widehat{f}_h^h\right), \left(\widehat{q}_l^h, \widehat{f}_l^h\right), \phi\right), \left(\left(\widehat{q}_h^l, \widehat{f}_h^l\right), \left(\widehat{q}_l^l, \widehat{f}_l^l\right), \phi\right)\right),$$

where $\left(\widehat{q}_{s'}^s, \widehat{f}_{s'}^s\right)$ is a pair of a quantity and a fixed fee offered by the rival downstream firm of type-s targeted to the upstream firm of s'-type, and ϕ is a null contract of contract rejection.[6] To characterize fully revealing equilibria, we restrict the set of conjectures to those satisfying the following two conditions:

Condition (SI). $q_{hh}^* \geq \widehat{q}_s^h > \widehat{q}_s^l \geq 0$ for each $s \in \{h, l\}$.

Condition (SC). For each $s \in \{h, l\}$, there exists $\widehat{u}^s \geq 0$ such that

$$\widehat{u}^s \equiv \widehat{f}_h^s - c\widehat{q}_h^s = \widehat{f}_l^s - c\widehat{q}_l^s.$$

Condition (SI) imposes the boundaries on quantities under conjectured offers and ensures signaling. That is, $\widehat{m}^h$ and $\widehat{m}^l$ are distinct in that

$$\max\{\widehat{q}_h^h, \widehat{q}_l^h\} \geq \widehat{q}_h^h > \widehat{q}_h^l \geq \min\{\widehat{q}_h^l, \widehat{q}_l^l\}.$$

[6] That is, $\widehat{f}^s\left(\widehat{q}_{s'}^s\right) = \widehat{f}_{s'}^s$ for each $s, s' \in \{h, l\}$ and $\widehat{f}^s(q) = -\infty$ for all other q's.

Because of separating offers, the upstream supplier learns the rival downstream firm's private information under condition (SI). The condition is intuitive since it is profitable to sell more quantities in the market when the demand signals are higher.

On the other hand, condition (SC) ensures screening. That is, the upstream firm is willing to accept an offer and choose a quantity targeted to its own type. In our framework in which the upstream firm's costs are separable across the downstream firms, this simple condition indeed guarantees such behaviors. If it is satisfied, we can alternatively write a pair of offers as, abusing notation,

$$\widehat{m} = \left(\widehat{m}^h, \widehat{m}^l\right) = \left(\left(\widehat{q}_h^h, \widehat{q}_l^h, \widehat{u}^h\right), \left(\widehat{q}_h^l, \widehat{q}_l^l, \widehat{u}^l\right)\right),$$

with the implicit understanding that $\widehat{f}_h^s = \widehat{u}^s + c\widehat{q}_h^s$ and $\widehat{f}_l^s = \widehat{u}^s + c\widehat{q}_l^s$. Consequently, we will focus on the set of offers $\widehat{m} = \left(\widehat{m}^h, \widehat{m}^l\right)$, denoted by $\mathcal{M}$, satisfying conditions (SI) and (SC) (i.e., $\widehat{u}^s \geq 0$ for $s \in \{h, l\}$).

Given conjectured menus $\widehat{m} \in \mathcal{M}$, the problem of the type-s downstream firm is then defined as follows:

$$\max_{\substack{0 \leq q_h, q_l \leq q_{hh}^* \\ f_h, f_l \in \mathbb{R}}} E_{\tilde{s}}\Big[P\left(q_{\tilde{s}}, \widehat{q}_s^{\tilde{s}}; s, \tilde{s}\right) q_{\tilde{s}} - f_{\tilde{s}}\Big]$$
$$s.t. \quad (IC_{-s}), (ICU), \text{ and } (IRU),$$

where

$$E_{\tilde{s}}\Big[P\left(\widehat{q}_{\tilde{s}}^{-s}, \widehat{q}_{-s}^{\tilde{s}}; -s, \tilde{s}\right) \widehat{q}_{\tilde{s}}^{-s} - \widehat{f}_{\tilde{s}}^{-s}\Big] \geq E_{\tilde{s}}\Big[P\left(q_{\tilde{s}}, \widehat{q}_s^{\tilde{s}}; -s, \tilde{s}\right) q_{\tilde{s}} - f_{\tilde{s}}\Big], \qquad (IC_{-s})$$

$$f_h - cq_h \geq f_l - cq_l \text{ and } f_l - cq_l \geq f_h - cq_h, \qquad (ICU)$$

and

$$f_h - cq_h \geq 0 \text{ and } f_l - cq_l \geq 0. \qquad (IRU)$$

Note that (IC_{-s}) is the incentive constraint for the downstream firm of type $-s \neq s$ not to mimic type s and ensures that private information is transmitted from a downstream firm to the upstream firm. When this constraint is binding, it generates distortion for separating types. On the other hand, (ICU) and (IRU) guarantee that the upstream firm is willing to accept a menu and choose a quantity targeted to its own type. In particular, since (ICU) reduces to $f_h - cq_h = f_l - cq_l = u$ for some u, we can alternatively write the type-s downstream firm's problem as

$$\max_{\substack{0 \leq q_h, q_l \leq q_{hh}^* \\ u \in \mathbb{R}}} E_{\tilde{s}}\Big[\left(P\left(q_{\tilde{s}}, \widehat{q}_s^{\tilde{s}}; s, \tilde{s}\right) - c\right) q_{\tilde{s}}\Big] - u \qquad \text{(P-}s\text{)}$$
$$s.t. \quad (IC_{-s}), \text{ and } (IRU'),$$

where

$$u \geq 0. \tag{IRU'}$$

Let $BR^s(\widehat{m})$ denote the set of solutions to the s-type problem. A fully revealing equilibrium we will focus on is then defined as follows.[7]

Definition 1. $m = (m^h, m^l) \in \mathcal{M}$ is a fully revealing equilibrium (henceforth, *equilibrium*) if $m^s \in BR^s(m)$ for each $s \in \{h, l\}$.

As we argue later, this notion is stronger than a perfect Bayesian equilibrium. That is, there exists a belief system under which an equilibrium, as defined in Definition 1, can be supported as a perfect Bayesian equilibrium, while the reverse does not hold.

4.2 Unique Equilibrium

In what follows, we will use λ_{-s} and λ_u to denote the nonnegative Lagrangian multipliers for (IC_{-s}) and (IRU'), respectively.[8] Also, we denote by

$$MR(q, q'; s, s') \equiv \frac{\partial P(q, q'; s, s')}{\partial q} q + P(q, q'; s, s') = \theta_{ss'} - \beta q' - 2q$$

the marginal revenue of a downstream firm at quantity q when the rival downstream firm sells q', and the demand signals are s and s'. Given these notations, the first-order necessary conditions with respect to q_h and q_l for problem (P-s) are given by

$$MR(q_h, \widehat{q}_s^h; s, h) - c - \lambda_{-s}\left(MR(q_h, \widehat{q}_s^h; -s, h) - c\right) = 0$$

and

$$MR(q_l, \widehat{q}_s^l; s, l) - c - \lambda_{-s}\left(MR(q_l, \widehat{q}_s^l; -s, l) - c\right) = 0.$$

If $\lambda_{-s} = 0$, each downstream firm equates the upstream marginal cost to its marginal revenue at given demand signals, but otherwise, the quantities sold in the market will be distorted due to information transmission. On the other hand, the downstream firms' screening efforts generate rents to the upstream firm only if $\lambda_u = 0$. However, given the constant returns to scale production technology—which implies separable upstream production costs across downstream firms—it turns out that downstream firms do not need to leave any rent to the upstream firm for screening.

Lemma 1. For each $s \in \{h, l\}$, we have $\lambda_u > 0$ at a solution to (P-s). Thus, $u^h = u^l = 0$ in equilibrium.

[7] A similar equilibrium notion is adopted in Lima and Moreira (2014) for political lobbying game.

[8] The details of the Kuhn-Tucker conditions can be found in the Appendix.

Next, consider the incentive compatibility condition (IC_{-s}), which requires that the offer made by the downstream firm of type s is not profitable for the other type $-s$ to mimic. Intuitively, each downstream firm makes more profits when the rival downstream firm sells less, which is the case when the rival downstream firm believes that the market demand is likely small. As a consequence, the downstream firm of l-type would have no incentive to mimic the offer made by the h-type downstream firm. The following result formalizes this intuition.

Lemma 2. In equilibrium, $\lambda_l = 0$ and $0 \leq \lambda_h < 1$. Therefore, equilibrium quantities must satisfy

$$q_h^h = q_{hh}^* = \frac{\theta_{hh} - c}{2+\beta},\ q_l^h = \frac{\theta_{hl} - c - \beta q_h^l}{2}.$$

Given the above observations, we now consider the reduced problems in which the h-type downstream firm solves

$$\max_{0 \leq q_l \leq q_{hh}^*} \left(P\left(q_l, \widehat{q}_h^l; h, l\right) - c\right) q_l \tag{RP-h}$$

and the l-type downstream firm solves

$$\begin{aligned} &\max_{0 \leq q_h, q_l \leq q_{hh}^*} E_{\tilde{s}}\Big[\left(P\left(q_{\tilde{s}}, \widehat{q}_l^{\tilde{s}}; l, \tilde{s}\right) - c\right) q_{\tilde{s}}\Big] \\ &\quad s.t. \quad (\widetilde{IC}_h), \end{aligned} \tag{RP-l}$$

where $(\widetilde{IC}_h)$ is the constraint obtained from (IC_h) by substituting q_{hh}^* into $\widehat{q}_h^h$:

$$E_{\tilde{s}}\Big[\left(P\left(\widehat{q}_{\tilde{s}}^h, \widehat{q}_h^{\tilde{s}}; h, \tilde{s}\right) - c\right) \widehat{q}_{\tilde{s}}^h \mid \widehat{q}_h^h = q_{hh}^*\Big] \geq E_{\tilde{s}}\Big[\left(P\left(q_{\tilde{s}}, \widehat{q}_l^{\tilde{s}}; h, \tilde{s}\right) - c\right) q_{\tilde{s}} \mid \widehat{q}_h^h = q_{hh}^*\Big]. \tag{$\widetilde{IC}_h$}$$

Let us denote the set of solutions to this reduced problem by $\widetilde{BR}^s\left(\widehat{q}_l^h, \widehat{q}_h^l, \widehat{q}_l^l\right)$ for each $s \in \{h, l\}$ and $\left(\widehat{q}_l^h, \widehat{q}_h^l, \widehat{q}_l^l\right) \in [0, q_{hh}^*]^3$. The following result then directly follows from the previous lemmas.

Proposition 1. Suppose $m = (m^h, m^l) = \left((q_h^h, q_l^h, u^h), (q_h^l, q_l^l, u^l)\right)$ is an equilibrium. Then,

(i) $u^h = u^l = 0$.

(ii) $q_h^h = \frac{\theta_{hh} - c}{2+\beta}$.

(iii) $\left(q_l^h, q_h^l, q_l^l\right)$ is a fixed point of $\widetilde{BR} \equiv \widetilde{BR}^h \times \widetilde{BR}^l$.

Hence, our analysis reduces to obtaining a fixed point of the reduced problem. From the first-order conditions with respect to quantities, it is straightforward to check that the equilibrium quantities should be of the following form:

$$\begin{pmatrix} q_h^h \\ q_l^h \\ q_h^l \\ q_l^l \end{pmatrix} = \begin{pmatrix} q_{hh}^* \\ q_{hl}^* \\ q_{hl}^* \\ q_{ll}^* \end{pmatrix} + \begin{pmatrix} 0 \\ \kappa_{hl} \\ -\kappa_{lh} \\ -\kappa_{ll} \end{pmatrix}, \tag{1}$$

where

$$\begin{aligned}
\kappa_{hl} =& \frac{\beta\tilde{\lambda}_h\left(\theta_{hh}-\theta_{hl}\right)}{\left(2-\beta\right)\left(2+\beta\right)\left(1-\tilde{\lambda}_h\right)} \geq 0, \\
\kappa_{lh} =& \frac{2\tilde{\lambda}_h\left(\theta_{hh}-\theta_{hl}\right)}{\left(2-\beta\right)\left(2+\beta\right)\left(1-\tilde{\lambda}_h\right)} \geq 0, \\
\kappa_{ll} =& \frac{\tilde{\lambda}_h\left(\theta_{hl}-\theta_{ll}\right)}{\left(2+\beta\right)\left(1-\tilde{\lambda}_h\right)} \geq 0,
\end{aligned}$$

and $\tilde{\lambda}_h \geq 0$ is the Lagrangian multiplier for $(\widetilde{IC}_h)$. As shown in the expression, the additional terms $\kappa_{ss'}$ measure the degree of quantity distortions, driven by signaling effects, from the Cournot-Nash equilibrium quantities under complete information.

Clearly, these quantities coincide with those under complete information when $\tilde{\lambda}_h = 0$, which turns out to be the case if the following inequality is satisfied:

$$E_{\tilde{s}}\left[\left(\theta_{h\tilde{s}}-\theta_{l\tilde{s}}\right)\left(\theta_{h\tilde{s}}-c-\left(1+\beta\right)\left(\theta_{l\tilde{s}}-c\right)\right)\right] \geq 0. \tag{2}$$

This condition is a simple rewriting of the constraint $(\widetilde{IC}_h)$ when each quantity is replaced by $q_{ss'}^*$. That is, under the condition (2), the h-type downstream firm has an incentive to separate itself from l-type even in the absence of quantity distortion. On the other hand, if it is violated, the distortions resulting from information spillover arise, leading to $\tilde{\lambda}_h > 0$ and $\kappa_{ss'} > 0$ for all $s, s' \in \{h, l\}$. Our main result is a complete characterization of the unique equilibrium in both cases.

Theorem 1. There exists a unique equilibrium. In the equilibrium, the quantities are given by the expression (1) for some $\tilde{\lambda}_h = \lambda^* \geq 0$ and ordered as $q_{hh}^* = q_h^h > q_l^h \geq q_h^l > q_l^l \geq 0$. In particular, quantity distortions arise if and only if condition (2) does not hold.

The sketch of the proof is as follows. We first write down $(\widetilde{IC}_h)$ as a function of $\tilde{\lambda}_h$, using the expression (1). Then, we show that if the condition (2) is violated, there exists a unique

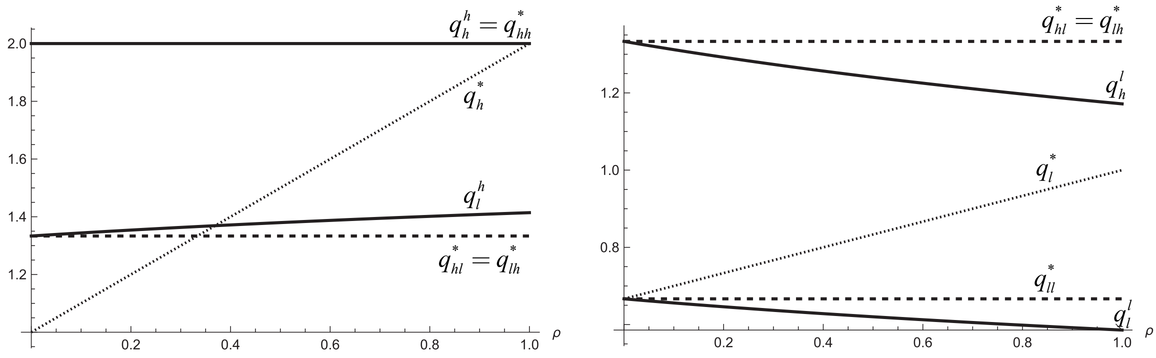

Figure 1: The quantities as ρ varies when $2 = \theta_{hh} - \theta_{hl} = \theta_{hl} - \theta_{ll} = \theta_{ll} - c$, $\beta = 1$, and $c = 1$.

$\lambda^* > 0$ such that $(\widetilde{IC}_h)$ is binding at the resulting offer with $\tilde{\lambda}_h = \lambda^*$, and both conditions (SI) and (SC) are satisfied. Finally, it is shown that the second-order sufficient conditions also hold at $\tilde{\lambda}_h = \lambda^*$. The formal proof is relegated to the Appendix.

There are several remarks to be made. First, the distortions of *individual* quantities, if they exist, are not always downward, although they are generated by the binding signaling constraint of the h-type. More specifically, a downstream firm sells *more* than q_{hl}^* when it receives h while its rival receives signal l; the firm that was initially more optimistic about the market demand than the other ends up supplying more, despite information spillover. This observation is driven by the fact that quantities are strategic substitutes. Without such an upward distortion, the l-type downstream firm will lack incentives to offer quantities distorted downward, $q_h^l < q_{hl}^*$, which is necessary to prevent the h-type from mimicking. In turn, the h-type downstream firm has the incentive to offer $q_h^h = q_{hh}^*$ and $q_l^h > q_{hl}^*$ because the potential distortions of the rival's quantities are only downward. Figure 1 illustrates how the equilibrium quantities respond to the change of ρ.

Second, the direction of *aggregate* distortions nonetheless remains negative whenever they exist. Letting $d_{ss'} \equiv q_{s'}^s + q_s^{s'} - 2q_{ss'}^*$ denote the distortion of aggregate quantities, we have $d_{hh} = 0$, $d_{ll} = -2\kappa_{ll} < 0$, and $d_{hl} = \kappa_{hl} - \kappa_{lh} = \frac{-\lambda^*(\theta_{hh}-\theta_{hl})}{(2+\beta)(1-\lambda^*)} < 0$. To illustrate this observation, suppose that, starting from the complete information Cournot-Nash outcomes, each quantity in the l-type offer is reduced by one unit to ensure the signaling constraint. From the perspective of the h-type downstream firm, this change reduces the rival firm's quantity by $1 - \rho$ unit on average because it is the probability that the rival is l-type. In addition, the best response against a unit decrease in the rival's quantity is to increase output by less than one unit. Therefore, the optimal increment of the h-type's quantity does not exceed the reduction of the l-type overall, and the aggregate quantity distortion remains negative.

Third, our result does not imply the uniqueness of a perfect Bayesain equilibrium in that the equilibrium notion we introduced is not equivalent to the definition of perfect Bayesian equilibrium. Nonetheless, it is supported as a perfect Bayesian equilibrium by properly constructing

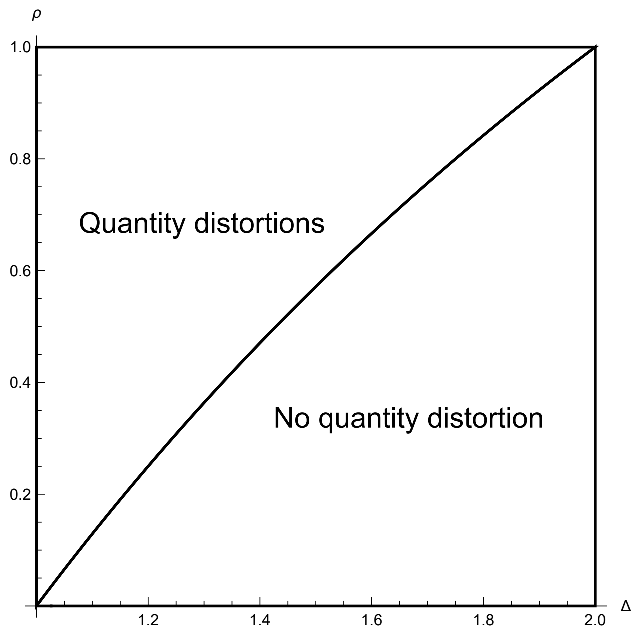

Figure 2: The region where quantity distortions occur when $\theta_{hh} - \theta_{hl} = \frac{3\Delta}{2} > \theta_{hl} - \theta_{ll} = \Delta \geq \theta_{ll} - c = 1$ and $\beta = 1$.

belief systems for D_i's off-path deviations. To see this, observe first that our construction ensures no incentive for on-path deviations of the downstream firms by (IC_h) and (IC_l). For off-path deviations, suppose that if D_i makes an off-path deviation, U believes $s_i = h$ and chooses D_{-i}'s quantity q_h^h or q_h^l depending on the menu offered by the rival, which is larger than q_l^h or q_l^l, respectively. Certainly, such deviations remain unprofitable to D_i. In this sense, our equilibrium concept is stronger than a perfect Bayesian equilibrium.

Finally, even though the information rent given to the upstream firm is zero—meaning there is no (direct) distortion for screening—the downstream firms' screening behaviors still play a critical role in shaping market outcomes in equilibrium through two channels. The first channel is direct: a downstream firm that successfully screens the upstream firm's type can condition its quantity on this information. The second channel is more indirect: a downstream firm that believes its *opponent* may successfully screen the upstream firm's type becomes concerned about information leakage to its rival. As a result, it may be tempted to manipulate the rival's belief by making a non-truthful offer. This highlights the close interdependence between signaling and screening behaviors in our model.

In Figure 2, we illustrate the cases where there are quantity distortions or not, when the gaps between $\theta_{ss'}$'s are parameterized by $\Delta > 0$. It shows that a smaller Δ or higher ρ induces these distortions, which can be explained as follows. In equilibrium, each downstream firm learns true demand signals, so they want to sell larger quantities when the signals indicate higher demand states. At the same time, there are incentives for the h-type downstream firm to mimic the l-type downstream firm to reduce the rival's quantity. To prevent it, the l-type downstream firm supplies sufficiently small outputs q_h^l and q_l^l. When Δ is high enough, the former effect dominates the latter, and the constraint $(\widetilde{IC}_h)$ becomes slack. On the other hand, if ρ is higher, each downstream firm believes that its rival is more likely h-type, supplying large quantities in the market. This creates a stronger incentive to induce lower quantities from the rival by mimicking the l-type.

5 Impact of Common Supplier

When the downstream firms rely on independent suppliers or produce their inputs in-house, the equilibrium quantities are determined by the unique Bayesian Cournot-Nash outcomes in Benchmark 2. In contrast, when the downstream firms share a common input supplier, as shown in Theorem 1, an equilibrium emerges in which information spillovers occur. In this equilibrium, the downstream firms perfectly learn the true demand signals through their shared supplier, but strategic incentives to mislead rivals can lead to quantity distortions.

Building on these observations, we now compare the two regimes—independent input sourcing versus a common supplier—to examine how the presence of a common supplier influences the firms' profits and consumer welfare. In doing so, recall first that the ex-post producer surplus is given by

$$PS\left(q;s\right)=\sum_{i=1,2}\left(P\left(q_i,q_{-i};s_i,s_{-i}\right)q_i-cq_i\right),$$

whereas the ex-post consumer surplus is

$$CS\left(q;s\right)=w\left(q;s\right)-\sum_{i=1,2}P\left(q_i,q_{-i};s_i,s_{-i}\right)q_i=\frac{1}{2}\left(q_1^2+q_2^2+2\beta q_1q_2\right),$$

where $q=(q_1,q_2)$ and $s=(s_1,s_2)$ are respectively quantity and signal profiles.

We will use notations CS, CS^*, and $\widehat{CS}$ to denote the ex-ante consumer surplus with independent input suppliers (Benchmark 2), the complete information (Benchmark 1), and the common input supplier (Theorem 1), respectively. Similarly, the ex-ante producer surplus under these regimes will be denoted by PS, PS^*, and $\widehat{PS}$. Since the upstream firm earns zero profit in all cases, these values also correspond to the whole vertical surplus of downstream-upstream pairs.

Note that the impact of a common supplier on welfare can be decomposed as

$$\widehat{CS}-CS=\underbrace{\widehat{CS}-CS^*}_{\text{Impact of quantity distortions}}+\underbrace{CS^*-CS,}_{\text{Impact of information spillover}}$$

and

$$\widehat{PS}-PS=\underbrace{\widehat{PS}-PS^*}_{\text{Impact of quantity distortions}}+\underbrace{PS^*-PS.}_{\text{Impact of information spillover}}$$

We first examine the case where competitive externalities are sufficiently small so that (2) holds. In this case,

$$\widehat{CS} - CS = CS^* - CS,$$

and

$$\widehat{PS} - PS = PS^* - PS.$$

In this scenario, each downstream firm behaves almost like a monopolist, and the information spillover enables them to determine equilibrium quantities as under complete information without causing any quantity distortions. Therefore, the downstream firms achieve higher profits with the common upstream supplier compared to the case with independent suppliers. Final consumers also benefit from the presence of the common upstream supplier because, given the linearity of demands, consumer surplus is convex in quantities, meaning that they have a preference for greater information and quantities that align more closely with the actual demand state. To summarize,

Proposition 2. If β is sufficiently small, we have $CS < \widehat{CS} = CS^*$ and $PS < \widehat{PS} = PS^*$.

If competitive externalities are large, quantity distortions may emerge due to the incentives to manipulate rivals' beliefs downward. In this scenario, the above conclusion that all market participants benefit from a common upstream supplier may no longer hold. To illustrate this, let us now consider the opposite extreme, where the final products are homogeneous ($\beta = 1$) and quantity distortions occur ($\lambda^* > 0$). We first compare the equilibrium outcomes with the common upstream supplier and the complete information to examine the welfare implications of such quantity distortions.

Note that the ex-post consumer surplus is given by $CS(q; s) = \frac{1}{2}(q_1 + q_2)^2$. Therefore, we have

$$\begin{aligned}
\widehat{CS} = E_{s\tilde{s}}\left[\frac{1}{2}\left(q^s_{\tilde{s}} + q^{\tilde{s}}_s\right)^2\right] &= E_{s\tilde{s}}\left[\frac{1}{2}\left(2q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)^2\right] \\
&< E_{s\tilde{s}}\left[\frac{1}{2}\left(2q^*_{s\tilde{s}}\right)^2\right] \\
&= CS^*,
\end{aligned}$$

which shows that quantity distortions have negative welfare impacts on final consumers.

Similarly, the ex-ante producer surplus can be rewritten as

$$\begin{aligned}
\widehat{PS} &= E_{s\tilde{s}}\left[\left(\theta_{s\tilde{s}} - \left(2q^*_{s\tilde{s}} + d_{s\tilde{s}}\right) - c\right)\left(2q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)\right] \\
&= E_{s\tilde{s}}\left[\left(q^*_{s\tilde{s}} - d_{s\tilde{s}}\right)\left(2q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)\right] \\
&= E_{s\tilde{s}}\left[\left(2q^*_{s\tilde{s}}\right)^2\right] - E_{s\tilde{s}}\left[d_{s\tilde{s}}\left(q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)\right] \\
&= PS^* - E_{s\tilde{s}}\left[d_{s\tilde{s}}\left(q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)\right],
\end{aligned}$$

where the second equation follows from $q^*_{s\tilde{s}} = \frac{\theta_{s\tilde{s}}-c}{3}$. Hence, we can conclude that $\widehat{PS} > PS^*$ whenever the following condition holds:

$$E_{s\tilde{s}}\left[d_{s\tilde{s}}\left(q^*_{s\tilde{s}} + d_{s\tilde{s}}\right)\right] < 0. \tag{3}$$

The proposition below summarizes these observations, which remain true as long as β is sufficiently close to 1 by continuity.

Proposition 3. Suppose β is sufficiently close to 1 and $\lambda^* > 0$. Then,

(i) $\widehat{CS} < CS^*$.

(ii) $\widehat{PS} > PS^*$ if the condition (3) holds.

Recall that the aggregate distortion $d_{ss'}$ is non-positive. Since $q^*_{hl}+d_{hl} > q^*_{hl}-\kappa_{lh} = q^l_h > 0$, we have $d_{hl}\left(q^*_{hl} + d_{hl}\right) < 0$, and the condition (3) is ensured when either it is less likely that both signals are low or the expected demand size θ_{ll} conditional on such event is large enough so that $q^*_{ll} + d_{ll} > 0$. More precisely,

Corollary 1. Suppose β is sufficiently close to 1 and $\lambda^* > 0$. Then, $\widehat{PS} > PS^*$ if either $\rho \geq \frac{1}{2}$ or $\frac{1}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right) \leq \frac{\theta_{ll}-c}{2\theta_{hl}-\theta_{ll}-c}$.

These results suggest that quantity distortions invariably harm final consumers while potentially benefiting the firms. This is intuitive as the aggregate quantity distortion is negative, though individual quantities may experience upward distortions. The suppressed aggregate supply leads to higher margins and increased profits for the firms, but reduces overall consumer surplus.

Finally, we examine the overall impact of having a common upstream supplier, which depends on the interplay between information spillover and quantity distortion effects. In general, this impact is ambiguous because the effects of quantity distortions and information spillover do not align in the same direction. To make progress, we thus consider a specific case where the expected demand size increases uniformly as the signal becomes more favorable, and the products are homogeneous. Under these conditions, we can fully characterize the equilibrium quantities and conduct welfare comparisons as follows.

Proposition 4. Suppose $\theta_{hh} - \theta_{hl} = \theta_{hl} - \theta_{ll} = \theta_{ll} - c = \Delta > 0$ and $\beta = 1$. Then,

(i) There exist $\bar{\rho}_h \in (0,1)$ such that if $\rho > \bar{\rho}_h$, then $q^h_h > \bar{q}_h > q^h_l$; if $\rho \leq \bar{\rho}_h$, then $q^h_h \geq q^h_l \geq \bar{q}_h$.

(ii) There exist $\bar{\rho}_l \in (0,1)$ such that if $\rho > \bar{\rho}_l$, then $\bar{q}_l > q^l_h > q^l_l$; if $\rho \leq \bar{\rho}_l$, then $q^l_h > \bar{q}_l > q^l_l$.

(iii) The Lagrangian multiplier is given by $\lambda^* = \frac{3+2\rho-3\sqrt{1+\rho}}{3+4\rho} > 0$ and strictly increasing in ρ.

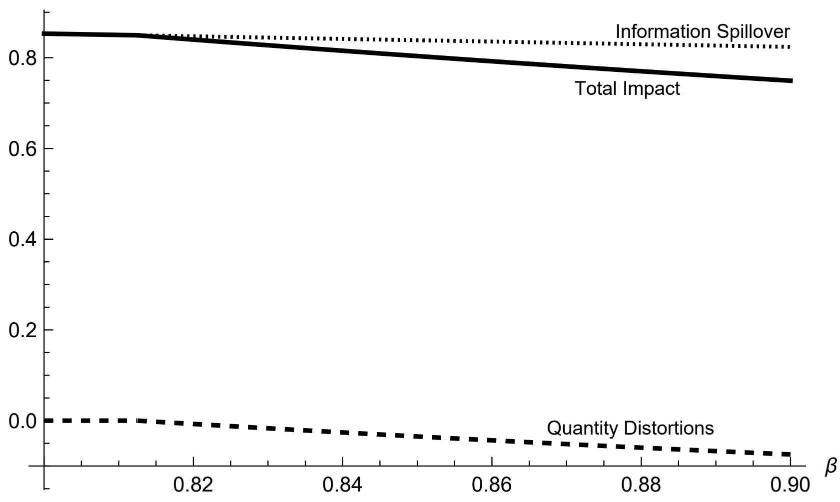

Figure 3: Impact of common supplier on consumer surplus (multiplied by 1000) when $\theta_{hh} - \theta_{hl} = \frac{3}{2} > 1 = \theta_{hl} - \theta_{ll} = \theta_{ll} - c > 0$ and $\rho = \frac{1}{2}$.

(iv) For all $\rho \in (0, 1)$, $CS < \widehat{CS}$ and $\widehat{PS} < PS$.

The first two parts, (i) and (ii), suggest that quantities are more dispersed when there is a common input supplier compared to when input suppliers are independent. Specifically, as the high-demand state occurs with sufficiently high probability, the downward quantity distortions become more severe. As a result, the quantities set by the h-type downstream firm often fall below the Bayesian Cournot-Nash quantity levels $\bar{q}_h$ that would arise with independent input suppliers. Consistent with this observation, item (iii) further shows that the degree of quantity distortion increases with ρ. Intuitively, a higher ρ leads each downstream firm to perceive its rival as more likely to be of the h-type, selling a larger quantity in the market. Through strategic substitutability, this creates an incentive for each downstream firm to offer a contract that induces smaller quantities from its own side.

Finally, the last part (iv) shows that consumer surplus continues to be higher when the input is supplied by the common upstream supplier, while the opposite holds for the downstream firms' profits. This is because the common input supplier allows the downstream firms to learn each other's private demand signal, so they sell larger quantities when the actual demand size is large. Although the reversed situation occurs when the market demand is small, the impact of such events on consumer welfare and the downstream profits is relatively small because the changes in the equilibrium quantities and resulting prices are limited by the demand size. In particular, the increase in average output due to information leakage can outweigh the negative distortions in aggregate quantities. In equilibrium, therefore, consumers are better off compared to the case where the upstream firms are independent, while the firms are worse off when downstream competition is intense.

Remark 1. While general results are hard to obtain, the observations in Proposition 4 are

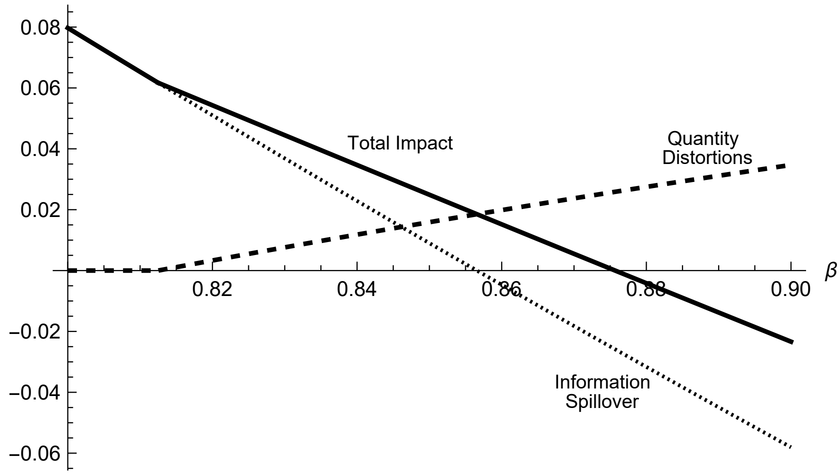

Figure 4: Impact of common supplier on producer surplus (multiplied by 1000) when $\theta_{hh} - \theta_{hl} = \frac{3}{2} > 1 = \theta_{hl} - \theta_{ll} = \theta_{ll} - c > 0$ and $\rho = \frac{1}{2}$.

likely robust to other parameter specifications. For instance, Figure 3 and Figure 4 respectively depict the impact of a common supplier on consumer surplus and on producer surplus for $\theta_{hh} - \theta_{hl} = \frac{3}{2} > 1 = \theta_{hl} - \theta_{ll} = \theta_{ll} - c > 0$ and $\rho = \frac{1}{2}$. In this case, $\beta \geq \frac{13}{16} \approx 0.81$ ensures that Assumption 1 holds but the condition (2) does not. Notably, it continues to hold that, with intense downstream competition, having a common upstream firm improves consumer surplus and reduces producer surplus. However, for the intermediate range of β where both total impact and the impact of quantity distortions are positive, the presence of a common upstream supplier yields the highest downstream profits among all possible cases, beyond those with independent suppliers or complete information. This is possible because, for such moderate competition, either the information spillover has positive impacts on profits or the downward signaling distortions in quantities dominate the negative profit effects of the information spillover.

Discussion: Informational Efficiency Gain of Upstream Mergers. Our results have immediate yet important policy implications regarding upstream mergers. Suppose that the downstream firms initially have purchased input from a competitive upstream input market or independent suppliers with a constant return to scale technology. In our model, the merger between upstream firms toward a common input supplier strictly benefits final consumers even when the production cost does not decrease post-merger, provided that (i) the post-merger production technology is still a constant return to scale and (ii) the downstream firms' ability to make offers is preserved.

As also argued in Marx and Shaffer (2016), this aligns with the views reflected in merger guidelines in many countries, which recognize that larger buyers can exert countervailing power to alleviate the potential harm of mergers to consumers. Building on the informational effects of such mergers, our results complement and strengthen this argument by demonstrating the possibility of a strict improvement in consumer surplus even when the direct efficiency gains from an upstream merger are *absent or even negative*.

6 Concluding Remarks

In this paper, we have revisited the standard Cournot competition model with asymmetric demand information. Departing from the existing literature, we have introduced a strategic common upstream firm that supplies the necessary input to both competing downstream firms. Our main results are twofold. First, we have constructed a unique equilibrium where information spillover occurs throughout the entire vertical chain. Second, we have analyzed the welfare implications of the presence of a common upstream supplier. Specifically, we have demonstrated that an upstream merger can lead to informational efficiency gains, ultimately benefiting final consumers even when direct efficiency improvements are absent.

Despite these contributions, our work has several limitations. Relaxing certain assumptions, such as increasing the number of downstream firms or demand signals, is conceptually straightforward, though it would significantly increase computational complexity. Other assumptions, however, are more foundational and would require novel approaches to address. For instance, if upstream production costs are non-separable across the downstream firms it serves, the upstream firm's acceptance of one firm's offer would depend in a non-trivial way on offers from the other. This interdependence could make constructing an equilibrium with positive quantities sold by all downstream firms particularly challenging due to the possibility of exclusion. Additionally, our model does not account for ex-ante asymmetries between downstream firms, whether in their informational or production technologies. These asymmetries are highly relevant for practical policy design and merit further investigation. We believe that exploring these important issues will be highly valuable and leave them for future research.

Appendix

A. Kuhn-Tucker First-Order Conditions

We begin our analysis by setting up the Lagrangian function for the s-type problem (P-s):

$$\begin{aligned}\mathcal{L}^s &= E_{\tilde{s}}\left[\left(P\left(q_{\tilde{s}}, \widehat{q}^{\tilde{s}}_s; s, \tilde{s}\right) - c\right) q_{\tilde{s}}\right] - u \\ &+ \lambda_{-s}\left(E_{\tilde{s}}\left[\left(P\left(\widehat{q_{\tilde{s}}}^{-s}, \widehat{q}^{\tilde{s}}_{-s}; -s, \tilde{s}\right) - c\right)\widehat{q_{\tilde{s}}}^{-s}\right] - E_{\tilde{s}}\left[\left(P\left(q_{\tilde{s}}, \widehat{q}^{\tilde{s}}_s; -s, \tilde{s}\right) - c\right) q_{\tilde{s}}\right] + u - \widehat{u}^{-s}\right) \\ &+ \lambda_u u,\end{aligned}$$

where λ_{-s} and λ_u are the multipliers for (IC_{-s}) and (IRU'), respectively. The boundary condition for the quantities, which we ignore in the above, will be checked later.

The Kuhn-Tucker first-order necessary conditions are then given as follows:

(i) Stationarity:

$$\frac{\partial \mathcal{L}^s}{\partial q_h} = MR\left(q_h, \widehat{q}^h_s; s, h\right) - c - \lambda_{-s}\left(MR\left(q_h, \widehat{q}^h_s; -s, h\right) - c\right) = 0, \tag{4}$$

$$\frac{\partial \mathcal{L}^s}{\partial q_l} = MR\left(q_l, \widehat{q}^l_s; s, l\right) - c - \lambda_{-s}\left(MR\left(q_l, \widehat{q}^l_s; -s, l\right) - c\right) = 0, \tag{5}$$

and

$$\frac{\partial \mathcal{L}^s}{\partial u} = -1 + \lambda_{-s} + \lambda_u = 0. \tag{6}$$

(ii) Complementary slackness:

$$\begin{aligned}&\lambda_{-s}\left(E_{\tilde{s}}\left[\left(P\left(\widehat{q_{\tilde{s}}}^{-s}, \widehat{q}^{\tilde{s}}_{-s}; -s, \tilde{s}\right) - c\right)\widehat{q_{\tilde{s}}}^{-s}\right] - E_{\tilde{s}}\left[\left(P\left(q_{\tilde{s}}, \widehat{q}^{\tilde{s}}_s; -s, \tilde{s}\right) - c\right) q_{\tilde{s}}\right] + u - \widehat{u}^{-s}\right) \\ &= 0,\end{aligned} \tag{7}$$

and

$$\lambda_u u = 0. \tag{8}$$

(iii) Dual feasibility:

$$\lambda_{-s} \geq 0, \text{ and } \lambda_u \geq 0. \tag{9}$$

(vi) Primal feasibility:

$$(IC_{-s}), \text{ and } (IRU'). \tag{10}$$

B. Omitted Proofs

Proof of Lemma 1. Consider the h-type problem. Suppose $\lambda_u = 0$. Then, by (6), we have $\lambda_l = 1$, implying that

$$MR\left(q_h, \widehat{q}_h^h; h, h\right) = MR\left(q_h, \widehat{q}_h^h; l, h\right) \iff \theta_{hh} - \theta_{hl} = 0,$$

from (4). This is a contradiction. The same argument applies to the l-type problem. □

Proof of Lemma 2. By Lemma 1, it is immediate that $0 \leq \lambda_{-s} < 1$. Now, suppose that $\lambda_l > 0$. From (4), we have

$$MR\left(q_h, \widehat{q}_h^h; h, h\right) - c - \lambda_{-s}\left(MR\left(q_h, \widehat{q}_h^h; l, h\right) - c\right) = 0$$
$$\iff \theta_{hh} - \beta\widehat{q}_h^h - 2q_h - c = \lambda_l\left(\theta_{hl} - \beta\widehat{q}_h^h - 2q_h - c\right).$$

The left-hand side is nonnegative because equilibrium quantities do not exceed $q_{hh}^* = \frac{\theta_{hh}-c}{2+\beta}$. To derive a contradiction, suppose that the left-hand side is zero. Then, the right-hand side is strictly negative since $\theta_{hh} > \theta_{hl}$. Suppose now that the left-hand side is strictly positive. Rearranging the terms, one can show that the equation is equivalent to

$$\frac{1}{\lambda_l}\left(\theta_{hh} - \beta\widehat{q}_h^h - 2q_h - c\right) = \theta_{hl} - \beta\widehat{q}_h^h - 2q_h - c.$$

Since $\lambda_l < 1$, this implies

$$\theta_{hh} - \beta\widehat{q}_h^h - 2q_h - c < \frac{1}{\lambda_l}\left(\theta_{hh} - \beta\widehat{q}_h^h - 2q_h - c\right) = \theta_{lh} - \beta\widehat{q}_h^h - 2q_h - c,$$

which is a contradiction. Therefore, $\lambda_l = 0$, and the remaining results follow from the conditions (4) and (5) of the high-type problem. □

Proof of Theorem 1. We will write $q_{s'}^s(\tilde{\lambda}_h)$ for the equilibrium quantity to emphasize its dependence on $\tilde{\lambda}_h$. The theorem is proved in several steps through Lemmas 3–7.

Lemma 3. Suppose $\tilde{\lambda}_h \leq \overline{\lambda}_h \equiv \frac{2-\beta}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right)$, where $\overline{\lambda}_h \leq \frac{1}{2}$. Then,

$$0 \leq q_l^l(\tilde{\lambda}_h) \leq q_h^l(\tilde{\lambda}_h) \leq q_l^h(\tilde{\lambda}_h) \leq q_h^h(\tilde{\lambda}_h) = q_{hh}^*.$$

Proof. A simple computation shows that

$$q_h^h(\tilde{\lambda}_h) \geq q_l^h(\tilde{\lambda}_h) \iff \tilde{\lambda}_h \leq \frac{2-\beta}{2},$$

$$q_l^l(\tilde{\lambda}_h) \geq 0 \iff \tilde{\lambda}_h \leq \frac{\theta_{ll}-c}{\theta_{hl}-c},$$

$$q_h^l(\tilde{\lambda}_h) \geq q_l^l(\tilde{\lambda}_h) \iff \tilde{\lambda}_h \leq \frac{2-\beta}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right).$$

Under Assumption 1, it is sufficient to satisfy the last inequality, yielding the result.

□

Now, consider $(\widetilde{IC}_h)$ where the quantities in the both sides are replaced by the candidate equilibrium quantities in (1). By tedious computations, one can show that it is equivalent to

$$\begin{aligned}
&\rho\left(q_{hh}^{*}\right)^{2}+(1-\rho)\left(q_{hl}^{*}\right)^{2}-\rho\left(\theta_{hh}-(1+\beta)\, q_{hl}^{*}-c\right) q_{hl}^{*}-(1-\rho)\left(\theta_{hl}-(1+\beta)\, q_{ll}^{*}-c\right) q_{ll}^{*} \\
&\geq\left((1-\rho)\left(\kappa_{hl}-\beta \kappa_{lh}\right)+\rho\left(\kappa_{lh}-\beta \kappa_{hl}\right)\right) q_{hl}^{*}-\rho\left(\theta_{hh}-(1+\beta)\, q_{hl}^{*}-c+\kappa_{lh}-\beta \kappa_{hl}\right) \kappa_{lh} \\
&-(1-\rho)\left(\theta_{hl}-(1+\beta)\, q_{hl}^{*}-c+\beta \kappa_{lh}-\kappa_{hl}\right) \kappa_{hl} \\
&-(1-\rho)\left(\theta_{hl}-2\,(1+\beta)\, q_{ll}^{*}-c+(1+\beta)\, \kappa_{ll}\right) \kappa_{ll}. \qquad (11)
\end{aligned}$$

Clearly, the left-hand side is independent of $\tilde{\lambda}_h$ and non-negative if and only if $(\widetilde{IC}_h)$ is satisfied at $q^*_{ss'}$.

Lemma 4. The right-hand side of (11) is zero at $\tilde{\lambda}_h = 0$, non-positive and strictly decreasing in $\tilde{\lambda}_h \in [0,1)$.

Proof. Letting $\mu = \frac{\tilde{\lambda}_h}{1-\tilde{\lambda}_h}$, we have

$$\begin{aligned}
\kappa_{hl} &= \frac{\beta\tilde{\lambda}_h\left(\theta_{hh}-\theta_{hl}\right)}{(2-\beta)\,(2+\beta)\,(1-\lambda)} = \mu\left(\frac{\beta}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right), \\
\kappa_{lh} &= \frac{2\tilde{\lambda}_h\left(\theta_{hh}-\theta_{hl}\right)}{(2-\beta)\,(2+\beta)\left(1-\tilde{\lambda}_h\right)} = \mu\left(\frac{2}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right), \\
\kappa_{ll} &= \frac{\tilde{\lambda}_h\left(\theta_{hl}-\theta_{ll}\right)}{(2+\beta)\left(1-\tilde{\lambda}_h\right)} = \mu\left(q_{hl}^{*}-q_{ll}^{*}\right).
\end{aligned}$$

Thus,

$$\kappa_{hl}-\beta\kappa_{lh} = \mu\left(q_{hh}^{*}-q_{hl}^{*}\right)\left(\frac{\beta}{2-\beta}-\frac{2\beta}{2-\beta}\right) = -\mu\left(\frac{\beta}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right),$$

and

$$\kappa_{lh}-\beta\kappa_{hl} = \mu\left(q_{hh}^{*}-q_{hl}^{*}\right)\left(\frac{2}{2-\beta}-\frac{\beta^2}{2-\beta}\right) = \mu\left(q_{hh}^{*}-q_{hl}^{*}\right)\left(\frac{2-\beta^2}{2-\beta}\right).$$

Therefore, the right-hand side of (11) can be written as

$$\begin{aligned}
&\mu\left(\rho\left(\frac{2-\beta^2}{2-\beta}\right)-(1-\rho)\left(\frac{\beta}{2-\beta}\right)\right)\left(q_{hh}^{*}-q_{hl}^{*}\right) q_{hl}^{*} \\
&-\rho\mu\left(\theta_{hh}-(1+\beta)\, q_{hl}^{*}-c+\mu\left(\frac{2-\beta^2}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right)\right)\left(\frac{2}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right) \\
&-(1-\rho)\,\mu\left(\theta_{hl}-(1+\beta)\, q_{hl}^{*}-c+\mu\left(\frac{\beta}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right)\right)\left(\frac{\beta}{2-\beta}\right)\left(q_{hh}^{*}-q_{hl}^{*}\right) \\
&-(1-\rho)\,\mu\left(\theta_{hl}-2\,(1+\beta)\, q_{ll}^{*}-c+\mu\,(1+\beta)\left(q_{hl}^{*}-q_{ll}^{*}\right)\right)\left(q_{hl}^{*}-q_{ll}^{*}\right).
\end{aligned}$$

Note that $\mu = \frac{\tilde{\lambda}_h}{1-\tilde{\lambda}_h}$ is zero at $\tilde{\lambda}_h = 0$, non-negative, and strictly increasing in $\tilde{\lambda}_h \in [0,1)$. Thus, it suffices to show that

$$\begin{aligned}
G \equiv & \left(\rho\left(\frac{2-\beta^2}{2-\beta}\right) - (1-\rho)\left(\frac{\beta}{2-\beta}\right)\right)\left(q^*_{hh} - q^*_{hl}\right) q^*_{hl} \\
& -\rho\left(\theta_{hh} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{2-\beta^2}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right)\left(\frac{2}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right) \\
& -(1-\rho)\left(\theta_{hl} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{\beta}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right)\left(\frac{\beta}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right) \\
& -(1-\rho)\left(\theta_{hl} - 2(1+\beta)\, q^*_{ll} - c + \mu(1+\beta)\left(q^*_{hl} - q^*_{ll}\right)\right)\left(q^*_{hl} - q^*_{ll}\right)
\end{aligned}$$

is non-positive and strictly decreasing in $\tilde{\lambda}_h \in [0,1)$. Clearly, G is strictly decreasing in μ, and so is in $\tilde{\lambda}_h$. To show that it is non-positive, observe that G is a linear function of ρ, which implies that it is enough to consider $\rho = 1$ and $\rho = 0$.

First, suppose $\rho = 1$. Then, G equals

$$\begin{aligned}
& \left[\left(\frac{2-\beta^2}{2-\beta}\right) q^*_{hl} - \left(\theta_{hh} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{2-\beta^2}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right)\left(\frac{2}{2-\beta}\right)\right]\left(q^*_{hh} - q^*_{hl}\right) \\
=& \frac{1}{2-\beta}\left[\left(2-\beta^2\right) q^*_{hl} - 2\left(\theta_{hh} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{2-\beta^2}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right)\right]\left(q^*_{hh} - q^*_{hl}\right).
\end{aligned}$$

This is strictly negative because

$$\begin{aligned}
& \left(2-\beta^2\right) q^*_{hl} - 2\left(\theta_{hh} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{2-\beta^2}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right) \\
\leq & \left(2-\beta^2\right) q^*_{hl} - 2\left(\theta_{hh} - (1+\beta)\, q^*_{hl} - c\right) \\
=& \frac{1}{2+\beta}\left(-4\theta_{hh} + \beta^2\left(c - \theta_{hl}\right) - 2\beta\left(\theta_{hh} - \theta_{hl}\right) + 4\theta_{hl}\right) \\
=& \frac{1}{2+\beta}\left(-4\left(\theta_{hh} - \theta_{hl}\right) - \beta\left(2\left(\theta_{hh} - \theta_{hl}\right) + \beta\left(\theta_{hl} - c\right)\right)\right) \\
<& 0.
\end{aligned}$$

Next, suppose $\rho = 0$. Then,

$$\begin{aligned}
G =& \left[-\frac{\beta}{2-\beta} q^*_{hl} - \left(\theta_{hl} - (1+\beta)\, q^*_{hl} - c + \mu\left(\frac{\beta}{2-\beta}\right)\left(q^*_{hh} - q^*_{hl}\right)\right)\frac{\beta}{2-\beta}\right]\left(q^*_{hh} - q^*_{hl}\right) \\
& -\left(\theta_{hl} - 2(1+\beta)\, q^*_{ll} - c + \mu(1+\beta)\left(q^*_{hl} - q^*_{ll}\right)\right)\left(q^*_{hl} - q^*_{ll}\right) \\
\leq & \left[-\frac{\beta}{2-\beta} q^*_{hl} - \left(\theta_{hl} - (1+\beta)\, q^*_{hl} - c\right)\frac{\beta}{2-\beta}\right]\left(q^*_{hh} - q^*_{hl}\right) \\
& -\left(\theta_{hl} - 2(1+\beta)\, q^*_{ll} - c\right)\left(q^*_{hl} - q^*_{ll}\right) \\
<& 0,
\end{aligned}$$

where the last inequality follows from

$$\theta_{hl} - (1+\beta)\, q_{hl}^* - c = (\theta_{hl} - c)\left(1 - \frac{1+\beta}{2+\beta}\right) > 0,$$

and

$$\begin{aligned}
\theta_{hl} - 2(1+\beta)\, q_{ll}^* - c &= \theta_{hl} - c - \frac{2(1+\beta)(\theta_{ll} - c)}{2+\beta} \\
&= \theta_{hl} - \theta_{ll} + \theta_{ll} - c - \frac{2(1+\beta)(\theta_{ll} - c)}{2+\beta} \\
&\geq 2(\theta_{ll} - c) - \frac{2(1+\beta)(\theta_{ll} - c)}{2+\beta} \\
&= (\theta_{ll} - c)\left(2 - \frac{2+2\beta}{2+\beta}\right) \\
&> 0.
\end{aligned}$$

This completes the proof of the lemma. □

Lemma 5. The left-hand side of (11) is non-negative if and only if condition (2) holds. Also, if condition (2) does not hold, there exists unique $\lambda^* \in \left(0, \overline{\lambda}_h\right]$ such that the inequality (11) holds as equality for $\tilde{\lambda}_h = \lambda^*$.

Proof. The first part follows directly from the computation, so we will only prove the second part. Observe that the right-hand side of (11) is zero when $\tilde{\lambda}_h = 0$. Thus, if (2) does not hold, then the left-hand side of (11) is strictly lower than the right-hand side of (11) when $\tilde{\lambda}_h = 0$. By the intermediate value theorem, it suffices to show that (11) is satisfied at $\tilde{\lambda}_h = \overline{\lambda}_h$ for the existence of $\tilde{\lambda}_h$ inducing equality of (11). Clearly, the uniqueness follows from Lemma 4.

For notational ease, we define

$$F(\lambda, \rho) \equiv \Big[\text{LHS of } (11) - \text{RHS of } (11)\Big]_{\tilde{\lambda}_h = \lambda}$$

By tedious computations, one can check that $F\left(\bar{\lambda}_h, \rho\right)$ is linear in ρ with

$$\begin{aligned}
\operatorname{sgn}\left[\frac{\partial}{\partial \rho} F\left(\overline{\lambda}_h, \rho\right)\right] = \operatorname{sgn}\Big[&2(\theta_{hh} - \theta_{hl})(\theta_{hh} - \theta_{ll}) - \beta^2 (\theta_{hl} - \theta_{ll})(\theta_{hl} - c) \\
&+ 2\beta\left(\theta_{hh}(\theta_{hl} - \theta_{ll}) - (\theta_{hh} - \theta_{hl})\theta_{ll} + c(\theta_{hh} + \theta_{ll} - 2\theta_{hl})\right)\Big].
\end{aligned}$$

Note that

$$\begin{aligned}
&2(\theta_{hh} - \theta_{hl})(\theta_{hh} - \theta_{ll}) - \beta^2 (\theta_{hl} - \theta_{ll})(\theta_{hl} - c) \\
&+2\beta\left(\theta_{hh}(\theta_{hl} - \theta_{ll}) - (\theta_{hh} - \theta_{hl})\theta_{ll} + c(\theta_{hh} + \theta_{ll} - 2\theta_{hl})\right) \\
&\geq 2\beta(\theta_{hh} - \theta_{hl})(\theta_{hh} - \theta_{ll}) - \beta(\theta_{hl} - \theta_{ll})(\theta_{hl} - c) \\
&+2\beta\left(\theta_{hh}(\theta_{hl} - \theta_{ll}) - (\theta_{hh} - \theta_{hl})\theta_{ll} + c(\theta_{hh} + \theta_{ll} - 2\theta_{hl})\right) \\
&= \beta\left(2(\theta_{hh} - \theta_{hl})(\theta_{hh} - \theta_{ll} - (\theta_{ll} - c)) + (\theta_{hl} - \theta_{ll})(\theta_{hh} - \theta_{hl} + \theta_{hh} - c)\right) \\
&\geq 0,
\end{aligned}$$

which shows that $F\left(\overline{\lambda}_h, \rho\right)$ is strictly increasing in ρ. To prove that (11) is satisfied at $\overline{\lambda}_h$, it is thus sufficient to show $F\left(\overline{\lambda}_h, 0\right) \geq 0$, which is indeed true since

$$F\left(\overline{\lambda}_h, 0\right) = \frac{\left(\theta_{hh} - \theta_{hl}\right)^2 \left(\theta_{hl} - \theta_{ll}\right)^2}{\left(2\theta_{hh} - \left(4 - \beta\right)\theta_{hl} + \left(2 - \beta\right)\theta_{ll}\right)^2} > 0.$$

This completes the proof. □

The previous results have proven the existence and uniqueness of the Lagrangian multiplier $\tilde{\lambda}_h$ (so, the candidate equilibrium quantities) satisfying the first-order necessary conditions. It is immediate that these quantities satisfy conditions (SI) and (SC). The remaining step is to show that the second-order sufficient conditions are satisfied as well, which can be done by noting the following result in general optimization problems.

Lemma 6. Consider the optimization problem

$$\max_{x,y\in\mathbb{R}} H\left(x,y\right)$$
$$s.t. g\left(x,y\right) \leq c$$

and the associated Lagrangian function

$$\mathcal{L}_\lambda\left(x,y\right) = H\left(x,y\right) - \lambda\left(g\left(x,y\right) - c\right),$$

where $H\left(\cdot\right)$ and $g\left(\cdot\right)$ are continuously differentiable functions. Suppose $\left(x^*, y^*, \lambda^*\right)$ satisfies $\frac{\partial \mathcal{L}_{\lambda^*}(x^*,y^*)}{\partial x} = \frac{\partial \mathcal{L}_{\lambda^*}(x^*,y^*)}{\partial y} = 0$, $g\left(x^*, y^*\right) \leq c$, $\lambda^*\left(g\left(x^*, y^*\right) - c\right) = 0$, $\lambda^* \geq 0$, and that $\mathcal{L}_{\lambda^*}\left(x,y\right)$ is strictly concave. Then, $\left(x^*, y^*\right)$ is the unique solution to the optimization problem.

Proof. Since $\mathcal{L}_{\lambda^*}\left(x,y\right)$ is strictly concave, $\mathcal{L}_{\lambda^*}\left(x^*,y^*\right) > \mathcal{L}_{\lambda^*}\left(x,y\right)$ for all $\left(x,y\right) \in S$. This directly implies that $H\left(x^*,y^*\right) - \lambda^*\left(g\left(x^*,y^*\right) - c\right) = H\left(x^*,y^*\right) > H\left(x,y\right) - \lambda^*\left(g\left(x,y\right) - c\right)$ for all $\left(x,y\right) \in \mathbb{R}^2$. Therefore, if $g\left(x,y\right) \leq c$, then $H\left(x^*,y^*\right) > H\left(x,y\right)$, implying that $\left(x^*,y^*\right)$ is the unique solution to the optimization problem. □

Now, we apply this lemma to our problem, thereby completing the proof of the theorem.

Lemma 7. Suppose (2) does not hold. Consider the reduced l-type problem (RP-l), in which $\widehat{q}^s_{s'} = q^s_{s'}(\lambda^*)$ with λ^* being defined in Lemma 5. Then, $\left(q_h, q_l\right) = \left(q^l_h\left(\lambda^*\right), q^l_l\left(\lambda^*\right)\right)$ is the unique solution to the optimization problem (RP-l).

Proof. By construction, $\left(q_h, q_l, \lambda\right) = \left(q^l_h\left(\lambda^*\right), q^l_l\left(\lambda^*\right), \lambda^*\right)$ satisfies (i) $\frac{\partial \mathcal{L}}{\partial q_h} = \frac{\partial \mathcal{L}}{\partial q_l} = 0$; (ii) $(\widetilde{IC}_h)$ is binding; and (iii) $\lambda = \lambda^* \in \left(0, \frac{1}{2}\right]$. Also, the Lagrangian function for this problem with $\lambda = \lambda^*$ is strictly concave since

$$D^2\mathcal{L} = \begin{bmatrix} \frac{\partial^2\mathcal{L}}{\partial q_h^2} & \frac{\partial^2\mathcal{L}}{\partial q_h \partial q_l} \\ \frac{\partial^2\mathcal{L}}{\partial q_l \partial q_h} & \frac{\partial^2\mathcal{L}}{\partial q_l^2} \end{bmatrix} = \begin{bmatrix} -2\rho(1-\lambda^*) & 0 \\ 0 & -2(1-\rho)(1-\lambda^*) \end{bmatrix} \tag{12}$$

By Lemma 6, we conclude that $\left(q_h, q_l\right) = \left(q^l_h\left(\lambda^*\right), q^l_l\left(\lambda^*\right), \lambda^*\right)$ is the unique solution to this problem. □

Proof of Proposition 2. By continuity, it suffices to consider $\beta = 0$. In this case, condition (2) is slack, and the equilibrium quantities with the common supplier are given by $q^*_{ss'} = \frac{\theta_{ss'}-c}{2}$. With independent suppliers, on the other hand, the equilibrium quantities are $\bar{q}_s = \frac{E_{\tilde{s}}[\theta_{s\tilde{s}}]-c}{2}$. The former is a mean-preserving-spread of the latter, which implies that both consumer surplus and producer surplus are strictly higher with the common supplier:

$$\widehat{CS} = E_{s\tilde{s}}\left[(q^*_{s\tilde{s}})^2\right] > CS = E_{s\tilde{s}}\left[(\bar{q}_s)^2\right],$$

and

$$\widehat{PS} = 2E_{s\tilde{s}}\left[(q^*_{s\tilde{s}})^2\right] > PS = 2E_{s\tilde{s}}\left[(\bar{q}_s)^2\right].$$

□

Proof of Corollary 1. First, observe that (3) is satisfied if

$$\begin{aligned} q^*_{ll} - 2\kappa_{ll} \geq 0 &\iff q^*_{ll} + d_{ll} = q^*_{ll} - 2\kappa_{ll} \geq 0 \\ &\iff \frac{\theta_{ll}-c}{3} - \frac{2\lambda^*}{3(1-\lambda^*)}(\theta_{hl}-\theta_{ll}) \geq 0 \\ &\iff \lambda^* \leq \frac{\theta_{ll}-c}{2\theta_{hl}-\theta_{ll}-c}, \end{aligned} \tag{13}$$

since

$$q^*_{hl} + d_{hl} = q^*_{hl} + \kappa_{hl} - \kappa_{lh} > q^*_{hl} - \kappa_{lh} = q^l_h > 0.$$

In the proof of Theorem 1, it is shown that the equilibrium λ^* is bounded above by $\overline{\lambda}_h = \frac{1}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right)$. Therefore, the inequality (13) is satisfied so long as

$$\overline{\lambda}_h = \frac{1}{2}\left(\frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-\theta_{hl}}\right) \leq \frac{\theta_{ll}-c}{2\theta_{hl}-\theta_{ll}-c}, \tag{14}$$

which is the second condition in the corollary.

Next, to show $\rho \geq \frac{1}{2}$ implies (3), let us rewrite (3) as:

$$\rho\left(\theta_{hh}-\theta_{hl}\right)\left(q^*_{hl}+\kappa_{hl}-\kappa_{lh}\right) + (1-\rho)\left(\theta_{hl}-\theta_{ll}\right)\left(q^*_{ll}-2\kappa_{ll}\right) > 0. \tag{15}$$

Also, notice that $q^*_{hl} + \kappa_{hl} - \kappa_{lh} > q^*_{ll} - 2\kappa_{ll}$ is equivalent to

$$\lambda^* < \frac{\theta_{hl}-\theta_{ll}}{\theta_{hh}-2\theta_{hl}+\theta_{ll}},$$

since

$$\begin{aligned} q^*_{hl} + \kappa_{hl} - \kappa_{lh} - q^*_{ll} + 2\kappa_{ll} &= \frac{\theta_{hl}-\theta_{ll}}{3} + \frac{2\lambda^*\left(\theta_{hl}-\theta_{ll}\right)}{3\left(1-\lambda^*\right)} - \frac{\lambda^*\left(\theta_{hh}-\theta_{hl}\right)}{3\left(1-\lambda^*\right)} \\ &= \frac{\theta_{hl}-\theta_{ll}}{3} + \frac{\lambda^*\left(3\theta_{hl}-\theta_{hh}-2\theta_{ll}\right)}{3\left(1-\lambda^*\right)} \\ &> 0. \end{aligned}$$

It follows that $q^*_{hl} + \kappa_{hl} - \kappa_{lh} > q^*_{ll} - 2\kappa_{ll}$ holds because

$$\bar{\lambda}_h = \frac{1}{2}\left(\frac{\theta_{hl} - \theta_{ll}}{\theta_{hh} - \theta_{hl}}\right) < \frac{\theta_{hl} - \theta_{ll}}{\theta_{hh} - 2\theta_{hl} + \theta_{ll}} \iff \theta_{ll} < \theta_{hh}.$$

This completes the proof since $\theta_{hh} - \theta_{hl} \geq \theta_{hl} - \theta_{ll}$ by Assumption 1. $\square$

Proof of Proposition 4. Under the assumptions in the proposition, one can compute to show that

$$\bar{q}_l = \frac{\Delta}{3} + \frac{\rho\Delta}{6} < \Delta\left(\frac{1+\rho}{2}\right) = \bar{q}_h.$$

In addition,

$$q^*_{hh} = \Delta > q^*_{hl} = q^*_{lh} = \frac{2\Delta}{3} > q^*_{ll} = \frac{\Delta}{3},$$

and

$$q^h_h = \Delta,\ q^l_h = \frac{2\Delta}{3} + \kappa_{hl},\ q^h_l = \frac{2\Delta}{3} - \kappa_{lh},\ q^l_l = \frac{\Delta}{3} - \kappa_{ll},$$

where $\kappa_{hl} = \frac{\lambda^*\Delta}{3(1-\lambda^*)}, \kappa_{lh} = \frac{2\lambda^*\Delta}{3(1-\lambda^*)}, \kappa_{ll} = \frac{\lambda^*\Delta}{3(1-\lambda^*)}$. Given these, one can directly compute to show the items (i) and (ii).

Now, we show (iii). Recall that λ^* is the unique solution to the binding constraint $(\widetilde{IC}_h)$. By tedious computations, one can check that it is either

$$\lambda^* = \frac{3 + 2\rho - 3\sqrt{1+\rho}}{3+4\rho}$$

or

$$\lambda^* = \frac{3 + 2\rho + 3\sqrt{1+\rho}}{3+4\rho}$$

Note that the first root lies in the interval $\left[0, \frac{1}{2}\right]$. To see this, observe that

$$\frac{3 + 2\rho - 3\sqrt{1+\rho}}{3+4\rho} \leq \frac{1}{2} \iff \frac{1}{2} \leq \sqrt{1+\rho},$$

which is true, and

$$\frac{3 + 2\rho - 3\sqrt{1+\rho}}{3+4\rho} \geq 0 \iff 1 + \frac{2\rho}{3} \geq \sqrt{1+\rho},$$

which also holds since $\rho > 0$. By Lemma 5, we thus conclude that $\lambda^* = \frac{3+2\rho-3\sqrt{1+\rho}}{3+4\rho}$ since $\lambda^* \leq \overline{\lambda}_h = \frac{1}{2}$. Furthermore, λ^* is strictly increasing in ρ because

$$\frac{\partial\lambda^*}{\partial\rho} = \frac{3}{2}\left(\frac{5 + 4\rho - 4\sqrt{1+\rho}}{(3+4\rho)^2\sqrt{1+\rho}}\right) > 0 \iff \frac{5}{4} + \rho > \sqrt{1+\rho}.$$

Finally, we prove part (iv). Let us first compute the ex-ante consumer surplus under the complete information benchmark:

$$
\begin{aligned}
CS^* &= \frac{1}{2}\left(\rho^2\left(2q^*_{hh}\right)^2 + 2\rho(1-\rho)\left(q^*_{hl}+q^*_{lh}\right)^2 + (1-\rho)^2\left(2q^*_{ll}\right)^2\right) \\
&= \Delta^2\left(2\rho^2 + \rho\left(1-\rho\right)\frac{16}{9} + \left(1-\rho\right)^2\frac{2}{9}\right) \\
&= \frac{2\Delta^2}{9}\left(1+6\rho+2\rho^2\right)
\end{aligned}
$$

Recall that the equilibrium with a common input supplier entails quantity distortions from those under complete information. Therefore,

$$
\begin{aligned}
&\widehat{CS} - CS^* \\
=&\Delta^2\left[\left(\frac{\lambda^*}{1-\lambda^*}\right)^2\left(\frac{2\left(1-\rho\right)\left(2-\rho\right)}{9}\right) - \left(\frac{\lambda^*}{1-\lambda^*}\right)\left(\frac{8\left(1-\rho^2\right)}{9}\right)\right] \\
=&-\Delta^2\left[\frac{2\left(1-\rho\right)\left(3+2\rho-3\sqrt{1+\rho}\right)\left(-6+10\rho^2+18\sqrt{1+\rho}+\rho\left(7+9\sqrt{1+\rho}\right)\right)}{9\left(2\rho+3\sqrt{1+\rho}\right)^2}\right].
\end{aligned}
$$

where the second equality follows from $\lambda^* = \frac{3+2\rho-3\sqrt{1+\rho}}{3+4\rho}$. It is straightforward to check that $\widehat{CS} - CS^*$ equals zero only when $\rho = 0$ or $\rho = 1$; for any $\rho \in (0,1)$, it is strictly negative. Also, we have

$$
\begin{aligned}
CS &= \frac{1}{2}\left(\rho^2\left(2q^*_h\right)^2 + 2\rho\left(1-\rho\right)\left(q^*_h+q^*_l\right)^2 + \left(1-\rho\right)^2\left(2q^*_l\right)^2\right) \\
&= \frac{\rho^2\Delta^2(1+\rho)^2}{2} + \frac{\rho(1-\rho)\Delta^2(5+4\rho)^2}{36} + \frac{(1-\rho)^2\Delta^2(2+\rho)^2}{18} \\
&= \left(8+17\rho+27\rho^2+16\rho^3+4\rho^4\right)\frac{\Delta^2}{36},
\end{aligned}
$$

which implies

$$
CS^* - CS = \left(31-11\rho-16\rho^2-4\rho^3\right)\frac{\Delta^2\rho}{36} > 0.
$$

Thus, letting

$$
\begin{aligned}
K(\rho) \equiv&\frac{1}{36}\left[\frac{8\left(1-\rho\right)\left(-3-2\rho+3\sqrt{1+\rho}\right)\left(-6+10\rho^2+18\sqrt{1+\rho}+\rho\left(7+9\sqrt{1+\rho}\right)\right)}{\left(2\rho+3\sqrt{1+\rho}\right)^2}\right] \\
&+\left(31-11\rho-16\rho^2-4\rho^3\right)\frac{\rho}{36},
\end{aligned}
$$

we have

$$
\widehat{CS} - CS = \Delta^2 K(\rho).
$$

Clearly, $K(1) = K(0) = 0$. We next argue that $K(\rho) > 0$ for all $\rho \in (0,1)$, which is equivalent to

$$\begin{aligned}&\rho\left(31-11\rho-16\rho^2-4\rho^3\right)\left(2\rho+3\sqrt{1+\rho}\right)^2\\ +&8\left(1-\rho\right)\left(-3-2\rho+3\sqrt{1+\rho}\right)\left(-6+10\rho^2+18\sqrt{1+\rho}+\rho\left(7+9\sqrt{1+\rho}\right)\right)>0.\end{aligned}$$

By rearranging terms, this further reduces to

$$\begin{aligned}&576+279\rho-532\rho^2-143\rho^3-64\rho^4-100\rho^5-16\rho^6\\ >&\sqrt{1+\rho}\left(576-240\rho-804\rho^2+228\rho^3+192\rho^4+48\rho^5\right).\end{aligned}$$

Note that the left-hand side is strictly positive since it is strictly positive for $\rho = 0$, equals zero for $\rho = 1$; and strictly concave in ρ. Similarly, one can check that the bracket in the right-hand side is strictly positive for $\rho = 0$; equals zero for $\rho = 1$; and strictly decreasing in ρ. This implies that it is also strictly positive for $\rho \in (0,1)$. Therefore, the inequality above is equivalent to

$$\begin{aligned}&\left(576+279\rho-532\rho^2-143\rho^3-64\rho^4-100\rho^5-16\rho^6\right)^2\\ >&\left(1+\rho\right)\left(576-240\rho-804\rho^2+228\rho^3+192\rho^4+48\rho^5\right)^2,\end{aligned}$$

which, after some routine calculations, is reduced to

$$29568+48073\rho+16674\rho^2-3895\rho^3-3896\rho^4-488\rho^5+64\rho^6+16\rho^7>0.$$

This is indeed satisfied since $29568 > 3895\rho^3$, $48073\rho > 3896\rho^4$, and $16674\rho^2 > 488\rho^5$. Therefore, we conclude that $\widehat{CS} > CS$ for all $\rho \in (0,1)$.

Lastly, we show that $\widehat{PS} < PS$ for all $\rho \in (0,1)$. Note that each firm's ex-ante profits are

$$\begin{aligned}\frac{\widehat{PS}}{2}=&\rho E_{\tilde{s}}\left[\left(P\left(q_{\tilde{s}}^h,q_h^{\tilde{s}};h,\tilde{s}\right)-c\right)q_{\tilde{s}}^h\right]+\left(1-\rho\right)E_{\tilde{s}}\left[\left(P\left(q_{\tilde{s}}^l,q_l^{\tilde{s}};l,\tilde{s}\right)-c\right)q_{\tilde{s}}^l\right]\\ =&\rho^2\left(\theta_{hh}-2q_h^h-c\right)q_h^h+\rho\left(1-\rho\right)\left(\theta_{hl}-q_l^h-q_h^l-c\right)q_l^h\\ &+\left(1-\rho\right)\rho\left(\theta_{lh}-q_h^l-q_l^h-c\right)q_h^l+\left(1-\rho\right)^2\left(\theta_{ll}-2q_l^l-c\right)q_l^l,\end{aligned}$$

and

$$\begin{aligned}\frac{PS}{2}=&\rho^2\left(\theta_{hh}-2\bar{q}_h-c\right)\bar{q}_h+\rho\left(1-\rho\right)\left(\theta_{hl}-\bar{q}_h-\bar{q}_l-c\right)\bar{q}_h\\ &+\left(1-\rho\right)\rho\left(\theta_{lh}-\bar{q}_l-\bar{q}_h-c\right)\bar{q}_l+\left(1-\rho\right)^2\left(\theta_{ll}-2\bar{q}_l-c\right)\bar{q}_l.\end{aligned}$$

Therefore, we have

$$\widehat{PS}-PS=2\Delta^2 Q(\rho)$$

where

$$
\begin{aligned}
Q(\rho) \equiv &- \frac{(1-\rho)\left(16\rho^5 + \rho\left(153 - 60\sqrt{1+\rho}\right) - 180\left(-1+\sqrt{1+\rho}\right)\right)}{36\left(2\rho + 3\sqrt{1+\rho}\right)^2} \\
&- \frac{(1-\rho)\left(16\rho^3\left(5+6\sqrt{1+\rho}\right) + 5\rho^2\left(5+12\sqrt{1+\rho}\right) + \rho^4\left(68+48\sqrt{1+\rho}\right)\right)}{36\left(2\rho + 3\sqrt{1+\rho}\right)^2}.
\end{aligned}
$$

Again, it is clear that $Q(1) = Q(0) = 0$. We next show that $Q(\rho) < 0$ for all $\rho \in (0,1)$, which follows so long as

$$\rho\left(153 - 60\sqrt{1+\rho}\right) - 180\left(-1+\sqrt{1+\rho}\right) + 16\rho^3\left(5+6\sqrt{1+\rho}\right) > 0.$$

By rearranging terms, this inequality is reduced to

$$153\rho + 180 + 80\rho^3 > \sqrt{1+\rho}\left(60\rho + 180 - 96\rho^3\right),$$

or, equivalently,

$$\left(153\rho + 180 + 80\rho^3\right)^2 > (1+\rho)\left(60\rho + 180 - 96\rho^3\right)^2.$$

A tedious computation shows that this is equivalent to

$$1080 - 1791\rho + 59760\rho^2 + 70560\rho^3 + 11520\rho^4 - 2816\rho^5 - 9216\rho^6 > 0.$$

Since $70560\rho^3 > 2816\rho^5$ and $11520\rho^4 > 9216\rho^6$, it is sufficient to check that

$$1080 - 1791\rho + 59760\rho^2 > 0.$$

Note that the left-hand side is strictly convex in ρ, attaining its minimum at $\rho = \frac{1791}{119520}$. Substituting this value, we can verify that the expression remains strictly positive, completing the proof. $\square$

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