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Redistribution and Government Commitment

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Abstract

I study a dynamic game of successive governments in an incomplete-markets economy, where governments set labor income taxes and lump-sum taxes/transfers depending on their commitment ability. I find that commitment enables the government to coordinate policies across periods, thereby internalizing the effects of current policy decisions on the evolution of factor prices and wealth distributions shaped by past history. Commitment facilitates substantial long-term taxes and transfers, incurring long-run welfare losses but yielding short-run welfare gains through front-loaded reductions in precautionary savings and favorable factor price adjustments for low-income individuals. Without commitment, the government overlooks this intertemporal trade-off, yielding smaller short-run gains.

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1 Introduction

Over the past four decades, many developed countries have increased public social spending and introduced broader, more generous social programs.[1] A large body of literature responds by analyzing various redistributive policies using heterogeneous-agent models with incomplete financial markets.[2] However, relatively few studies have addressed redistributive policies in the context of *time inconsistency*. This gap is notable, given the longstanding emphasis in the literature on the critical role of time inconsistency in designing effective government policies (Kydland and Prescott, 1977; Calvo, 1978; Barro and Gordon, 1983; Lucas and Stokey, 1983; Chari and Kehoe, 1990; Krusell and Ríos-Rull, 1996; Klein et al., 2008; Azzimonti, 2011; Bassetto et al., 2024).

This paper contributes to filling this void by exploring how and to what extent government commitment affects optimal tax-transfer schemes in the standard incomplete-markets model of Bewley (1986), Huggett (1993), and Aiyagari (1994). To this end, I undertake the following steps. First, in a two-period economy, I analytically characterize the equilibria of a dynamic game between successive governments in an incomplete-markets economy, depending on the presence of government commitment. Second, to quantify my theoretical findings, I develop a numerical solution algorithm that can solve infinite-horizon heterogeneous-agent models in which the government lacks commitment. Finally, applying this numerical method to a calibrated economy, I quantitatively examine variations in optimal tax-transfer schemes along the transition path under different government commitment technologies.

In the theoretical analysis, I find that without commitment, the government overlooks the effects of its current policy decisions on the evolution of factor prices and wealth distributions shaped by past outcomes. In contrast, commitment enables the government to internalize these retrospective impacts by coordinating policies across periods. Such intertemporal coordination increases wages and reduces the capital income share through lower precautionary savings, thereby improving welfare for low-income individuals. In quantitative exercises in a calibrated economy, this distinction yields sizable implications. Commitment induces substantial long-term taxes and transfers, resulting in long-run welfare losses but delivering significant short-run welfare gains through front-loaded reductions in precautionary savings and favorable shifts in factor prices for low-income individuals. Without commitment, the government fails to recognize this intertemporal trade-off, resulting in smaller short-run gains. The welfare gain, measured by the consumption-equivalent variation, is larger under commitment (+2.19%) than without it (+1.46%).

[1] The average proportion of social spending out of GDP among the OECD countries increased from 14.4% in 1980 to 23% in 2020. For the U.S. case, Ben-Shalom et al. (2011) investigated its expansions in more detail.

[2] Building on the framework of Aiyagari (1994), numerous papers have explored related themes. For relatively recent examples, Conesa et al. (2009); Heathcote et al. (2017); Boar and Midrigan (2022); Ferriere et al. (2023) investigate the optimality of progressive taxation. Chatterjee et al. (2007) explores the implications of consumer bankruptcy policies, Guner et al. (2023) examines the welfare effects of Universal Basic Income.

Throughout the above exercises, I assume a utilitarian government that finances predetermined spending through lump-sum and labor income taxes, redistributing any surplus, if one exists, via lump-sum transfers. This policy environment is designed to isolate the time inconsistency arising purely from redistributive motives and market incompleteness, serving as a necessary step before studying conventional fiscal instruments, which already carry canonical time inconsistency in representative-agent models. By excluding capital income taxes, it avoids related time inconsistency issues (Chari, 1988; Atkeson et al., 1999; Domínguez, 2007a). Lump-sum taxes allow avoiding concerns about government debt maturity management (Lucas and Stokey, 1983) and high initial debt near the peak of the Laffer curve (Debortoli et al., 2021). In a representative-agent economy, the optimal policy—zero labor tax, with government spending fully funded by lump-sum taxation—is unaffected by commitment. Therefore, any time inconsistency that arises in this setting stems from economic forces linked to individual heterogeneity.

Given the shared policy environment, I compare two economies that differ in the availability of government commitment. In the case with commitment, following the spirit of Ramsey (1927), the government (a Ramsey planner) can commit to future tax and transfer policies, choosing an entire sequence of labor income taxes and lump-sum taxes/transfers along the transition path. In contrast, in the case without commitment, the economy is in Markov-perfect equilibrium (MPE), as in Krusell et al. (1996); Krusell and Ríos-Rull (1999); Klein et al. (2008). The government can only set a labor income tax rate and a lump-sum tax or transfer for the current period and cannot commit to future decisions. It chooses tax and transfer policies sequentially each period, resulting in a *time-consistent policy*.

For the theoretical analysis in a two-period economy, I employ the generalized Euler equation (GEE) approach from Klein et al. (2008), the first-order condition (FOC) of the government's choice. The GEE reveals the trade-offs that the government confronts when making policy decisions. I analytically characterize these trade-offs by adopting the Constant Absolute Risk Aversion (CARA) utility function, following Wang (2003); Park (2018); Acharya et al. (2023), that provides closed-form solutions for individual decisions and aggregate laws of motion in a standard incomplete-markets economy.

Through this characterization, I obtain three main findings. First, in the GEE, individual decision conditions for consumption, saving, and labor supply are canceled out by the Envelope theorem. These cancellations imply that the government cannot influence welfare through these margins, as, given a set of prices and policies, individuals make efficient decisions in a decentralized equilibrium. Second, instead, when considering welfare, the government accounts for three types of externalities: (i) the *income redistribution* through taxes and transfers, (ii) the *fiscal externalities* from reduced transfer sizes due to tax distortions, and (iii) the *pecuniary externalities* from factor price adjustments affecting factor income composition, as explored by Davila, Hong, Krusell and Ríos-Rull (2012).[3]

These externalities affect welfare by altering individuals' disposable income. For instance, an increase in the labor income tax rate distorts labor supply, reducing tax revenue and lump-sum transfers—the *fiscal externalities*—and lowering overall welfare. The other two externalities impact income groups differently. The *income redistribution* from increased transfers following higher labor income taxes benefits low-income individuals, but harms high-income individuals. The pecuniary externalities arise because, with capital held fixed, a reduction in labor supply—following an increase in the labor tax—raises the capital-to-labor ratio. This increases wages and lowers interest rates in equilibrium, benefiting (hurting) low- (high-) income individuals, whose income is biased toward labor (capital).

Lastly, in the theoretical analysis, I find that when making policy decisions, the government balances these externalities differently depending on its ability to commit. In particular, the role of pecuniary externalities differs significantly depending on the government's commitment technology. Without commitment, although the current government can fully control the other types of externalities—income redistribution and fiscal externalities—through the government budget constraint, it cannot exert full control over the pecuniary externalities, as some components are predetermined by the preceding government.

The pecuniary externalities consist of three components: relative individual labor supply, relative individual asset holdings, and the aggregate capital-to-labor ratio. With a utilitarian government, the pecuniary externalities improve welfare when the income composition of low-income individuals is more (less) biased toward labor (capital) and when the capital-to-labor ratio is higher, implying higher wages. Among these components, the current government influences only the component related to relative individual labor supply, while treating the other savings-related components as predetermined from the previous period. In contrast, the preceding government internalizes the successor's perspective and influences the successor's policy decisions by reshaping individual savings, which are determined during its own tenure. However, the successor government considers only forward-looking effects and not the impact of its policies on past outcomes.

With commitment, the government, equivalent to a Ramsey planner, internalizes the effects of future redistributive policies on past and present economic outcomes. Specifically, commitment allows the government to exploit the components of pecuniary externalities related to relative individual asset holdings and the aggregate capital-to-labor ratio by intertemporally coordinating taxes and transfers. It commits to increasing future transfers, thereby reducing precautionary savings and lowering the capital income share of low-income individuals in the current period. Additionally, by coordinating lower income taxes in the past and higher taxes in the current period, the government boosts the savings of middle- and high-income individuals shaped in the previous pe-

[3]They thoroughly investigated the pecuniary externalities on constrained efficiency in incomplete-markets models.

riod. This, in turn, raises the current capital-to-labor ratio, generating larger pecuniary externalities through higher wages. These findings imply that commitment aligns the allocation more closely with the constrained-efficient allocation in Davila et al. (2012) by accounting for both intertemporal consumption allocations and the pecuniary externalities, whereas without commitment, the government neglects the intertemporal allocation margins.

To examine the quantitative implications of the above theoretical findings, I extend the model to an infinite horizon with the Constant Relative Risk Aversion (CRRA) utility function. I calibrate the model to the U.S. economy as a starting point and compare the differences in taxes and transfers based on the availability of government commitment, in the spirit of Lucas and Stokey (1983).

These quantitative exercises show that government commitment—and the resulting time inconsistency—significantly affects the aggregate economy, inequality, and welfare. With commitment, the Ramsey planner sets higher labor income taxes and larger transfers over time than the government without commitment. Specifically, in the case with (without) commitment, the labor tax rate rises by 25 (15) percentage points, increasing transfers by 13.7 (5.4) percentage points as a share of initial GDP. While the case without commitment shows higher aggregate consumption, capital, and output, the case with commitment has lower inequality. The welfare gain, measured by consumption-equivalent variation, is larger under commitment (+2.19%) than without it (+1.46%), as the equity benefits outweigh the efficiency losses.

I find that with commitment, the Ramsey planner achieves front-loaded welfare gains through two forces. The first force is changes in factor prices that favor low-income individuals—higher wages and lower interest rates—early in the transition. Although substantial increases in taxes and transfers reduce both aggregate capital and labor, slower adjustments in capital cause temporarily higher capital-to-labor ratios, thereby generating the short-run *positive pecuniary externalities*.

The second force is substantial income redistribution early in the transition, driven by a rapid decline in income inequality resulting from reduced precautionary savings. The long-term provision of substantial transfers enables low-income individuals to reduce their precautionary savings early in the transition through the *retrospective distributional effects*, resulting in a decrease in the proportion of their capital income. Since capital income is more unequally distributed than labor income and lump-sum transfers, this shift leads to a rapid decline in after-tax income inequality, thereby facilitating redistribution toward low-income individuals. In contrast, the saving behavior of middle- and high-income individuals is primarily influenced by the substitution effect, with their capital income share closely tracking changes in the market interest rate.

In exchange for these front-loaded welfare gains, the government bears long-run welfare losses from stagnant income redistribution and negative pecuniary externalities—lower wages and higher interest rates.[4] The gap between short- and long-run welfare effects implies that the Ramsey

[4]This long-run immiseration is similar to the finding in the literature on endogenous incomplete markets with pri-

planner intertemporally coordinates policies to internalize the retrospective distributional effects, thereby balancing these externalities across periods.

Without commitment, this intertemporal balancing strategy is not credible because the government cannot commit to providing substantial long-term insurance. If the government without commitment stands in the long-run equilibrium of the commitment case, it disregards any upfront welfare gains and evaluates welfare only from the implementation period onward. Then, the government implements a one-time reduction in taxes and transfers, its only available option without commitment, in an attempt to improve welfare. Households rationally anticipate this government incentive and increase their precautionary savings, thereby deterring front-loaded reductions in income inequality. These changes mitigate the welfare trade-off between the short and long runs (+1.34 pp vs. +4.36 pp), resulting in a smaller short-run welfare gain than in the commitment case.

Finally, note that the global solution method for the case without commitment is applicable to a broader class of MPE games under incomplete financial markets, including those involving optimal policies or political processes. Besides their inherent complexity, as demonstrated by Krusell and Ríos-Rull (1999), such as the interplay of endogenous government policy functions, a key computational challenge is handling one-shot deviations that generate off-equilibrium paths over distributions, which must be fully specified to solve subgame-perfect Nash equilibria under incomplete markets. I address these computational challenges using a modified backward induction method from Reiter (2010), accommodating these features in MPE.

Related Literature: This paper belongs to the stream of macroeconomic literature that examines the implications of time-inconsistent features for government policies, following the seminal study of Kydland and Prescott (1977). A branch of this literature conducts this task by investigating the features of time-consistent policies in MPEs that depend on the fundamental economic state variables (Cohen and Michel, 1988; Krusell et al., 1996; Krusell and Ríos-Rull, 1996, 1999; Klein and Ríos-Rull, 2003; Klein et al., 2008; Corbae et al., 2009; Azzimonti, 2011; Song et al., 2012; Laczó and Rossi, 2020; Bassetto et al., 2024).[5] This paper contributes to the literature by examining the effects of government commitment on optimal policies in a standard incomplete-markets economy.

This paper is also related to macroeconomic studies on constrained efficiency in dynamic general equilibrium models with incomplete markets that focus on pecuniary externalities. For example, Davila et al. (2012) analyzes constrained efficiency in models presented in Aiyagari's (1994) model; Nuño and Moll (2018) conducts the same analysis in continuous space. Park (2018) conducts a similar analysis with human capital; and Itskhoki and Moll (2019) characterizes the opti-

vate information (Atkeson and Lucas, 1992), despite differences in the underlying sources of market incompleteness.

[5] Another branch of this literature has focused on designing policies to overcome time-inconsistency issues through the use of rules (Barro and Gordon, 1983; Rogoff, 1985), a richer range of policy instruments (Lucas and Stokey, 1983; Debortoli et al., 2017), and reputational equilibria (Chari and Kehoe, 1990; Domínguez, 2007a,b) using representative-agent models.

mal distribution of development policies between workers and entrepreneurs in an economy with financial constraints. This paper contributes to this literature by examining the interactions of these pecuniary externalities with government commitment.

This paper is relevant to a strand of literature that examines the optimality of taxes and transfers using heterogeneous-agent models (Conesa and Krueger, 2006; Conesa et al., 2009; Heathcote et al., 2017; Heathcote and Tsujiyama, 2021; Boar and Midrigan, 2022; Dyrda and Pedroni, 2023; Ferriere et al., 2023). They focus on the trade-offs between the progressivity or size of taxes and the size of transfers when designing optimal policies. This paper complements these studies by exploring the impacts of government commitment on these trade-offs.

The global solution method in this paper is an independent contribution to the literature. A method often used to solve these games is the approach of Phelan and Stacchetti (2001), which identifies the entire set of sustainable equilibria. However, it cannot be used to find the MPE. In continuous space, Dávila and Schaab (2023) solve a Ramsey planner's problem with finite-period commitment and view the Ramsey economy in the limit as the commitment period approaches zero as consistent with the MPE without commitment. In discrete space, models in MPE are often solved using either the local method of Klein et al. (2008), which is efficient but unsuitable for heterogeneous-agent models with incomplete markets, or simulation-based adaptations of Krusell and Smith's (1998) method, which are computationally costly in handling economies with incomplete markets.[6] The approach in this paper is more computationally feasible in discrete space, following a non-simulation-based solution method, as in Reiter (2010).

The remainder of this paper is organized as follows. Section 2 introduces a two-period model for analytical representation, defines the equilibria, and characterizes them using the GEE. Section 3 extends the model to an infinite-horizon framework for quantitative analysis. Section 4 outlines the calibration strategy and the core ideas of the numerical solution algorithm for the quantitative exercises. Section 5 presents the quantitative results of the policy analysis. Section 6 concludes the paper. Finally, Appendix B provides a detailed description of the numerical solution algorithm.

2 Two-Period Model

In this chapter, I conduct an analytical investigation of the time inconsistency problem related to government redistributive concerns, using a two-period version of an incomplete-markets model. In this model, given a tax-transfer policy, heterogeneous households make decisions on consumption, savings, and labor supply at the intensive margin, as in standard incomplete-markets models. A notable difference from the standard models is the manner in which the tax-transfer policy is

[6] To apply Krusell and Smith's (1998) method to economies in MPE, strong assumptions are often required. For example, Corbae et al. (2009) used this approach with the GHH preference, which reduces the computational burden.

set. It is determined endogenously, according to the government's commitment technology. In an equilibrium state, the tax-transfer policy, individual decisions, and the evolution of the distribution are all consistent with one another, given the government's commitment constraint.

2.1 Environment

The model economy is populated by a continuum of households with a two-period lifespan. Their preferences follow

$$E_1\left[\sum_{t=1}^{2}\beta^{t-1}u(c_t,l_t)\right] \tag{1}$$

where c_t is consumption, l_t is leisure in period t, and β is the discount factor. Preferences are represented by the form of CARA:

$$u(c_t,l_t) = -\frac{e^{-\sigma c_t}}{\sigma} - \chi e^{-\frac{l_t}{\chi}} \tag{2}$$

where σ is the coefficient of absolute risk aversion for consumption and $1/\chi$ is for leisure.

The CARA utility function is employed to leverage several theoretical advantages that work with the standard incomplete-markets economy framework. First, the CARA utility provides a closed-form solution for individual decisions under economies with incomplete financial markets, as demonstrated by Wang (2003); Park (2018); Acharya et al. (2023) in deriving their theoretical results. Second, the CARA utility generates a closed-form of the law of motion for household distribution. This closed-form law of motion is attainable by aggregating individual decisions rules because they are linear in individual state variables and their coefficients are identical across all individuals. Therefore, this closed-form law of motion enables an analytical representation as to how policies affects individuals' welfare through changes in household distributions. I rely on these theoretical advantages for my analytical examinations in the subsequent sections.

The representative firm produces output with constant returns to scale. The firm's technology is represented by

$$Y_t = zF(K_t,N_t) = zK_t^{\theta}N_t^{1-\theta} \tag{3}$$

where z is the total factor productivity (TFP), K_t is the quantity of aggregate capital, N_t is the quantity of aggregate labor, and θ is the capital income share. I assume full capital depreciation, $\delta = 1$, for the theoretical derivations in this two-period model.[7]

In each period, households confront an uninsurable, idiosyncratic shock on their leisure en-

[7]This assumption will be relaxed in the quantitative analysis later.

dowments ϵ_t, which follows:

$$log(\epsilon_t) \sim N(0, \sigma_\epsilon^2). \tag{4}$$

The labor supply of households n_t is subjected to ϵ_t such that $n_t = \epsilon_t - l_t$. Consequently, individuals with a higher ϵ_t can allocate longer hours to both work and leisure. These shocks to leisure endowment enable closed-form solutions for individual decisions for models with incomplete financial markets and endogenous labor supply, as shown by Acharya et al. (2023).

The government can obtain tax revenue from two sources over time: a lump-sum tax ($T_t < 0$) and a proportional labor income tax ($\tau_t > 0$). With the tax revenue, the government finances a constant level of government spending G. The government may implement lump-sum transfers $T_t > 0$ or lump-sum taxes $T_t < 0$. The government ensures a balanced budget in each period:

$$G + T_t = \tau_t w_t N_t. \tag{5}$$

This government budget is deliberately chosen to distinguish the time inconsistency problem driven by redistributive concerns from the canonical time inconsistency issues examined in representative-agent economies. First, by excluding a capital income tax, the government avoids the time inconsistency associated with taxing capital income (Rogers, 1987; Chari, 1988; Atkeson et al., 1999; Domínguez, 2007a). Second, despite the absence of government debt, the availability of lump-sum taxes makes the government problem unrelated to time inconsistency arising from limited instruments for managing government debt maturity (Lucas and Stokey, 1983). Finally, by assuming no initial debt, the government is freed from time inconsistency caused by a large initial debt exceeding the peak of the Laffer curve (Debortoli et al., 2021).

Note that, in the absence of individual heterogeneity, commitment technologies do not matter for the design of government policies: optimal is always a zero labor income tax rate ($\tau = 0$) with the full financing of G through a lump-sum tax ($T = -G$). However, when individual heterogeneity is introduced, the government has an incentive to implement positive lump-sum transfers $T_t > 0$, supported by a positive rate of distortionary labor income tax $\tau > 0$, due to redistributive concerns. Therefore, if government commitment technologies lead to some differences in the optimal design of labor income taxes and lump-sum transfers under this government budget, this divergence reflects a time-inconsistency problem arising not from the aforementioned reasons but from the government's redistributive concerns interacting with individual heterogeneity.

2.2 Competitive Equilibrium for the Two-Period Economy, Exogenous Policy

In this section, I define the competitive equilibrium, given an exogenous tax and transfer policy. I start with a setting to address the economy with commitment. To describe problems with commitment (the Ramsey problem), household dynamic problems need to be represented in a sequential manner. At the beginning of each period, households differ from one another in asset holdings a_t and leisure endowment ϵ_t. $\mu_t(a_t, \epsilon_t)$ denotes the distribution of households in period $t = 1, 2$. Given a sequence of factor prices $\{r_t, w_t\}_{t=1}^2$, labor income taxes $\{\tau_t\}_{t=1}^2$, and lump-sum taxes/transfers $\{T_t\}_{t=1}^2$, households solve

$$v(a_1, \epsilon_1) = \max_{\{c_t,\, n_t\}_{t=1}^2, a_2} u(c_1(a_1, \epsilon_1), \epsilon_1 - n_1(a_1, \epsilon_1)) + \beta E_{\epsilon_{i,2}} \Big[u(c_2(a_2, \epsilon_2), \epsilon_2 - n_2(a_2, \epsilon_2)) \Big] \tag{6}$$

such that

$$c_1 + a_2 = (1 - \tau_1) w_1 n_1 + r_1 a_1 + T_1$$
$$c_2 = (1 - \tau_2) w_2 n_2 + r_2 a_2 + T_2$$

where r_t is the rental rate of capital in period t, implying the gross return on savings is $1 + r_t - \delta = r_t$ under the assumption of full capital depreciation. In terminal period 2, individuals do not save and consume all disposable income. This economy lies in a sequential competitive equilibrium as follows.

Definition 2.1. ***Sequential Competitive Equilibrium (SCE) for the Two-Period Economy, Given a Sequence of Labor Income Taxes and Lump-sum Taxes/Transfers***
Given G, an initial distribution $\mu_1(\cdot)$, and a sequence of labor income taxes and lump-sum taxes/transfers $\{\tau_t, T_t\}_{t=1}^2$, an SCE is a sequence of factor prices $\{w_t, r_t\}_{t=1}^2$, a sequence of allocations $\{\{c_t, n_t, K_t, N_t\}_{t=1}^2,\ a_2\}$, a household value function $v(\cdot)$, and a distribution over the state space $\mu_2(\cdot)$, such that for all $t = 1, 2$

(i) Given $\{\tau_t, T_t\}_{t=1}^2$ and $\{w_t, r_t\}_{t=1}^2$, the decision rules $\left(\{c_t, n_t\}_{t=1}^2, a_2\right)$ solve the household problem in (6), and $v(a_1, \epsilon_1)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w_t = (1 - \theta) z \left(\frac{K_t}{N_t}\right)^{\theta} \tag{7}$$
$$r_t = \theta z \left(\frac{K_t}{N_t}\right)^{\theta - 1}. \tag{8}$$

(iii) The factor markets clear:

$$K_t = \int a_t \, \mu_t(\mathbf{d}(a_t \times \epsilon_t)) \tag{9}$$

$$N_t = \int n_t(a_t, \epsilon_t) \, \mu_t(\mathbf{d}(a_t \times \epsilon_t)) \tag{10}$$

(iv) The government budget constraint (5) is satisfied.

(v) Let $\mathcal{B}(A \times E)$ denote the Borel σ-algebra on $A \times E$. For any $B \in \mathcal{B}(A \times E)$, the sequence of distributions over individual $\{\mu_t(\cdot)\}_{t=1}^{2}$ satisfies

$$\mu_2(B) = \int_{\{(a_1,\epsilon_1)|(a_2(a_1,\epsilon_1),\epsilon_2)\in B\}} \left[\int f(\epsilon_2)\mathbf{d}(\epsilon_2) \right] \mu_1(\mathbf{d}(a_1 \times \epsilon_1)) \tag{11}$$

where $f(\cdot)$ is the probability density function of ϵ_2.

In contrast, to handle problems without commitment, it is convenient to present the household dynamic problems in a recursive manner. In addition to the individual state variables a_t and ϵ_t, there is an aggregate state variable,the distribution of households $\mu_t(a_t, \epsilon_t)$ over a_t and ϵ_t.

Let $v_t(a_t, \epsilon_t; \mu_t)$ denote the value of households associated with a state of $(a_t, \epsilon_t; \mu_t)$ in period t. In terminal period 2, they solve

$$v_2(a_2, \epsilon_2; \mu_2) = \max_{c_2>0,\, 0\leq n_2 \leq \epsilon_2} u(c_2, \epsilon_2 - n_2) \tag{12}$$

such that

$$c_2 = (1 - \tau_2)\, w_2(K_2, N_2)\, n_2 + r_2(K_2, N_2)\, a_2 + T_2$$

$$\tau_2 = \Psi_2(\mu_2).$$

where $\tau_2 = \Psi_2(\mu_2)$ is the perceived law of motion of labor taxes in period 2.[8]

In periods 1, households solve

$$v_1(a_1, \epsilon_1; \mu_1) = \max_{c_1>0,\, a_2\geq \underline{a},\, 0\leq n_1\leq \epsilon_1} u(c_1, \epsilon_1 - n_1) + \beta E_{\epsilon_{i,2}} v_2(a_2, \epsilon_2; \mu_2) \tag{13}$$

such that

$$c_1 + a_2 = (1 - \tau_1)\, w_1(K_1, N_1)\, n_1 + r_1(K_1, N_1)\, a_1 + T_1$$

$$\tau_1 = \Psi_1(\mu_1)$$

$$\mu_2 = \Gamma_1(\mu_1, \tau_1) = \Gamma_1(\mu_1, \Psi_1(\mu_1))$$

[8] Note that either τ or T needs to represented by a perceived law of motion because once one of them chosen, the other is fixed in the balanced government budget, $G + T_t = \tau_t w_t N_t$. Here, I choose τ_t as the perceived law of motion because this choice provides a better representation when GEE is applied.

where $\underline{a} \leq 0$ is a borrowing limit, and $\mu_2 = \Gamma_1(\mu_1, \tau_1)$ is the law of motion for the distribution over households. $\Gamma_1(\cdot, \cdot)$ has τ_1 as an input because individual savings decisions are influenced by the current labor income rate. Note that households here solve the above problem given an exogenous tax policy function, $\tau_1 = \Psi_1(\mu_1)$. This economy lies in a recursive competitive equilibrium as follows.

Definition 2.2. ***Recursive Competitive Equilibrium (RCE) for the Two-Period Model, Given a Law of Motion for Labor Income Tax.***
Given G, $\mu_1(a_1, \epsilon_1)$, and $\{\tau_t = \Psi_t(\mu_t)\}_{t=1}^2$, an RCE is a set of factor prices $\{w_t(\cdot), r_t(\cdot)\}_{t=1}^2$, a set of decision rules for households $a_2 = g_1^a(a_1, \epsilon_1; \mu_1)$ and $\{n_t = g_t^n(a_t, \epsilon_t; \mu_t)\}_{t=1}^2$, value functions $\{v_t(a_t, \epsilon_t; \mu_t)\}_{t=1}^2$, a distribution of households $\mu_2(a_2, \epsilon_2)$ over the state space, and the law of motion for the distribution of households $\mu_2 = \Gamma_1(\mu_1, \Psi_1(\mu_1))$ such that

(i) Given $\{w_t(K_t, N_t), r_t(K_t, N_t)\}$ and $\tau_t = \Psi_t(\mu_t)$, the decision rules $a_2 = g_1^a(a_1, \epsilon_1; \mu_1)$ and $n_t = g_t^n(a_t, \epsilon_t; \mu_t)$ solve the household problem in (12) and (13), and $v_t(a_t, \epsilon_t; \mu_t)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w_t(K_t, N_t) = (1-\theta)z\left(\frac{K_t}{N_t}\right)^{\theta} \tag{14}$$

$$r_t(K_t, N_t) = \theta z\left(\frac{K_t}{N_t}\right)^{\theta-1}. \tag{15}$$

(iii) The factor markets clear:

$$K_t = \int a_t\, \mu_t(\mathbf{d}(a_t \times \epsilon_t)) \tag{16}$$

$$N_t = \int g_t^n(a_t, \epsilon_t; \mu_t)\, \mu_t(\mathbf{d}(a_t \times \epsilon_t)) \tag{17}$$

(iv) The government budget constraint (5) is satisfied.

(v) Let $\mathcal{B}(A \times E)$ denote the Borel σ-algebra on $A \times E$. For any $B \in \mathcal{B}(A \times E)$, the sequence of distributions over individual $\{\mu_t(\cdot)\}_{t=1}^2$ satisfies

$$\mu_2(B) = \int_{\{(a_1,\epsilon_1)|(a_2(a_1,\epsilon_1),\epsilon_2)\in B\}} \left[\int f(\epsilon_2)\mathbf{d}(\epsilon_2)\right] \mu_1(\mathbf{d}(a_1 \times \epsilon_1)) \tag{18}$$

where $f(\cdot)$ is the probability density function of ϵ_2.

2.3 Competitive Equilibrium for the Two-Period Economy, Endogenous Policy

In this section, I define competitive equilibria where labor income taxes and lump-sum taxes/transfers are endogenously determined. I model the tax-transfer choice in two ways: the optimal labor income tax and lump-sum taxes/transfers with commitment (Ramsey problem) and the optimal labor income taxes and lump-sum taxes/transfers without commitment.

In both cases, the policy reform follows the same timing: a distribution of households in period 1 $\mu_1(\cdot,\cdot)$ is given. At the beginning of period 1, the government unexpectedly announces an optimal policy path for labor income taxes and lump-sum taxes/transfers, $\{\tau_t, T_t\}_{t=1}^2$. The government internalizes the effects of tax and transfer changes on overall welfare through individual budget constraints. Substituting the government budget (5), the individual budget constraint is given by

$$\begin{aligned} c_t + a_{t+1} &= (1-\tau_t)w_t n_t + r_t a_t + T_t \\ &= w_t n_t + r_t a_t + (\tau_t w_t (N_t - n_t) - G). \end{aligned} \tag{19}$$

The last term, $\tau_t w_t(N_t - n_t) - G$, highlights one of the government main policy concerns: income redistribution. Without this term, the household budget constraint is identical to the case without government intervention. Moreover, aside from the case of a lump-sum tax $T = -G$ with $\tau = 0$, $\tau_t w_t(N_t - n_t) - G$ suggests that the government, the objective function of which is concave, favors redistribution toward individuals with a lower n_t—corresponding to a lower ϵ_t. In other words, this term acts as an effective borrowing constraint managed by the government that benefits low-income individuals. These findings suggest that, given the limited policy instruments, $T_t + G = \tau_t w_t N_t$, the government uses tax-transfer policies as the management of individual borrowing constraints—favoring low-income individuals.

I analyze how this redistributive concern varies with the government's commitment technology by comparing economies with and without commitment. Going forward, I reformulate the government's problem as choosing only labor income tax rates, $\{\tau_t\}_{t=1}^2$, by substituting the government budget constraint (5) into the individual constraints, as in (19). I begin with the economy with commitment—the Ramsey problem.

Definition 2.3. ***The Ramsey Problem for the Two-Period Economy: An SCE with the Optimal Labor Income Taxes and Lump-Sum Taxes/Transfers with Commitment***

Given μ_1, the government chooses $\{\tau_t\}_{t=1}^2$ such that

$$\{\tau_t\}_{t=1}^2 = \underset{\{\tilde{\tau}_t \geq 0\}_{t=1}^2}{\operatorname{argmax}} \int E_1 \sum_{\hat{t}=1}^{2} \beta^{\hat{t}-1} u\big(c_{\hat{t}}(a_t, \epsilon_t | \{\tilde{\tau}_{\hat{t}}\}_{t=1}^2), \epsilon_{\hat{t}} - n_{\hat{t}}(a_t, \epsilon_t | \{\tilde{\tau}_{\hat{t}}\}_{t=1}^2)\big) \mu_1(\mathbf{d}(a_1 \times \epsilon_1))$$

$$\text{such that}$$

$$c_1 + a_2 = w_1\, n_1 + r_1 a_1 + [\tilde{\tau}_1 w_1 (N_1 - \, n_1) - G], \quad \text{and}$$

$$c_2 = w_2\, n_2 + r_2 a_2 + [\tilde{\tau}_2 w_2 (N_2 - \, n_2) - G]$$

where the allocations and prices constitute a competitive equilibrium as defined in Definition (2.1) in period $\hat{t}$, given $\{\tilde{\tau}_t, \tilde{T}_{\hat{t}} = \tilde{\tau}_{\hat{t}} w_{\hat{t}} N_{\hat{t}} - G\}_{\hat{t}=1}^2$.

Note that the decision rules for $\{c_t, n_t\}_{t=1}^2$ and $a_{t+1} = g_t^a(\cdot)$ are affected not only by the policy in period t but also by the entire sequence of $\{\tau_{\hat{t}}\}_{\hat{t}=1}^2$. Therefore, the decisions at time 2 are influenced by taxes and transfers in period 1, which might lead to time inconsistency issues.

For the case without commitment, I have employed the definition of the MPE in Klein et al. (2008).

Definition 2.4. ***An RCE with the Optimal Labor Income Taxes and Lump-Sum Taxes/Transfers without Commitment***

(i) A set of functions $\left(\{w_t(\cdot), r_t(\cdot), g_t^n(\cdot), v_t(\cdot)\}_{t=1}^2\,, g_1^a(\cdot), \Gamma_1(\cdot)\right)$ satisfies Definition (2.2).

(ii) For each t and μ_t, the government chooses $\psi_t(\mu_t)$ such that

$$\psi_t(\mu_t) = \underset{\tilde{\tau}_t}{\operatorname{argmax}} \int \hat{V}_t(a_t, \epsilon_t; \mu_t, \tilde{\tau}_t) \mu_t(\mathbf{d}(a_t \times \epsilon_t)) \tag{20}$$

$$\text{where}$$

$$\hat{V}_2(a_2, \epsilon_2; \mu_2, \tilde{\tau}_2) = \max_{c_2 > 0,\ 0 \leq n_2 \leq \epsilon_2} u(c_2, \epsilon_2 - n_2) \tag{21}$$

$$\text{such that}$$

$$c_2 = w_2(K_2, N_2) n_2 + r_2(K_2, N_2)\, a_2 + [\tilde{\tau}_2 w_2(K_2, N_2)(N_2 - n_2) - G]$$

$$\tau_2 = \tilde{\tau}_2, \quad \text{and}$$

$$\hat{V}_1(a_1, \epsilon_1; \mu_1, \tilde{\tau}_1) = \max_{c_1 > 0,\ a_2 \geq \underline{a}.\ 0 \leq n_1 \leq \epsilon_1} \Big[u(c_1, \epsilon_1 - n_1) + \beta E_{\epsilon_{i,2}} v_2(a_2, \epsilon_2; \mu_2) \Big] \tag{22}$$

$$\text{such that}$$

$$c_1 + a_2 = w_1(K_1, N_1) n_1 + r_1(K_1, N_1)\, a_1 + [\tilde{\tau}_1 w_1(K_1, N_1)(N_1 - n_1) - G]$$

$$\tau_1 = \tilde{\tau}_1, \ \text{and thereafter}\ \tau_2 = \Psi_2(\mu_2) \tag{23}$$

$$\mu_2 = \Gamma_1(\mu_1, \tilde{\tau}_1). \tag{24}$$

(iii) $a_2 = \hat{g}_1^a(a_1, \epsilon_1; \mu_1, \tilde{\tau}_1)$ and $\{n_t = \hat{g}_t^n(a_t, \epsilon_t; \mu_t, \tilde{\tau}_t)\}_{t=1}^2$ solve (21) and (22) at prices that clear markets and satisfy the government budget constraint, and Γ_1 is consistent with individual decisions and the stochastic process of ϵ_2.

(iv) For each t and μ_t, the policy outcome function satisfies $\psi_t(\mu_t) = \Psi_t(\mu_t)$.

The above government's solution to the problem is consistent with the sub-perfect Nash equilibrium obtained via backward induction. In the economy without commitment, the government implements a time-consistent optimal policy: a contemporary labor income tax rate is sequentially chosen for each period while maximizing its utilitarian welfare under this commitment constraint. The government cannot commit to the future tax rate from the next period. Thus, if the chosen tax rate $\tilde{\tau}_1$ deviates from the equilibrium tax policy function $\Psi_1(\cdot)$, future tax rates will return to follow the equilibrium tax policy function $\Psi_2(\cdot)$ because of the lack of commitment. (23) presents such a commitment constraint. Moreover, since changes in labor income taxes affect individual savings decisions $a_2 = g_1^a(\cdot)$, the law of motion for the distribution of households $\Gamma_1(\cdot)$ has to capture all the changes in the evolution of distributions caused by the deviation of the labor income tax from the equilibrium tax function, as shown in (24). In equilibrium, for each period t and aggregate state μ_t, the government's choice of the tax rate, $\psi_t(\mu_t)$, should be equal to the households' perceived tax function $\Psi_t(\mu_t)$, as presented in (iv).

Note that the first-period government can influence its successor's policy decision in period 2 by reshaping the distribution μ_2. For example, a change in τ_1 implemented by the first-period government affects individual savings a_2 and, consequently, the distribution μ_2, as represented by (24). This change in μ_2 then influences the second-period government's decision on τ_2, as shown in (20). However, because the second-period government takes the current wealth distribution, μ_2, as given, it does not internalize the impact of its policy choice for τ_2 on the first-period economy—an effect that is accounted for in the case with commitment. The subsequent section provides further details.

2.4 Characterization of the Equilibria for the Two-Period Model

In this section, I employ the GEE approach, introduced by Klein et al. (2008), to characterize the equilibria, thereby demonstrating the economic trade-offs considered by the government in making decisions on tax-transfer policies.

Specifically, I aim to deliver four points. First, in the government's GEE, all individual optimality conditions—related to decisions on savings and labor supply—cancel out because the economy lies in a decentralized competitive equilibrium. Under this setting, the government has no room to improve welfare through these individual decision margins. Second, instead, the government considers three types of externalities that reshape individual disposable income:

$$(1-\tau_t)w_t n_{i,t} + r_t a_{i,t} + T_t. \tag{25}$$

The first force is the *income redistribution* via changes in τ_t and T_t. The second force is the *fiscal externalities* through changes in the size of lump-sum transfers T caused by a reduction in tax revenue, $\tau_t w_t N_t$, following labor income tax distortions. The last force is the *pecuniary externalities* arising from shifts in the factor composition of individual income, driven by changes in w_t and r_t—general equilibrium effects. Third, these externalities differ across individuals, requiring the government to strike a balance between them when making policy decisions on taxes and transfers. Finally, government commitment alters how the pecuniary externalities are weighed in policy decisions over time, resulting in time inconsistency.

I will first analyze the case without commitment and then proceed to the case with commitment.

2.4.1 The Case without Commitment

For the case without commitment, the government's solution is consistent with the sub-perfect Nash equilibrium. I characterize this equilibrium employing the GEE approach that reveals the economic trade-offs through the government's FOC. To derive the FOC, I take the partial derivative of the government's value, represented by $\hat{V}_t$, with respect to the current labor income tax rate, $\tilde{\tau}_t$, in its vicinity of the equilibrium value τ_t.

Because the sub-perfect Nash equilibrium can be obtained using backward induction, I begin with the terminal-period problem:

$$\begin{aligned} 0 =& \frac{d}{d\tilde{\tau}_2}\Big|_{\tilde{\tau}_2=\tau_2} \int \hat{V}_2(a_2, \epsilon_2; \mu_2, \tilde{\tau}_2)\mu_2(d(a_2 \times \epsilon_2)) \\ =& \frac{d}{d\tilde{\tau}_2}\Big|_{\tilde{\tau}_2=\tau_2} \int u(\tilde{c}_2, \epsilon_2 - \tilde{n}_2)\mu_2(d(a_2 \times \epsilon_2)) \\ & \text{where} \\ & \tilde{c}_2 = \tilde{w}_2(K_2, \tilde{N}_2)\tilde{n}_2 + \tilde{r}_2(K_2, \tilde{N}_2)a_2 + \tilde{\tau}_2\tilde{w}_2(K_2, \tilde{N}_2)(\tilde{N}_2 - \tilde{n}_2) - G. \end{aligned} \tag{26}$$

Note that the tilde over individual decisions and factor prices indicates that the deviation of $\tilde{\tau}_2$ from its equilibrium value τ_2 causes them to differ from their equilibrium counterparts. n_2 and N_2 have a tilde because individual labor supply n_2 instantaneously adjusts to a change in τ_2, leading to a change in aggregate labor supply N_2. This adjustment in N_2, in turn, affects the factor prices w_2 and r_2, making them different from their equilibrium values, which are marked with a tilde. However, K_2 and a_2 do not have a tilde because they are predetermined from the previous period.

Going forward, I normalize the TFP z to 1 for clarity. Also I will simplify the notation by using sequential terms. For example, $y_{i,t}$ is referred to as the variable for y individual i in period

t. And I also assume that the substitution effect of labor supply dominates the income effect, as is standard in most macroeconomic applications. In the quantitative analysis, this dominance is required only at the aggregate level. However, in this section, for theoretical clarity, I impose a stronger assumption, as stated below.

Assumption 1. *The substitution effect of labor supply dominates the income effect. Under the CARA utility function, this condition implies*

$$\sigma > \frac{1}{\chi}. \tag{27}$$

The above assumption restricts the sign of $\frac{dn_{i,t}}{d\tau_t}$ to non-positive, as the effective wage, $(1-\tau_t)w_t$, decreases with an increase in τ_t.

The derivative of $\hat{V}_2(a_2, \epsilon_2; \mu_2, \tilde{\tau}_2)$ in $\tilde{\tau}_2$ at τ_2 is given by:

$$\begin{aligned}
\frac{d\hat{V}_{i,2}}{d\tilde{\tau}_2}\Big|_{\tilde{\tau}_2=\tau_2} &= u_{c,i,2} \cdot \frac{dc_{i,2}}{d\tau_2} + u_{l,i,2}\Big(-\frac{dn_{i,2}}{d\tau_2}\Big) && (28)\\
&= u_{c,i,2} \cdot \Big((N_2 - n_{i,2})(w_2 + \tau_2\frac{dw_2}{d\tau_2})\Big) + \underbrace{\Big(u_{c,i,2} \cdot (1-\tau_2)w_2 - u_{l,i,2}\Big)}_{\textit{Consumption-Leisure Optimal Condition}} \cdot \frac{dn_{i,2}}{d\tau_2} \\
&\quad + u_{c,i,2} \cdot (\frac{dw_2}{d\tau_2}n_{i,2} + \frac{dr_2}{d\tau_2}a_{i,2} + \tau_2 w_2 \frac{dN_2}{d\tau_2}) \\
&= u_{c,i,2} \cdot \Big((N_2 - n_{i,2})(w_2 + \tau_2\frac{dw_2}{d\tau_2}) + \frac{dw_2}{d\tau_2}n_{i,2} + \frac{dr_2}{d\tau_2}a_{i,2} + \tau_2 w_2 \frac{dN_2}{d\tau_2}\Big) && (29)
\end{aligned}$$

where $u_{c,i,t} = \frac{\partial u(c_{i,t}, l_{i,t})}{\partial c_{i,t}}$ and $u_{l,i,t} = \frac{\partial u(c_{i,t}, l_{i,t})}{\partial l_{i,t}}$.

The term $\Big(u_{c,i,2} \cdot (1-\tau_2)w_2 - u_{l,i,2}\Big) \cdot \frac{dn_{i,2}}{d\tau_2}$ is the product of the consumption-leisure optimal condition and the individual's labor response to a change in τ_2. This term is always zero because individual labor supply decisions lie either at an inner solution $(n_2 < \epsilon_2)$—where the consumption-leisure condition holds—or at the corner solution $(n_2 = \epsilon_2)$, where $\frac{dn_{i,2}}{d\tau_2} = 0$. Furthermore, $\frac{dw_2}{d\tau_2}n_{i,2} + \frac{dr_2}{d\tau_2}a_{i,2}$ is arranged to:

$$\begin{aligned}
\frac{dw_2}{d\tau_2}n_{i,2} + \frac{dr_2}{d\tau_2}a_{i,2} &= \Big(F_{NN,2} \cdot n_{i,2} + F_{KN,2} \cdot a_{i,2}\Big) \cdot \frac{dN_2}{d\tau_2} && (30)\\
&= \Big(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}\Big)\big(\frac{K_2}{N_2}\big)^{\theta}(-\theta)(1-\theta) \cdot \frac{dN_2}{d\tau_2} && (31)
\end{aligned}$$

where $F_{NN,t} = \frac{\partial^2 F(K_t, N_t)}{\partial N_t^2}$ and $F_{KN,t} = \frac{\partial^2 F(K_t, N_t)}{\partial N_t \partial K_t}$. Because F is homogeneous of degree 1, $F_{KN,t}K_t + F_{NN,t}N_t = 0$. (31) is obtained by substituting this condition into (30).

Then, substituting (31) into (29), $\frac{d\hat{V}_{i,2}}{d\tilde{\tau}_2}\Big|_{\tilde{\tau}_2=\tau_2}$ is given by:

$$\frac{d\hat{V}_{i,2}}{d\tilde{\tau}_2}\Big|_{\tilde{\tau}_2=\tau_2} = \underbrace{u_{c,i,2} \cdot \tau_2 w_2 \frac{dN_2}{d\tau_2}}_{\textit{Fiscal Externalities } \left[\textit{(-)}\right]}. + \underbrace{u_{c,i,2} \cdot (w_2 + \tau_2 \cdot F_{NN,2} \cdot \frac{dN_2}{d\tau_2})(N_2 - n_{i,2})}_{\textit{Income Redistribution } \left[\textit{(+) or (-)}\right]} \tag{32}$$
$$+ \underbrace{u_{c,i,2} \cdot \left(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}\right) \cdot \left(\frac{K_2}{N_2}\right)^{\theta} \cdot (-\theta)(1-\theta) \cdot \frac{dN_2}{d\tau_2}}_{\textit{Pecuniary Externalities } \left[\textit{(+) or (-)}\right]}$$

The change in the government's perceived value for each individual consists of three forces: (i) *fiscal externalities*, (ii) *income redistribution*, and (iii) *pecuniary externalities*. All three forces are weighted with the marginal utility of consumption, which is higher for low-income individuals. Therefore, the marginal utility terms suggest that the government prioritizes the interests of low-income individuals over those of other income groups.

The first type of force, the *fiscal externalities*, measures individual welfare changes resulting from decreases in lump-sum transfers caused by distortions in labor supply due to an increase in τ_2. An increase in τ affects the government budget, $\tau w N + G = T$, in two ways. Although it proportionally increases T given N, a rise in τ distorts individuals' labor supply, thereby reducing aggregate labor supply N and, consequently, the size of lump-sum transfers T. This force is always negative, but its magnitude is greater for low-income individuals whose marginal utility of consumption is high.

The second type of force, the *income redistribution*, measures individual welfare changes resulting from the difference between after-tax and before-tax incomes. Its impact varies across individuals and is particularly beneficial for low-income individuals with $n_{i,2} < N_2$. The extent of income redistribution increases with the relative individual labor supply compared to the aggregate labor supply, $N_2 - n_{i,2}$, given that $u_{c,i,2} > 0$ and $w_2 + \tau_2 F_{NN,2} \cdot \frac{dN_2}{d\tau_2} > 0$ for all individuals. Consequently, its effects differ across individuals and are favorable to low-income individuals $(n_2 < N_2)$.[9] This result is intuitive: low-income individuals experience an increase in after-tax income relative to their before-tax income when τ_2 and T_2 rise. In contrast, high-income individuals $(n_2 > N_2)$ face a reduction in after-tax income in response to an increase in τ_2 and T_2, leading to a negative welfare impact.

The last type of force, the *pecuniary externalities*, indicates individual welfare changes driven by variations in the factor composition between capital and labor income, caused by changes in w_2 and r_2—general equilibrium effects—following a change in τ_2.[10] The impact of pecuniary

[9] $\frac{dN_2}{d\tau_2} < 0$ holds when the substitute effect of leisure outweighs its income effect, as assumed here.

[10] Davila et al. (2012) conducts a thorough investigation for pecuniary externalities in a centralized economy. My paper shows that this force appears in a decentralized economy.

externalities differs across individuals and is favorable for low-income individuals whose income composition is biased toward labor, $\frac{n_{i,2}}{N_2} > \frac{a_{i,2}}{K_2}$. Given that $u_{c,i,2} > 0$, $(K_2/N_2)^\theta > 0$ and $(-\theta)(1-\theta)\frac{dN_2}{d\tau_2} > 0$ for all individuals, the extent of pecuniary externalities is positively related to the degree of income biased to labor $\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}$; therefore, they have positive welfare implications for those with $\frac{n_{i,2}}{N} > \frac{a_{i,2}}{K_2}$, which are prevalent among low-income individuals.

The economic intuition is as follows. For example, in response to an increase in τ_2, individuals reduce their labor supply $n_{i,2}$, leading to a decrease in aggregate labor supply N_2 while K_2 remains unchanged. This shift increases the capital-to-labor ratio (K_2/N_2), thereby raising w_2 while reducing r_2. This general equilibrium effect has a positive welfare impact on low-income individuals whose income composition is biased toward labor.

Note that, in contrast to the other two forces, pecuniary externalities are not fully controlled by the government in period 2. These externalities consist of three components: those related to relative labor supply, $n_{i,2}/N_2$; relative asset holdings, $a_{i,2}/K_2$; and the aggregate capital-to-labor ratio, K_2/N_2. Since individual asset holdings $a_{i,2}$ and the aggregate capital stock K_2 are predetermined in period 1, the second-period government, when making policy decisions, considers that it influences only the component related to relative labor supply, $n_{i,2}/N_2$. The first-period government anticipates how its successor will treat $a_{i,2}$ and K_2 and thus influences the successor's policy decisions through its impact on the distribution shaped by individuals' savings in period 1. However, due to the lack of commitment, the second-period government does not internalize the effects of its policy decisions on economic outcomes in period 1, an effect that would be taken into account under commitment. Further details will be provided in the following paragraphs.

(26) and (32) imply the following proposition.

Proposition 1. *In the terminal period* $(t = 2)$, *the government without commitment makes a decision in* (τ_2, T_2) *satisfying*

$$
\begin{aligned}
&- \underbrace{\tau_2 w_2 \frac{dN_2}{d\tau_2} \cdot \int [u_{c,i,2}] \mu_2(d(a_{i,2} \times \epsilon_{i,2}))}_{\text{Aggregate Fiscal Externalities}} \\
&= \underbrace{(w_2 + \tau_2 \cdot F_{NN,2} \cdot \frac{dN_2}{d\tau_2}) \int [u_{c,i,2} \cdot (N_2 - n_{i,2})] \mu_2(d(a_{i,2} \times \epsilon_{i,2}))}_{\text{Aggregate Income Redistribution}} \\
&+ \underbrace{(-\theta)(1-\theta) \cdot \frac{dN_2}{d\tau_2} \cdot \left(\frac{K_2}{N_2}\right)^\theta \int \left[u_{c,i,2}\left(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}\right)\right] \mu_2(d(a_{i,2} \times \epsilon_{i,2}))}_{\text{Aggregate Pecuniary Externalities}}.
\end{aligned}
$$

Since the government places greater weight on the welfare of low-income individuals, who have a large $u_{c,i,2}$, the income redistribution and the pecuniary externalities, which benefit low-income individuals, are considered benefits of raising τ_2 and T_2, while a reduction in lump-sum transfers due to the fiscal externalities is regarded as a cost. Thus, as Proposition 1 shows, the government balances the total benefit of increasing these variables—*aggregate income redistribution* and *aggregate pecuniary externalities*—against the total cost—*aggregate fiscal externalities*. Furthermore, Proposition 1 implies the following:

Corollary 1. *In the terminal period ($t = 2$), the government without commitment makes a policy decision as follows:*

1. *If aggregate fiscal externalities are **greater** than the sum of aggregate income redistribution and aggregate pecuniary externalities, the government **reduces** τ_2 and T_2.*
2. *If aggregate fiscal externalities are **less** than the sum of aggregate income redistribution and aggregate pecuniary externalities, the government **increases** τ_2 and T_2.*
3. *As the asset holdings and labor supply of low-income individuals approach their averages, the government **reduces** τ_2 and T_2.*
4. *In the absence of individual heterogeneity ($n_{i,2} = N_2$ and $a_{i,2} = K_2, \forall i$), the government sets the labor income tax rate to zero, $\tau_2 = 0$, with a lump-sum tax of $T_2 = -G$.*

In period 1, the government's policy decision satisfies

$$\begin{aligned}
0 =& \frac{d}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} \int \hat{V}_1(a_1, \epsilon_1; \mu_1, \tilde{\tau}_1)\mu_1(d(a_1 \times \epsilon_1)) \\
=& \frac{d}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} \int \Big[u(\tilde{c}_1, \epsilon_1 - \tilde{n}_1) + \beta E_{\epsilon_{i,2}} v_2(\tilde{a}_2, \epsilon_2; \tilde{\mu}_2)\Big]\mu_1(d(a_1 \times \epsilon_1)) \\
& \text{where} \\
& \tilde{c}_1 + \tilde{a}_2 = \tilde{w}_1(K_1, \tilde{N}_1)\tilde{n}_1 + \tilde{r}_1(K_1, \tilde{N}_1)a_1 + \tilde{\tau}_1\tilde{w}_1(K_1, \tilde{N}_1)(\tilde{N}_1 - \tilde{n}_1) - G \\
& \tau_1 = \tilde{\tau}_1 \text{ and thereafter } \tau_2 = \Psi(\tilde{\mu}_2) \\
& \mu_2 = \tilde{\mu}_2 = \Gamma(\mu_1, \tilde{\tau}_1)
\end{aligned} \tag{33}$$

As in the terminal period ($t = 2$), the tilde over individual decisions and factor prices indicates that the deviation of $\tilde{\tau}_1$ from its equilibrium value τ_1 causes these variables to differ from their equilibrium counterparts. The derivative of $\hat{V}_1(a_1, \epsilon_1; \mu_1, \tilde{\tau}_1)$ in $\tilde{\tau}_1$, $\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}$, at τ_1 is given by:

$$
\begin{aligned}
\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} = \;& \underbrace{u_{c,i,1} \cdot (w_1 + \tau_1 F_{NN,1} \cdot \frac{\partial N_1}{\partial \tau_1})(N_1 - n_{i,1})}_{\text{Income Redistribution}} + \underbrace{u_{c,i,1} \cdot \left(\frac{n_{i,1}}{N_1} - \frac{a_{i,1}}{K_1}\right) \cdot \left(\frac{K_1}{N_1}\right)^{\theta} \cdot (-\theta)(1-\theta)\frac{\partial N_1}{\partial \tau_1}}_{\text{Pecuniary Externalities}} \\
& + \underbrace{u_{c,i,1} \cdot \tau_1 w_1 \frac{\partial N_1}{\partial \tau_1}}_{\textit{Fiscal Externalities}} + \underbrace{(u_{c,i,1}(1-\tau_1)w_1 - u_{l,i,1})}_{\text{Consumption-Leisure Optimal Condition}} \frac{\partial n_{i,1}}{\partial \tau_1} \\
& + \underbrace{(-u_{c,i,1} + \beta E_{\epsilon_{i,2}} r_2(K_2, N_2) u_{c,i,2})}_{\text{Consumption Euler Equation}} \frac{\partial a_{i,2}}{\partial \tau_1} + \underbrace{\beta E_{\epsilon_{i,2}}\left[\frac{dv_{i,2}}{d\mu_2} \cdot \frac{\partial \Gamma_1(\mu_1, \tau_1)}{\partial \tau_1}\right]}_{\text{Expected Distributional Effect}} \qquad (34)
\end{aligned}
$$

Many features of the terminal period's solution are repeated in the case of the second period. As in (32), the first three forces—*income redistribution*, *pecuniary externalities*, and *fiscal externalities*—arise from individual welfare changes following a change in τ_1. In addition, the product of the consumption-leisure optimal condition and the change in individual labor supply in response to a change in τ_1—$(u_{c,i,1}(1-\tau_1)w_1 - u_{l,i,1}) \cdot \frac{\partial n_{i,1}}{\partial \tau_1}$—appears. This term implies that this MPE encompasses a competitive equilibrium. Recall that its value is always equal to zero because this term accounts for both inner and corner solutions for labor supply.

In contrast to the terminal period case, the presence of a subsequent period introduces two additional dynamic terms. The first term is the product of the consumption Euler equation and the change in individual savings due to a change in τ_1: $-u_{c,i,1} + \beta E_{\epsilon_{i,2}} r_2(K_2, N_2) u_{c,i,2}$. Its value is always zero because this product accounts for both interior and corner solutions for individual savings, just as the product term associated with the consumption-leisure optimality condition does. The disappearance of these individual optimality conditions occurs because the economy is decentralized and in competitive equilibrium, leaving the government with no room to influence individual welfare through these margins. Therefore, $\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1}$ can be rewritten as:

$$
\begin{aligned}
\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} = \;& \underbrace{u_{c,i,1} \cdot (w_1 + \tau_1 F_{NN,1} \cdot \frac{\partial N_1}{\partial \tau_1})(N_1 - n_{i,1})}_{\text{Income Redistribution}} + \underbrace{u_{c,i,1} \cdot \left(\frac{n_{i,1}}{N_1} - \frac{a_{i,1}}{K_1}\right) \cdot \left(\frac{K_1}{N_1}\right)^{\theta} \cdot (-\theta)(1-\theta)\frac{\partial N_1}{\partial \tau_1}}_{\text{Pecuniary Externalities}} \qquad (35) \\
& + \underbrace{u_{c,i,1} \cdot \tau_1 w_1 \frac{\partial N_1}{\partial \tau_1}}_{\text{Fiscal Externalities}} + \underbrace{\beta E_{\epsilon_{i,2}}\left[\frac{dv_{i,2}}{d\mu_2} \cdot \frac{\partial \Gamma_1(\mu_1, \tau_1)}{\partial \tau_1}\right]}_{\text{Expected Distributional Effect}}.
\end{aligned}
$$

The second dynamic term—the *expected distributional effect*— measures individual welfare changes driven by variations in the future expected value $v_{i,2}$ due to changes in the future distribution μ_2 following a change in τ_1. Computing the derivative of $v_{i,2}$ with respect to μ_2 is generally intractable. However, under the CARA utility, the distributional effects can be restricted to the impacts on the first-order moment of the distribution, K_2, because the individual savings decision rule $a_{i,2}$ is linear in $a_{i,1}$ and $\epsilon_{i,1}$, with identical coefficients across all individuals.[11] Consequently, higher-order moments of the distribution μ_2 are unaffected by changes in τ_1. These properties imply that the sum of the closed-form decision rules across individuals exactly determines the law of motion for the first-order moment K_2. More specifically,

Lemma 1. *Under the CARA preference setting, the individual decision rules for savings, $a_2 = g_1^a(a_1, \epsilon_1)$ are linear in assets, a_1, and idiosyncratic shock, ϵ_1, with identical coefficients across individuals. That is,*

$$a_2 = g_1^a(a_1, \epsilon_1) = \kappa_a(\tau_1) \cdot a_1 + \kappa_\epsilon(\tau_1) \cdot \epsilon_1 + Q(\tau_t)$$

where, given τ_1, $\kappa_a(\tau_1)$, $\kappa_\epsilon(\tau_1)$, and $Q(\tau_1)$ are constants and depend on τ_1. Hence,

$$K_2 = \Gamma_1(K_1, \tau_1) = \kappa_a(\tau_1) \cdot K_t + \kappa_\epsilon(\tau_1) \cdot e^{\frac{\sigma_\epsilon^2}{2}} + Q(\tau_1).$$

Proof. See Online Appendix A. □

And Lemma 1 results in the following:

Corollary 2. *Under the CARA preference setting, the future distributional effect can be fully captured by the first-order moment effects. That is,*

$$\frac{dv_2(a_2, \epsilon_2; \mu_2)}{d\mu_2} \cdot \frac{\partial \Gamma_1(\mu_1, \tau_1)}{\partial \tau_1} = \frac{dv_2(a_2, \epsilon_2; K_2)}{dK_2} \cdot \frac{\partial \Gamma_1(K_1, \tau_1)}{\partial \tau_1}.$$

Because Proposition 1 implies that changes in its first-order moment can represent the law of motion for household distributions, the *expected distributional effect* can also be measured by the impact of changes in the first-order moment of the distribution on individual values.

[11] Of course, N_t is required to determine factor prices, which influence individual welfare. Going forward, I will account for the equilibrium effects of changes in N_t. However, for notational purposes, I do not include N_t as an argument of μ_t, since it is not an aggregate state variable. Instead, N_t is the sum of all individual n_t, which is not a state variable but a choice variable determined contemporaneously in equilibrium.

Then, $\frac{dv_{i,2}}{dK_2}$ is given by:

$$\begin{aligned}\frac{dv_{i,2}}{dK_2} &= \frac{\partial v_{i,2}}{\partial K_2} + \frac{\partial v_{i,2}}{\partial \tau_2} \cdot \frac{\partial \Psi_2}{\partial K_2} \qquad (36)\\ &= u_{c,i,2} \cdot \left[\left(\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} \right) \cdot n_{i,2} + \left(\frac{\partial r_2}{\partial K_2} + \frac{\partial r_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} \right) \cdot a_{i,2} \right.\\ &\left. + \tau_2 \left(\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} \right) (N_2 - n_2) + \tau_2 w_2 \frac{\partial N_2}{\partial K_2} \right] + \frac{\partial v_{i,2}}{\partial \tau_2} \cdot \frac{\partial \Psi_2}{\partial K_2}\\ &+ \underbrace{\Big(- u_{l,i,2} + (1-\tau_2) u_{c,i,2} w_2 \Big)}_{\text{Consumption-Leisure Optimality Condition}} \cdot \frac{\partial n_{i,2}}{\partial K_2}. \qquad (37)\end{aligned}$$

As discussed earlier, the last product term associated with the consumption-leisure optimality condition is zero because both inner and corner solutions for labor supply choice apply. Furthermore, the expected pecuniary externalities—$\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2}$ and $\frac{\partial r_2}{\partial K_2} + \frac{\partial r_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2}$—have a zero value.

Lemma 2. $\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = \frac{\partial r_2}{\partial K_2} + \frac{\partial r_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = 0$ *under the Cobb-Douglas production function.*

Proof. See Online Appendix A. □

Lemma 2 implies that the government does not consider welfare changes caused by relative variations in the expected factor price, w_2 and r_2—the *expected pecuniary externalities*—as their relative variations cancel out under the Cobb-Douglas production function.[12] Lemma 2 streamlines (37) as follows:

$$\frac{dv_{i,2}}{dK_2} = u_{c,i,2} \cdot \tau_2 \, w_2 \, \frac{\partial N_2}{\partial K_2} + \frac{\partial v_{i,2}}{\partial \tau_2} \cdot \frac{\partial \Psi_2}{\partial K_2}. \qquad (38)$$

By taking similar steps in (32), $\frac{\partial v_{i,2}}{\partial \tau_2}$ is given by

$$\begin{aligned}\frac{\partial v_{i,2}}{\partial \tau_2} &= u_{c,i,2} \cdot \tau_2 w_2 \frac{dN_2}{d\tau_2} + u_{c,i,2} \cdot (N_2 - n_{i,2}) \cdot (w_2 + \tau_2 \cdot F_{NN,2} \cdot \frac{dN_2}{d\tau_2})\\ &+ u_{c,i,2} \cdot \left(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2} \right) \cdot N_2 \cdot F_{NN,2} \cdot \frac{dN_2}{d\tau_2} \qquad (39)\end{aligned}$$

Substituting (39) into (38), the expected distributional effect, $\beta E_{\epsilon_{i,2}} \left[\frac{dv_{i,2}}{dK_2} \cdot \frac{\partial \Gamma_1(K_1, \tau_1)}{\partial \tau_1} \right]$ is given

[12] It is worth noting that this canceling-out relies on the production function setting. *Expected pecuniary externalities* may play crucial roles under other general production settings.

by:

$$\beta E_{\epsilon_{i,2}}\left[\frac{dv_{i,2}}{dK_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}\right]= \tag{40}$$

$$\beta E_{\epsilon_{i,2}}\Bigg[\underbrace{u_{c,i,2}\cdot\tau_2\cdot w_2\cdot\Big(\frac{\partial N_2}{\partial K_2}+\frac{dN_2}{d\tau_2}\cdot\frac{\partial\Psi_2}{\partial K_2}\Big)}_{\textit{Expected Fiscal Externalities}}+\underbrace{u_{c,i,2}\cdot(N_2-n_{i,2})\cdot\Big(w_2+\tau_2\cdot F_{NN,2}\cdot\frac{dN_2}{d\tau_2}\Big)\cdot\frac{\partial\Psi_2}{\partial K_2}}_{\textit{Expected Income Redistribution}}$$
$$+\underbrace{u_{c,i,2}\cdot\Bigg[\Big(\frac{n_{i,2}}{N_2}-\frac{a_{i,2}}{K_2}\Big)\cdot\Big(\frac{K_2}{N_2}\Big)^{\theta}\cdot(-\theta)(1-\theta)\cdot\frac{dN_2}{d\tau_2}\Bigg]\cdot\frac{\partial\Psi_2}{\partial K_2}}_{\textit{Expected Pecuniary Externalities}}\Bigg]\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}.$$

To explore welfare implications in (40), the signs of $\frac{\partial N_2}{\partial K_2}$, $\frac{\partial\Psi_2}{\partial K_2}$, and $\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}$ are required. The term $\frac{\partial N_2}{\partial K_2}$ is positive when the substitution effect on labor supply dominates the income effect, as stated in Assumption 1. Specifically, $\frac{\partial N_2}{\partial K_2}$ can be decomposed into $\frac{\partial N_2}{\partial w_2}\cdot\frac{\partial w_2}{\partial K_2}$, which is positive with Assumption 1 because $\frac{\partial w_2}{\partial K_2}$ is positive in general equilibrium. In addition, $\frac{\partial\Psi_2}{\partial K_2}$ is negative because a stronger wealth effect diminishes disparities in the marginal utility of consumption, thereby reducing the need for redistribution through taxes and transfers. The sign of $\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}$ is revealed through the following Lemma:

Lemma 3. $\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}<0$ *when savings driven by precautionary motives and patience are sufficiently large. Specifically,* β *and* σ_ϵ^2 *must satisfy*

$$\Delta(\beta,\sigma_\epsilon^2)>r_2\Big(r_1K_1+T_1+\chi(1-\tau_1)\ln[(1-\tau_1)w_1]\Big)$$

where

$$\Delta(\beta,\sigma_\epsilon^2)=\Big(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\Big)\ln(\beta r_2)+\frac{1}{2}\cdot\frac{\sigma((1-\tau_2)w_2)^2\sigma_\epsilon^2}{1+(1-\tau_2)w_2\sigma\chi}+r_2(1-\tau_1)w_1e^{\sigma_\epsilon^2}.$$

Proof. See Online Appendix A. □

The logic behind $\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}<0$ is as follows. In response to an increase in τ_1, most individuals experience a reduction in permanent labor income and its after tax uncertainty, $(1-\tau_1)w_1n_1$, thereby reducing their permanent and precautionary savings a_2, in turn, aggregate capital stock K_2. As a result, the product term $\frac{\partial\Psi_2}{\partial K_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}$ is positive, reflecting an increase in future taxes and transfers to offset larger differences in marginal utility caused by reduced savings following a higher current-period tax.

$\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1}$ can be arranged as:

$$
\begin{aligned}
\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} = \; & \underbrace{u_{c,i,1}\cdot\tau_1 w_1\frac{\partial N_1}{\partial\tau_1}}_{\textit{Fiscal Externalities}} + \underbrace{u_{c,i,1}\cdot(w_1+\tau_1 F_{NN,1}\cdot\frac{\partial N_1}{\partial\tau_1})(N_1-n_{i,1})}_{\textit{Income Redistribution}} \\
& + \underbrace{u_{c,i,1}\Big(\frac{n_{i,1}}{N_1}-\frac{a_{i,1}}{K_1}\Big)\Big(\frac{K_1}{N_1}\Big)^{\theta}(-\theta)(1-\theta)\frac{\partial N_1}{\partial\tau_1}}_{\textit{Pecuniary Externalities}} + \underbrace{\beta E_{\epsilon_{i,2}}\Big[u_{c,i,2}\cdot\tau_2 w_2\cdot\frac{\partial N_2}{\partial K_2}\Big]\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}}_{\textit{Expected Fiscal Externalities due to Substitution Effect}} \\
& \beta E_{\epsilon_{i,2}}\Bigg[\underbrace{u_{c,i,2}\cdot\tau_2 w_2\cdot\frac{dN_2}{d\tau_2}}_{\textit{Expected Fiscal Externalities through the Successor}} + \underbrace{u_{c,i,2}\cdot(N_2-n_{i,2})\cdot\Big(w_2+\tau_2\cdot F_{NN,2}\cdot\frac{dN_2}{d\tau_2}\Big)}_{\textit{Expected Income Redistribution through the Successor}} \\
& + \underbrace{u_{c,i,2}\cdot\Big[\Big(\frac{n_{i,2}}{N_2}-\frac{a_{i,2}}{K_2}\Big)\cdot\Big(\frac{K_2}{N_2}\Big)^{\theta}\cdot(-\theta)(1-\theta)\cdot\frac{dN_2}{d\tau_2}\Big]}_{\textit{Expected Pecuniary Externalities through the Successor}}\Bigg]\cdot\frac{\partial\Psi_2}{\partial K_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}. \qquad (41)
\end{aligned}
$$

The expected fiscal externalities driven by the substitution effect reduce the welfare of all individuals, since $\frac{\partial N_2}{\partial K_2} > 0$ and $\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1} < 0$. This mechanism reflects a dynamic inefficiency: lower aggregate savings reduce total labor supply—due to the dominant substitution effect—which, in turn, diminishes government tax revenue and transfers in the next period.

More interestingly, the last three expected externalities-related terms associated with $\frac{\partial\Psi_2}{\partial K_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}$ capture how the first-period government influences the second-period government through a distributional channel. Specifically, the first-period government affects the successor government's policy decisions by shaping the period-2 wealth distribution, which is determined by individual savings decisions made in period 1. This individual savings-related motive operates through the last term of (41): the expected pecuniary externalities transmitted through the successor government.

As mentioned earlier, although the pecuniary externalities in period 2 play a role in improving welfare when low-income individuals hold a smaller share of capital income, the second-period government treats the distribution of $a_{i,2}$ as given, inherited from the previous period. As a result, the second-period government can only exploit pecuniary externalities associated with relative labor supply margins, $n_{i,2}/N_2$. Recognizing this limitation, the first-period government uses taxes and transfers in period 1 to induce low-income individuals to hold less capital income in period 2 (i.e., to reduce a_2/K_2) by reducing their precautionary savings. In addition, the first-period government has an incentive to increase the second-period capital-to-labor ratio, K_2/N_2, through its policy instruments, thereby raising the wage rate in period 2. However, the inverse channel does not operate: without commitment, the second-period government cannot internalize the impact of its policy choices on the distribution inherited from period 1, as shown in (32).

Note that the three expected externality-related terms associated with $\frac{\partial\Psi_2}{\partial K_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}$ are equal to $\frac{d\hat{V}_{i,2}}{d\tau_2}$. Thus, $\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1}$ can be rewritten as:

$$\begin{aligned}\frac{d\hat{V}_{i,1}}{d\tilde{\tau}_1}\Big|_{\tilde{\tau}_1=\tau_1} = & \underbrace{u_{c,i,1}\cdot\tau_1 w_1\frac{\partial N_1}{\partial \tau_1}}_{\textit{Fiscal Externalities}} + \underbrace{u_{c,i,1}\cdot(w_1+\tau_1 F_{NN,1}\cdot\frac{\partial N_1}{\partial \tau_1})(N_1-n_{i,1})}_{\textit{Income Redistribution}} \\ & + \underbrace{u_{c,i,1}\Big(\frac{n_{i,1}}{N_1}-\frac{a_{i,1}}{K_1}\Big)\Big(\frac{K_1}{N_1}\Big)^{\theta}(-\theta)(1-\theta)\frac{\partial N_1}{\partial \tau_1}}_{\textit{Pecuniary Externalities}} + \underbrace{\beta E_{\epsilon_{i,2}}\Big[u_{c,i,2}\cdot\tau_2 w_2\cdot\frac{\partial N_2}{\partial K_2}\Big]\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}}_{\textit{Expected Fiscal Externalities due to Substitution Effect}} \\ & \underbrace{\beta E_{\epsilon_{i,2}}\Big[\frac{d\hat{V}_{i,2}}{d\tau_2}\Big]\cdot\frac{\partial\Psi_2}{\partial K_2}\cdot\frac{\partial\Gamma_1(K_1,\tau_1)}{\partial\tau_1}}_{\textit{Expected Externalities via the Successor}} \end{aligned} \tag{42}$$

Recall that the sum of the expected externalities via the successor government across individuals in period 2 indicates zero in MPE, as shown in (26). For the subsequent discussion, let me introduce some notations. I refer to $\mathcal{L}_{i,t}$ as $\frac{\partial v_{i,t}}{\partial K_t} = u_{c,i,t}\cdot\Big[\tau_t w_t\frac{\partial N_t}{\partial K_t}\Big]$. Then, $E_{\epsilon_{i,t+1}}[\mathcal{L}_{i,t+1}]\cdot\frac{\partial\Gamma_t(K_t,\tau_t)}{\partial\tau_t} = E_{\epsilon_{i,t+1}}\Big[u_{c,i,t+1}\cdot\tau_{t+1}w_{t+1}\frac{\partial N_{t+1}}{\partial K_{t+1}}\Big]\cdot\frac{\partial\Gamma_t(K_t,\tau_t)}{\partial\tau_t}$ indicates *individual's expected net loss* of τ_t:

$$E_{\epsilon_{i,t+1}}[\mathcal{L}_{i,t+1}]\cdot\frac{\partial\Gamma_t(K_t,\tau_t)}{\partial\tau_t} = \underbrace{E_{\epsilon_{i,t+1}}\Big[u_{c,i,t+1}\cdot\tau_{t+1}w_{t+1}\frac{\partial N_{t+1}}{\partial K_{t+1}}\Big]\cdot\frac{\partial\Gamma_t(K_t,\tau_t)}{\partial\tau_t}}_{\textit{Expected Fiscal Externalities due to Substitution Effect }\big[(\text{-})\big]}. \tag{43}$$

Let $G_{i,t}$ denote *individual contemporary net gain* of τ_t:

$$\mathcal{G}_{i,t} = \underbrace{u_{c,i,t}\cdot(w_t+\tau_t F_{NN,t}\cdot\frac{\partial N_t}{\partial\tau_t})(N_t-n_{i,t})}_{\textit{Income Redistribution }\big[(+)\text{ or }(\text{-})\big]} + \underbrace{u_{c,i,t}\cdot\Big(\frac{n_{i,t}}{N_t}-\frac{a_{i,t}}{K_t}\Big)\Big(\frac{K_t}{N_t}\Big)^{\theta}(-\theta)(1-\theta)\frac{\partial N_t}{\partial\tau_t}}_{\textit{Pecuniary Externalities }\big[(+)\text{ or }(\text{-})\big]} + \underbrace{u_{c,i,t}\cdot\tau_t w_t\frac{\partial N_t}{\partial\tau_t}}_{\textit{Fiscal Externalities }\big[(\text{-})\big]}.$$

Utilizing (32) and (42), the GEEs, (26) and (33) , generates the following proposition.

Proposition 2. *When making decisions on $\{\tau_t, T_t\}_{t=1}^{2}$, the government without commitment satisfies the following conditions:*

$$\begin{aligned}\underbrace{\int\mathcal{G}_{i,t}\ \mu_t(d(a_{i,t}\times\epsilon_{i,t}))}_{\textit{Aggregate Contemporary Net Gain of }\tau_t} &= \underbrace{-\int\beta E_{\epsilon_{i,t+1}}\Big[\mathcal{L}_{i,t+1}\Big]\cdot\frac{\partial\Gamma_t(K_t,\tau_t)}{\partial\tau_t}\mu_t(d(a_{i,t}\times\epsilon_{i,t}))}_{\textit{Aggregate Expected Net Loss for }\tau_t},\ \textit{for } t=1 \\ \underbrace{\int\mathcal{G}_{i,t}\ \mu_t(d(a_{i,t}\times\epsilon_{i,t}))}_{\textit{Aggregate Contemporary Net Gain of }\tau_t} &= 0,\ \textit{for } t=2.\end{aligned}$$

Proposition 2 suggests that the government strikes a balance between the *current* marginal benefit of raising τ_t—*aggregate contemporary net gain for* τ_t—and the *expected* marginal cost—*aggregate expected net loss for* τ_t. Since these forces are weighted with the marginal utility of consumption, the government's policy decision process can be reinterpreted as a cost-benefit analysis from the perspective of low-income individuals. For low-income individuals, $\mathcal{G}_{i,t}$ represents the welfare gain from an increase in τ_t and T_t, a gain that driven by *income redistribution* and *pecuniary externalities* after accounting for welfare losses due to *fiscal externalities*. All of these effects occur in the *current period*. However, an increase in τ_t in period t induced an *expected fiscal externalities via the substitution effect* force via a change in K_{t+1}, which are related to $E_{\epsilon_{i,t+1}}[\mathcal{L}_{i,t+1}]\frac{\partial\Gamma_t}{\partial\tau_t}$. All individuals are worse off due to a decrease in the size of T_{t+1}, driven by a reduction in K_{t+1} and N_{t+1}—*expected fiscal externalities*. This net expected loss is represented by $E_{\epsilon_{i,t}}[\mathcal{L}_{i,t+1}]\frac{\partial\Gamma_t}{\partial\tau_t}$. Reflecting the interest of low-income individuals further, the government without commitment balances between *aggregate contemporary net gain* and *aggregate expected net loss*. In the terminal period, the government sets its policies to ensure that *aggregate contemporary net gain* is zero, as it no longer needs to consider the future expected effects.

2.4.2 The Case with Commitment

Regarding the case with commitment (the Ramsey problem), to derive comparable conditions to the case without commitment, I assume that the government has already implemented an optimal sequence of taxes and transfers, $\{\tau_t, T_t\}_{t=1}^2$ and now considers a policy change in period 2, $\{\tau_2, T_2\}$.

For further discussion, let me introduce some notations. Under commitment, when deciding a policy in period t, the government has the ability to coordinate policies across different periods to enhance their effectiveness. Let $\frac{\partial\tau_k}{\partial\tau_t}$ denote *coordinated changes in* τ_k *in response to a change* τ_t, where $k \neq t$. Note that in the absence of commitment, the government cannot coordinate policy changes across different periods, implying that $\frac{\partial\tau_k}{\partial\tau_t} = 0$.

I also refer to $\frac{\Delta K_2}{\Delta\tau_2}$ as *retrospective distributional effects of* τ_2:

$$\frac{\Delta K_2}{\Delta\tau_2} = \frac{\partial K_2}{\partial\tau_1} \cdot \frac{\partial\tau_1}{\partial\tau_2}. \tag{44}$$

The *retrospective distributional effects of* τ_2 capture the *indirect* effect of the current labor income tax τ_2 on the current aggregate capital K_2—representing μ_2 under the CARA utility—through a coordinated change in the previous labor income tax rate, τ_1. Since the *retrospective distributional effect* relies on coordinated changes in past policies, it is zero when the government lacks commitment.

Then, by repeating the calculations shown in the case without commitment, I obtain the following:

Proposition 3. *When deciding τ_2, the government with commitment satisfies*

$$\int E_{i,1}\left[\frac{\partial \tau_1}{\partial \tau_2}\cdot \mathcal{G}_{i,1}+\beta\Big(1+\frac{\partial \Psi_2}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2}\Big)\cdot \mathcal{G}_{i,2}\right]\mu_1(d(a_{i,1}\times \epsilon_{i,1}))=-\int E_{i,1}\left[\beta\,\frac{\Delta K_2}{\Delta \tau_2}\cdot \mathcal{L}_{i,2}\right]\mu_1(d(a_{i,1}\times \epsilon_{i,1}))$$

where $\frac{\partial \tau_1}{\partial \tau_2}$ and $\frac{\Delta K_2}{\Delta \tau_2}$ are non-zero.

Proof. See Online Appendix A. □

2.4.3 Commitment, No Commitment, and Constrained Efficiency

Comparing Propositions 2 with 3 shows how government commitment is related to time inconsistency issues in the presence of individual heterogeneity. Notably, the key differences arise from two terms: $\frac{\partial \tau_1}{\partial \tau_2}$—representing *intertemporally coordinated tax changes*—and $\frac{\Delta K_2}{\Delta \tau_2}$—*retrospective distributional effects*. In fact, when turning off these two types of forces, the government's solution with commitment becomes similar to that under a lack of commitment.

Corollary 3. *When $\frac{\partial \tau_1}{\partial \tau_2}=\frac{\Delta K_2}{\Delta \tau_2}=0$, the government's solution in deciding τ_2 satisfies*

$$\int E_{i,1}\Big[\mathcal{G}_{i,2}\Big]\mu_1(d(a_{i,1}\times \epsilon_{i,1}))=0.$$

Corollary 3 implies that without the two types of forces, the government with commitment chooses the same policies as that without commitment.The only difference is the timing of expectation. In the case with commitment, the expectation is taken at $t=1$, while it is taken at $t=2$ in the case without commitment. This difference occurs because the Ramsey planner decides all policies at the initial period, while under a lack of commitment, the government sets them for each period, implying a time-consistent policy. However, since there is no aggregate uncertainty, their solutions are equivalent. In other words, the source of time inconsistency lies in the two types of forces: *intertemporally coordinated tax changes* and *retrospective distributional effects*.

Intertemporally coordinated tax changes, $\frac{\partial \tau_1}{\partial \tau_2}$, are already a well-known source for the time inconsistency of optimal policies in the previous studies. For example, this force had shown up as the Lagrangian multipliers for budgets across different periods, λ_0 and λ_1 in Lucas and Stokey (1983).

A novel channel arising from individual heterogeneity is that these coordinated policy changes not only have direct impacts but also drive an additional indirect force, *retrospective distributional effects*—represented by $\frac{\Delta K_2}{\Delta \tau_2}$. These indirect forces imply that under full commitment, the government considers the impact of policies on the concurrent distribution, a mechanism enabled through coordinated tax changes in the past. For example, when increasing τ_2, the government with commitment may reduce τ_1, $\frac{\partial \tau_1}{\partial \tau_2} < 0$, to maximize the effect of the change in τ_2.

$$E_{\epsilon_{i,2}}\left[\underbrace{u_{c,i,2} \cdot \left[\left(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}\right) \cdot \left(\frac{K_2}{N_2}\right)^{\theta} \cdot (-\theta)(1-\theta) \cdot \frac{dN_2}{d\tau_2}\right]}_{\textit{Expected Pecuniary Externalities in period 2}}\right. \tag{45}$$

This intertemporal policy coordination generates two types of effects on pecuniary externalities in period 2. The first operates through adjustments in factor prices, w_2 and r_2—captured by the capital-to-labor ratio, K_2/N_2. A reduction in τ_1 increases the savings of middle- and high-income individuals, $a_{i,2}$, whose saving behavior is primarily driven by the substitution effect. This, in turn, raises aggregate capital K_2, while N_2 declines due to the increase in τ_2. As a result, the capital-to-labor ratio K_2/N_2 rises in period 2, leading to higher wages w_2 and lower returns on capital r_2. This shift improves the welfare of low-income individuals, whose income is predominantly derived from labor.

The second effect arises from changes in the composition of factor income for low-income individuals, captured by $(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2})$. An increase in T_2, resulting from the higher τ_2, reduces precautionary savings $a_{i,2}$ for low-income individuals—determined in period 1. This change in savings behavior lowers the capital income share of low-income individuals in period 2, thereby improving their welfare via the second-period pecuniary externalities. These two additional effects, represented by $\frac{\partial \Psi_2}{\partial K_2} \cdot \frac{\Delta K_2}{\Delta \tau_2} \cdot \mathcal{G}_{i,2}$ in Proposition 3, are not internalized when the government lacks commitment.

Comparing the economy under constrained efficiency with economies with and without government commitment is helpful for understanding the role of commitment in achieving efficient allocations. Davila et al. (2012) assume that the social planner faces the same market incompleteness as individuals but is able to implement individual-specific income taxes and transfers. This setting renders the constrained-efficient allocation immune to externalities that operate through the government budget constraint—income redistribution and fiscal externalities. Consistent with earlier notation, the *individual contemporaneous net gain* from $\tau_{i,t}$, denoted by $\mathcal{G}_{i,t}$, includes only pecuniary externalities, while the *individual's expected net loss* from $\tau_{i,t}$, denoted by $\mathcal{L}_{i,t}$, is zero. This key difference leads to the following implications.

Corollary 4. *For each individual i, the constrained-efficient allocation satisfies*

$$u_{c,i,1} = \beta \cdot r_2 \cdot E_{\epsilon_{i,2}}\Big[u_{c,i,2}\Big] + \int \Big[\mathcal{G}_{i,2}\Big]\mu_2(d(a_{i,2} \times \epsilon_{i,2}))$$

where

$$\mathcal{G}_{i,2} = u_{c,i,2}\Big(\frac{n_{i,2}}{N_2} - \frac{a_{i,2}}{K_2}\Big)\Big(\frac{K_2}{N_2}\Big)^{\theta}(\theta)(1-\theta).$$

The consumption-saving Euler equation is preserved at the individual level because the planner can manipulate individual allocations through individual-specific policies. In contrast, in the decentralized economies studied in this paper, this term cancels out because they operate in competitive equilibrium, where either the consumption-saving Euler equation holds or the borrowing constraint binds, $\Big(-u_{c,i,1} + \beta \cdot r_2 \cdot E_{\epsilon_{i,2}}[u_{c,i,2}]\Big) \cdot \frac{\partial a_{i,2}}{\partial \tau_1} = 0$ in (34).

The constrained-efficient allocation described above more closely resembles the allocation under commitment, as characterized in Proposition 3. In both cases, the social planner and the government account for intertemporal consumption allocations as well as pecuniary externalities in period 2. Under constrained efficiency, intertemporal consumption is governed by individuals' Euler equations, as their decisions are not restricted by a uniform policy rule. In Proposition 3, intertemporal allocation is instead shaped by the policy transmission term $\frac{\partial \tau_1}{\partial \tau_2} \cdot \mathcal{G}_{i,1}$, reflecting the constraint imposed by the linear labor income tax. Moreover, the social planner internalizes the average pecuniary externalities in period 2, just as the government does under commitment in Proposition 3. In contrast, without commitment, the period-2 government overlooks the intertemporal consumption allocations, as shown in Proposition 2. These findings suggest that government commitment brings the allocation closer to the constrained-efficient allocation by considering both intertemporal consumption allocations and the pecuniary externalities.

3 Infinite Horizon Model

In this section, I extend the model period to an infinite horizon and introduce partial capital depreciation, idiosyncratic shocks to labor productivity instead of leisure, and the CRRA utility function, model components commonly used in quantitative macroeconomic studies. Aside from these changes, all other model settings are preserved as in the two-period case.

3.1 Environment

The model economy is populated by a continuum of infinitely lived households. Their preferences follow

$$E\Big[\sum_{t=0}^{\infty}\beta^t u(c_t, 1-n_t)\Big] \tag{46}$$

where c_t is consumption, $n_t \in [0,1]$ is labor supply in period t ($(1-n_t)$ refers to leisure), and β is the discount factor. Preferences are represented by

$$u(c_t, 1-n_t) = \frac{{c_t}^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n_t)^{1-1/\chi}}{1-1/\chi} \tag{47}$$

where σ is the coefficient of relative risk aversion, B is the utility of leisure, and χ is the Frisch elasticity of labor supply. Note that the preferences here capture the wealth effects of labor supply. Such wealth effects are crucial for a welfare analysis, closely related to efficiency loss. An increase in transfers, for example, decreases the overall labor supply, shrinking the size of the aggregate economy and hence playing a role in reducing welfare.

In each period, households confront an uninsurable, idiosyncratic shock ϵ_t to wage, rather than their leisure endowment. And this shock follows an AR-1 process:

$$\log(\epsilon_{t+1}) = \rho_\epsilon \log(\epsilon_t) + \eta^{\epsilon}_{t+1} \tag{48}$$

where $\eta^{\epsilon}_{t+1} \sim N(0, \sigma^2_\epsilon)$. Using the method in Rouwenhorst (1995), I approximate the AR-1 process as a finite-state Markov chain with transition probabilities $\pi_{\epsilon,\epsilon'}$ from state ϵ to state ϵ'. Households earn $w_t\epsilon_t n_t$ as their labor income, where w_t is the market equilibrium wage. They can self-insure through assets a_t. Such households have capital income of as much as $(r_t - \delta)a_t$ where $r_t - \delta$ is the equilibrium risk-free interest rate that is the difference between the rental rate of capital r_t and the capital depreciation rate δ.[13] In contrast to the previous two-period model, δ is between 0 and 1. That means that capital partially, rather than fully, depreciates in each period. This partial depreciation has quantitatively important implications, generating a slow adjustment of capital over time.

3.2 Competitive Equilibrium, Exogenous Policy

In this section, I define the competitive equilibrium, given an exogenous tax and transfer policy. I start with a setting to address the economy with commitment. To describe problems with commitment (the Ramsey problem), household dynamic problems need to be represented in a sequential

[13]The gross return on savings is $1 + r_t - \delta$. Recall that it was r_t in the two-period case where $\delta = 1$.

manner. At the beginning of each period, households differ from one another in asset holdings a and labor productivity ϵ. $\mu_t(a,\epsilon)$ denotes the distribution of households in period t. Given a sequence of factor prices $\{r_t, w_t\}_{t=0}^{\infty}$, labor income taxes $\{\tau_t\}_{t=0}^{\infty}$, and lump-sum taxes/transfers $\{T_t\}_{t=0}^{\infty}$, households in period t solve

$$v_t(a,\epsilon) = \max_{c_t,\, a_{t+1},\, n_t} \frac{c_t(a,\epsilon)^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n_t(a,\epsilon))^{1-1/\chi}}{1-1/\chi} + \beta \sum_{\epsilon_{t+1}} \pi_{\epsilon_t,\epsilon_{t+1}} v_{t+1}(a_{t+1}(a,\epsilon),\epsilon_{t+1}) \tag{49}$$

such that

$$c_t + a_{t+1} = (1-\tau_t) w_t \epsilon_t n_t + (1 + r_t - \delta) a + T_t$$

where r_t is the rental rate of capital in period t. This economy lies in a sequential competitive equilibrium as follows.

Definition 3.1. ***Sequential Competitive Equilibrium (SCE) for the Inifintite Horizon Economy, Given a Sequence of Taxes and Transfers***

Given G, an initial distribution $\mu_0(\cdot)$, and a sequence of labor income taxes and lump-sum taxes/transfers $\{\tau_t, T_t\}_{t=0}^{\infty}$, an SCE is a sequence of factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, a sequence of allocations $\{c_t, n_t, a_{t+1}, K_t, N_t\}_{t=0}^{\infty}$, a sequence of value functions $\{v_t(\cdot)\}_{t=0}^{\infty}$, and a sequence of distributions over the state space $\{\mu_t(\cdot)\}_{t=1}^{\infty}$, such that for all t

(i) Given $\{\tau_t, T_t\}_{t=0}^{\infty}$ and $\{w_t, r_t\}_{t=0}^{\infty}$, the decision rules $a_{t+1}(a,\epsilon)$ and $n_t(a,\epsilon)$ solve the household problem in (49), and $v_t(a,\epsilon)$ is the associated value function.

(ii) The representative agent firm engages in competitive pricing:

$$w_t = (1-\theta) z \left(\frac{K_t}{N_t}\right)^{\theta} \tag{50}$$

$$r_t = \theta z \left(\frac{K_t}{N_t}\right)^{\theta-1}. \tag{51}$$

(iii) The factor markets clear:

$$K_t = \int a\, \mu_t(\mathbf{d}(a\times\epsilon)) \tag{52}$$

$$N_t = \int \epsilon\, n_t(a,\epsilon)\, \mu_t(\mathbf{d}(a\times\epsilon)) \tag{53}$$

(iv) The government budget constraint (5) is satisfied.

(v) Let $\mathcal{B}(A \times E)$ denote the Borel σ-algebra on $A \times E$. For any $B \in \mathcal{B}(A \times E)$, the sequence of distributions over individual $\{\mu_t(\cdot)\}_{t=1}^{\infty}$ satisfies

$$\mu_{t+1}(B) = \int_{\{(a,\epsilon)|(a_{t+1}(a,\epsilon),\epsilon_{t+1})\in B\}} \sum_{\epsilon_{t+1}} \pi_{\epsilon,\epsilon_{t+1}} \mu_t(\mathbf{d}(a \times \epsilon)). \tag{54}$$

In contrast, to handle problems without commitment, it is convenient to present the household dynamic problems in a recursive manner. In addition to the individual state variables a and ϵ, there is an aggregate state variable, the distribution of households $\mu(a, \epsilon)$ over a and ϵ. A variable with a prime symbol denotes its value in the next period.

Let $v(a, \epsilon; \mu)$ denote the value of households associated with a state of $(a, \epsilon; \mu)$. They solve

$$v(a, \epsilon; \mu) = \max_{c>0,\ a' \geq \underline{a},\ 0 \leq n \leq 1} \left[\frac{c^{1-\sigma}}{1-\sigma} + B\,\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} v(a', \epsilon'; \mu') \right] \tag{55}$$

such that

$$c + a' = (1-\tau)\, w(K,N)\, \epsilon\, n + (1 + r(K,N) - \delta)\, a + T$$
$$\tau = \Psi(\mu)$$
$$\mu' = \Gamma(\mu, \tau) = \Gamma(\mu, \Psi(\mu))$$

where $\underline{a} \leq 0$ is a borrowing limit, $\tau = \Psi(\mu)$ is the perceived law of motion of labor income taxes, and $\mu' = \Gamma(\mu, \tau)$ is the law of motion for the distribution over households.and labor income tax τ.[14] $\Gamma(\cdot, \cdot)$ has τ as an input because individual savings decisions are influenced by labor income taxes. Note that households here solve the above problem given an exogenous tax policy function $\tau = \Psi(\mu)$. This economy lies in a recursive competitive equilibrium as follows.

Definition 3.2. ***Recursive Competitive Equilibrium (RCE) for the Infinite Horizon Economy, given a Law of Motion for Tax.***
Given G and $\tau = \Psi(\mu)$, an RCE is a set of factor prices $\{w(\mu), r(\mu)\}$, a set of decision rules for households $g^a(a, \epsilon; \mu)$ and $g^n(a, \epsilon; \mu)$, a value function $v(a, \epsilon; \mu)$, a distribution of households $\mu(a, \epsilon)$ over the state space, and the law of motion for the distribution of households $\Gamma(\mu, \Psi(\mu))$ such that

(i) Given $\{w(\mu), r(\mu)\}$ and $\tau = \Psi(\mu)$, the decision rules $a' = g^a(a, \epsilon; \mu)$ and $n = g^n(a, \epsilon; \mu)$ solve the household problem in (55), and $v(a, \epsilon; \mu)$ is the associated value function.

[14] Recall that after a policy reform, either τ or T needs to a perceived law of motion because once one of them chosen, the other is fixed in the balanced government budget, $G + T = \tau w N$. Here, I have chosen τ, as in the two-period model.

(ii) The representative agent firm engages in competitive pricing:

$$w(K,N) = (1-\theta)z\left(\frac{K}{N}\right)^{\theta} \tag{56}$$

$$r(K,N) = \theta z\left(\frac{K}{N}\right)^{\theta-1}. \tag{57}$$

(iii) The factor markets clear:

$$K = \int a\,\mu(\mathbf{d}(a\times\epsilon)) \tag{58}$$

$$N = \int \epsilon\, g^n(a,\epsilon;\mu,\tau)\,\mu(\mathbf{d}(a\times\epsilon)) \tag{59}$$

(iv) The government budget constraint (5) is satisfied.

(v) The law of motion for the distribution of households $\mu' = \Gamma(\mu, \Psi(\mu,\tau))$ is consistent with the individual decision rules and the stochastic process of ϵ.

3.3 Competitive Equilibrium, Endogenous Policy

In this section, I define competitive equilibria where labor income taxes and lump-sum taxes/transfers are endogenously determined. I model the tax-transfer choice in two ways: the optimal labor income tax and lump-sum taxes/transfers with commitment (Ramsey problem) and the optimal labor income taxes and lump-sum taxes/transfers without commitment (time-consistent case).

The timing of this policy reform is as follows: until period -1, the economy remains in a steady state. At the beginning of period 0, the government unexpectedly announces an optimal policy path for income taxes and lump-sum taxes/transfers, which differs based on the availability of government commitment. I begin by addressing the Ramsey problem.

Definition 3.3. ***The Ramsey Problem:***
An SCE with the Optimal Income Tax and Lump-Sum Taxes/transfers with Commitment
Given μ_0, the government chooses $\{\tau_t\}_{t=0}^{\infty}$ such that

$$\{\tau_t\}_{t=0}^{\infty} = \underset{\{\tilde{\tau}_t,\}_{t=0}^{\infty}}{\operatorname{argmax}} \int E_0 \sum_{\hat{t}=0}^{\infty} \beta^{\hat{t}} u\big(c^*_{\hat{t}}(a,\epsilon|\{\tilde{\tau}_{\hat{t}},\tilde{T}_{\hat{t}}\}_{t=0}^{\infty}), 1-n^*_{\hat{t}}(a,\epsilon|\{\tilde{\tau}_{\hat{t}},\tilde{T}_{\hat{t}}\}_{t=0}^{\infty})\big)\mu_0(\mathbf{d}(a\times\epsilon))$$

such that

$$c^*_t + a^*_{t+1} = w_t\epsilon_t n^*_t + (1+r_t-\delta)a + \tilde{\tau}_t(w_t(N_t - \epsilon_t n^*_t) - G)$$

where $\left(c_{\hat{t}}^*(a,\epsilon|\{\tilde{\tau}_{\hat{t}},\tilde{T}_{\hat{t}}\}_{t=0}^{\infty}), n_{\hat{t}}^*(a,\epsilon|\{\tilde{\tau}_{\hat{t}},\tilde{T}_{\hat{t}}\}_{t=0}^{\infty})\right)$ is an allocation in Definition (3.1) in period $\hat{t}$, given $\{\tilde{\tau}_t, \tilde{T}_{\hat{t}} = \tilde{\tau}_t(w_t(N_t - \epsilon_t n_t^*) - G)\}_{t=0}^{\infty}$.

Note that the consumption and labor decisions at time t, (c_t^*, n_t^*), are affected not only by the policy in that period t but also by the entire sequence of labor income taxes and transfers. Therefore, the decisions at time t are influenced by taxes and transfers in periods after t, which can lead to time inconsistency issues.

Recall that the last term, $\tau_t w_t(N_t - \epsilon_t n_t) - G$, indicates that this government, with the concave objective function, considers income redistribution by increasing the after-tax income of low-income individuals with lower $\epsilon_t n_t$. In other words, as discussed in the two-period model, given the policy restriction, $T_t + G = \tau_t w_t N_t$, the government implements tax-transfer policies in a way that manages individual borrowing constraints to benefit low-income individuals.

For the case without commitment, I have borrowed the definition of the MPE in Klein et al. (2008).

Definition 3.4. ***An RCE with the Optimal Income Tax and Lump-Sum Taxes/transfers without Commitment***

(i) A set of functions $\{w(\cdot), r(\cdot), g^a(\cdot), g^n(\cdot), v(\cdot), \Gamma(\cdot)\}$ satisfy Definition (3.2).

(ii) For each μ, the government chooses $\psi(\mu)$ such that

$$\psi(\mu) = \underset{\tilde{\tau}}{\operatorname{argmax}} \int \hat{V}(a,\epsilon;\mu,\tilde{\tau})\mu(\mathbf{d}(a\times\epsilon)) \tag{60}$$

where

$$\hat{V}(a,\epsilon;\mu,\tilde{\tau}) = \max_{c>0,\, a'\geq \underline{a},\, 0\leq n\leq 1}\left[\frac{c^{1-\sigma}}{1-\sigma} + B\frac{(1-n)^{1-1/\chi}}{1-1/\chi} + \beta\sum_{\epsilon'}\pi_{\epsilon,\epsilon'}v(a',\epsilon';\tilde{\mu}')\right]$$

such that

$$c + a' = (1-\tilde{\tau})\, w(K,N)\,\epsilon\, n + (1 + r(K,N) - \delta)\, a + T$$

$$\tau = \tilde{\tau},\ \text{and thereafter } \tau' = \Psi(\tilde{\mu}') \tag{61}$$

$$\tilde{\mu}' = \Gamma(\mu, \tau = \tilde{\tau}),\ \text{and thereafter } \mu'' = \Gamma(\tilde{\mu}', \tau' = \Psi(\tilde{\mu}')) \tag{62}$$

(iii) $a' = \hat{g}_a(a,\epsilon;\mu,\tilde{\tau})$ and $n = \hat{g}_n(a,\epsilon;\mu,\tilde{\tau})$ solve (60) at prices that clear markets and satisfy the government budget constraint, and Γ is consistent with individual decisions and the stochastic process of ϵ.

(iv) For each μ, the policy outcome function satisfies $\psi(\mu) = \Psi(\mu)$.

Note that the government's solution to the problem is consistent with the sub-perfect Nash equilibrium of the infinite repeated game, as derived using the one-shot deviation principle. In the case without commitment, the government implements a time-consistent policy: a contemporary labor income tax rate is sequentially chosen for each period while maximizing its utilitarian welfare under this commitment constraint. The government cannot commit to the future tax rate from the next period. Thus, if the chosen tax rate $\tilde{\tau}$ deviates from the equilibrium tax policy function $\Psi(\cdot)$, future tax rates will follow the equilibrium tax policy function $\Psi(\cdot)$ because of the lack of commitment. (61) presents such a commitment constraint. The law of motion for the distribution of households $\Gamma(\cdot)$ has to capture all the changes in the evolution of distributions caused by the deviation of the income tax from the equilibrium tax function, as shown in (62). In equilibrium, for each aggregate state μ, the government's choice of the tax rate, $\psi(\mu)$, should be equal to the households' perceived tax function $\Psi(\mu)$, as presented in (iv).

4 Numerical Solution Algorithm and Calibration

4.1 Numerical Solution Algorithm

Here, I focus on conveying the key ideas of the numerical solution algorithm. Appendix B demonstrates each step of the algorithm with details, including outcomes related to its efficiency and accuracy.

Solving models with heterogeneous agents in MPE entails a substantial computational burden. The law of motion for the distribution of households $\Gamma(\cdot)$ has to be consistent with individual decisions. Furthermore, perhaps the most challenging part is finding the equilibrium policy function $\Psi(\cdot)$ that must be consistent with individual decisions and the law of motion for the distribution of households. That is, three equilibrium objects—specifically, individual decisions, $g^n(\cdot)$ and $g^a(\cdot)$, the law of motion for the distribution, $\Gamma(\cdot)$, and the policy function, $\Psi(\cdot)$—interact and have to be consistent with one another in an MPE.

To address the above computational issues, I propose a global solution method by drawing on ideas from the backward induction method of Reiter (2010).[15] However, I wish to ensure that I cannot directly apply the Reiter's (2010) method to the model in this paper because of the presence of off-equilibrium paths after one-shot deviations that are required to find the sub-perfect Nash equilibrium. In an economy with aggregate uncertainty, for which the Reiter's (2010) method is originally designed, the distribution of aggregate shocks (TFP) Z is ergodic. Thus, with a positive probability, all the states in Z are realized on the equilibrium path. However, cases in the MPE do not have this property. For example, in the economy without commitment, the government chooses a tax rate by comparing one-time deviation policies, as in (60). Some tax paths will not be reached on the equilibrium path but the corresponding values need defining to solve the

[15]This method is originally designed to solve an incomplete-markets economy with aggregate uncertainty.

government problem, as in (60).

To cope with this issue, I make three changes to the original backward induction method of Reiter (2010). First, as mentioned above, I approximate not only the aggregate law of motion for the distribution of households but also the endogenous tax policy function. I find these mappings in a nonparametric way as in Reiter (2010). Second, I arrange distributions for all types of off-the-equilibrium paths by taking the initial distribution of the simulations as the previous proxy distribution for each finite grid point of the aggregate state. Finally, I modify the way of constructing the reference distributions, which is required to update the proxy distributions in Reiter (2010), by reflecting the features of the MPE, in which how many times a tax rate off the equilibrium takes place is unknown before simulation.

As with all global solution methods, including those by Krusell and Smith (1998) and Reiter (2010), my approach also requires a reduction in the dimension of distributions in the law of motion $\Gamma(\cdot)$ to some finite moments of the distribution. In solving the MPE, I represent the evolution of household distributions, $\mu' = \Gamma(\mu, \tau)$, by that of their first-order moment, $K' = \Gamma(K, \tau)$. This representation in the numerical approach aligns with the theoretical analysis because all the theoretical results rely on the effects of the first-order moment. Under this assumption, the numerical errors are not significant. Using the accuracy statistics proposed by Den Haan (2010), I find this gap to be within a reasonable range, as reported in Table 5 in Appendix B.

Regarding the case with commitment (the Ramsey problem), I employ the approach in Dyrda and Pedroni (2023) by parameterizing the transition path of income taxes as follows:

$$\tau_t = \left(\sum_{i=0}^{m_{x0}} \alpha_i^x P_i(t) \right) \exp(\lambda t) + (1 - \exp(-\lambda t)) \left(\sum_{j=0}^{m_{xF}} \beta_j^x P_j(t) \right), \quad t \le t_F \tag{63}$$

where $\{P_i(t)\}_{i=1}^{m_{x0}}$ and $\{P_j(t)\}_{j=0}^{m_{xF}}$ are families of the Chebyshev polynomial; m_{x0} and m_{xF} are the orders of the polynomial approximation for the short- and long-run dynamics, respectively; $\{\alpha_i^x\}_{i=0}^{m_{x0}}$ and $\{\beta_j^x\}_{j=0}^{m_{xF}}$ are weights on the consecutive elements of the family; and λ controls the convergence rate of the fiscal instrument. This setting assumes that the economy has the long-run steady state at the latest in period t_F.[16] I choose $m_{x0} = m_{xF} = 2$ and $t_F = 250$, and seek $\{\alpha_0^x, \alpha_1^x, \alpha_2^x, \beta_0^x, \beta_1^x, \beta_2^x, \lambda\}$ maximizing welfare, as evaluated by a utilitarian government at $t = 0$.

4.2 Calibration

I calibrate the model to capture the features of the U.S. economy. I divide the parameters into two groups. The first set of parameters is determined outside the model. I take the values of these

[16]There have been debates about the existence of a Ramsey steady state (Acikgoz et al., 2018; Straub and Werning, 2020; Chien and Wen, 2024; Auclert et al., 2024). Instead of focusing on long-run outcomes, this paper examines the transition dynamics directly, following the approach of Dyrda and Pedroni (2023).

parameters from the macroeconomic literature and policies. The other set of parameters requires solving the stationary distribution of the model to match the moments generated by the model with their empirical counterparts.

Table 1: Parameter Values of the Baseline Economy

Parameters	Description	Target	Data Value	Model Value
Externally chosen parameters				
$\sigma = 2$	Relative risk aversion			
$\chi = 0.75$	Frisch elasticity of labor supply			
$\rho_\epsilon = 0.955$	Persistence of wage shocks			
$\underline{a} = 0$	Borrowing constraint			
$z = 1$	TFP			
$\theta = 0.36$	Capital income share			
$\delta = 0.08$	Depreciation rate			
$\tau = 0.31$	AVG labor income tax			
Internally chosen parameters				
$\beta = 0.9392$	Discount factor	K/Y	3	3
$B = 3.615$	Utility of leisure	AVG Wrk Hrs	1/3	1/3
$\sigma_\epsilon = 0.2$	STD of wage shocks	Wage Gini Coefficient	0.359	0.360
$T = 0.0291$	Lump-sum transfers	T/Y	0.044	0.044

Table 1 displays the parameters. Eight parameters are determined outside the model. The coefficient of relative risk aversion σ is set to 2. The Frisch elasticity of labor supply χ is taken to be 0.75. I set the borrowing constraint as $\underline{a} = 0$. The TFP z is set as 1, and the capital income share θ is chosen to reproduce the empirical finding that the share of capital income is 0.36. The annual depreciation rate δ is 8 percent. The value of ρ_ϵ is taken from Jang et al. (2023), which estimates the annual autocorrelation of idiosyncratic wage shocks using data from the Panel Study of Income Dynamics (PSID). The labor income tax rate is chosen as 0.31 in the baseline economy.

Internally, I calibrate four parameters. The discount factor β is selected to match a capital-to-output ratio of 3, and the utility of leisure B is chosen to reproduce an average of hours worked of 8 hours a day. The standard deviation of wage shocks σ_ϵ is chosen to match the Gini coefficient of hourly wages in the Panel Study of Income Dynamics (PSID), which is 0.359. The size of lump-sum transfers T is set to match the total transfers-output ratio of 0.044, which is the time-series average of the ratio of transfers (excluding Social Security and Medicare) to output over the years 1961-2016 according to the Bureau of Economic Analysis (BEA) data.

Government spending G is determined as the difference between the implied total tax revenue and the total size of transfers, which amounts to 15.4 percent of the baseline GDP. Throughout the subsequent counterfactual exercises, G remains fixed at this level.

5 Quantitative Results

In this section, I quantitatively explore how commitment technologies affect the optimal design of labor income tax and lump-sum tax/transfers and, hence, their resulting consequences for the aggregate economy, inequality, and welfare. To do so, I compare the economy without commitment to the economy with commitment. I assume that all the economies are at the calibrated steady-state until period 0, and each government unexpectedly announces a policy reform at the beginning of period 1. I compare their equilibrium results along the transition path.

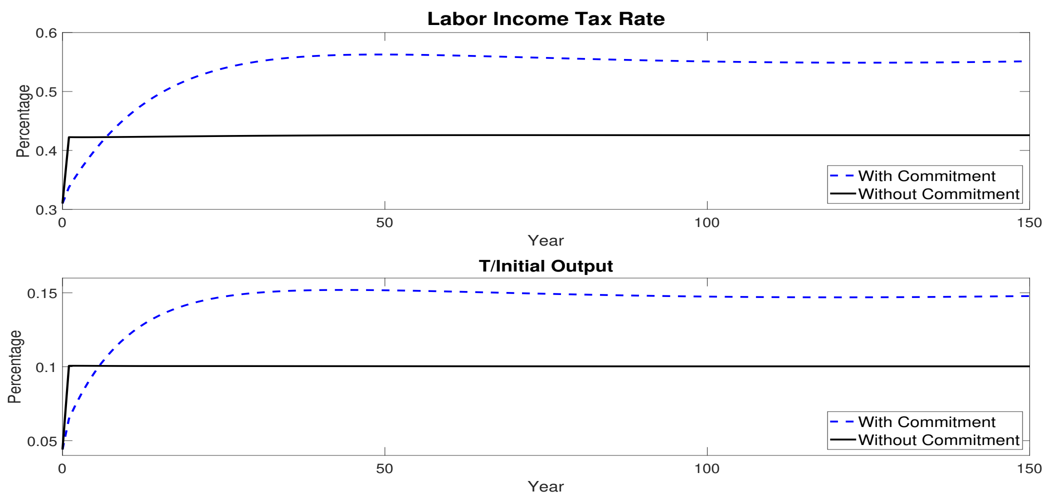

Figure 1: Tax-transfer Transition Paths According to Government Commitment

Figure 1 illustrates the optimal tax policies and the implied ratio of transfers to initial output, depending on the government's ability to commit. The top panel of Figure 1 suggests that a government with commitment implements more gradual adjustments and larger increases in both taxes and transfers compared to a government without commitment during the transition. The Ramsey planner incrementally raises income taxes by 25 percentage points before slightly reducing them later. In contrast, a government without commitment immediately increases taxes by 15 percentage points and maintains that level thereafter. These differences in tax policies lead to variations in the evolution of transfers. The economy with commitment experiences more gradual and substantial increases in transfers, whereas the economy without commitment sees a more abrupt but smaller increase. Specifically, the ratio of transfers to initial GDP rises gradually by 13.7 percentage points in the commitment case, whereas, in the no-commitment case, it jumps by 5.4 percentage points immediately.

Table 2: Welfare Outcomes According to Government Commitment

	With Commitment	Without Commitment
Welfare (CEV)	+2.19%	+1.46%

This distinction in income taxes and transfers leads to different welfare outcomes. Table 2 shows that welfare, measured by the consumption-equivalent variation (CEV) of the utilitarian welfare function, is higher in the case with commitment. Compared to the baseline economy, the tax-transfer policy with commitment increases welfare by 2.19 percent, whereas the policy without commitment improves welfare by 1.46 percent. To understand this disparity in welfare outcomes, I examine how the inputs of the social welfare function evolve over time depending on the availability of commitment. Note that welfare increases when the overall level of consumption and leisure increases, and their inequalities are reduced.

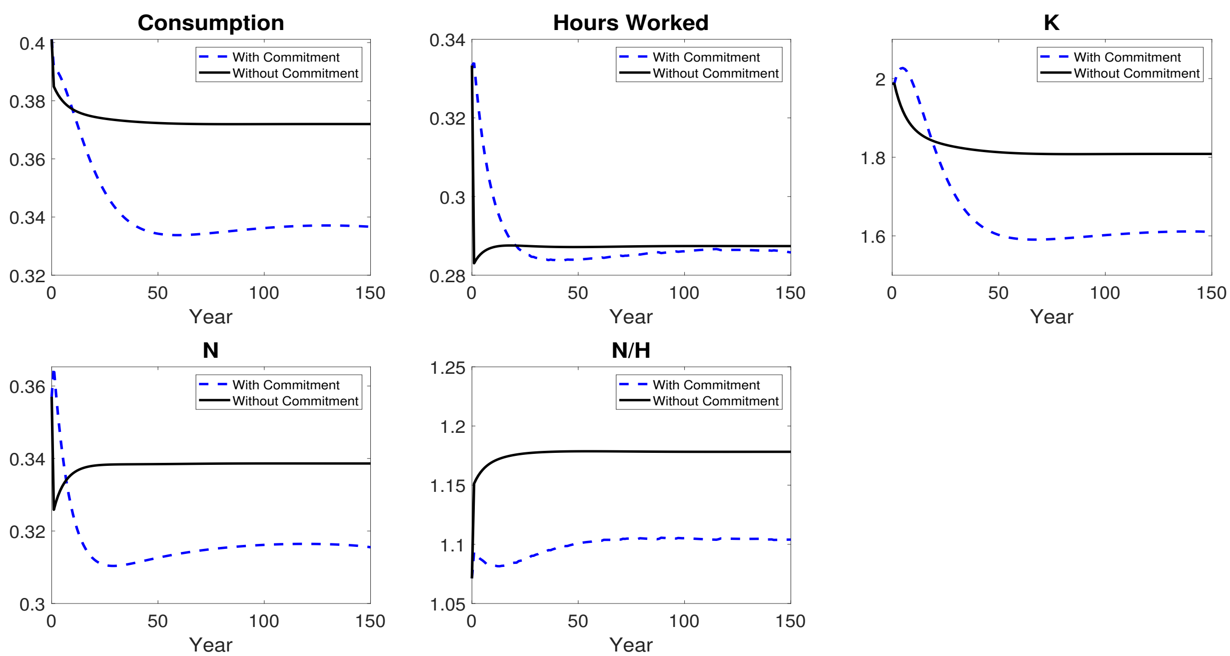

Figure 2: Aggregate Outcomes According to Government Commitment

Figure 2 illustrates changes in the levels of the aggregate variables. The figure suggests that the case without commitment generates more efficient outcomes. All the aggregate variables are larger in the economy without commitment than that with commitment. This result may seem obvious because lower taxes in the case without commitment cause fewer distortions. However, this finding might be unclear when examining the welfare consequences. Despite the fact that consumption is substantially larger in the case without commitment, the gaps in hours worked/leisure are not significant and do not offer a complete understanding of the welfare implications.

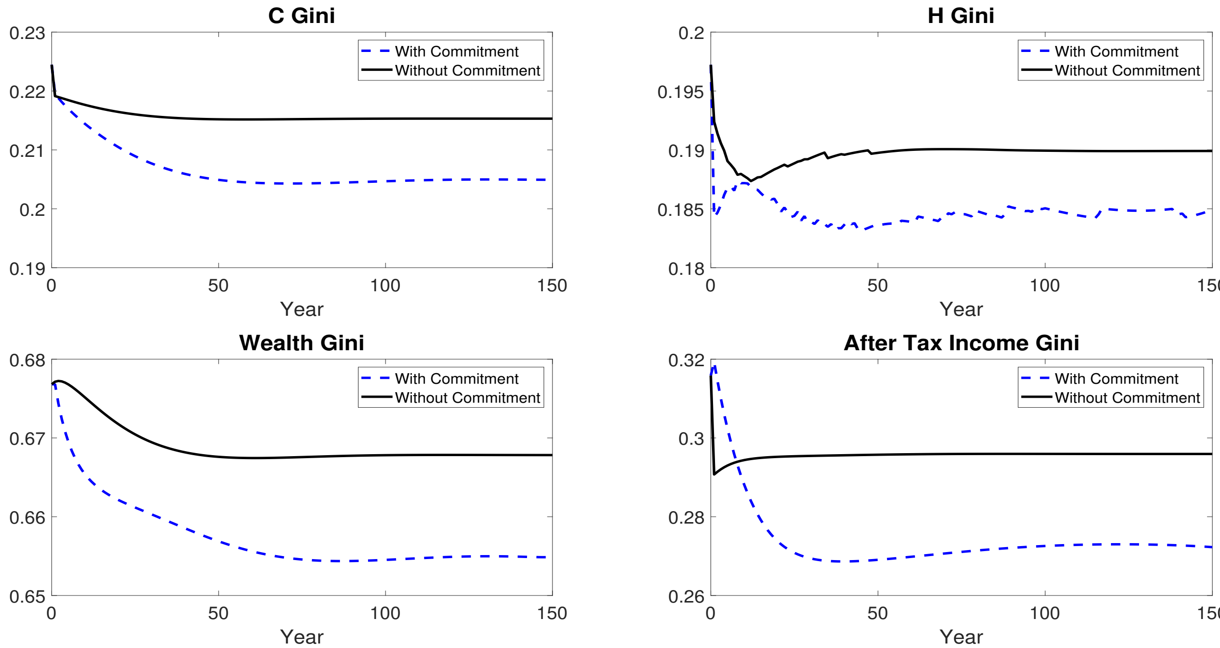

Figure 3: Distributional Outcomes According to Government Commitment

Figure 3 illustrates inequalities in consumption, hours worked, wealth, and after-tax income. The figure suggests that the more significant welfare improvements in the case with commitment are mainly driven by larger reductions in inequalities in consumption and leisure. While consumption inequality, measured by the Gini coefficient, decreases near by 3 percent with the time-consistent optimal policy (without commitment), it is reduced by approximately 9 percent with the time-inconsistent optimal tax (with commitment). Similarly, inequality in hours worked is also reduced more in the case with commitment. These findings imply that the commitment instrument allows the Ramsey planner to better manage the evolution of inequality, leading to a more favorable welfare outcome. However, this explanation does not provide a clear economic logic as to how the availability of government's commitment plays a role in shaping these quantitative disparities. To better understand the economic logic behind these differences, I employ the theoretical findings in the previous section to interpret these quantitative outcomes.

Figure 4 shows the dynamics of the factor prices w and r depending on the availability of commitment. This figure suggests that the government with commitment (the Ramsey planner) benefits from pecuniary externalities in the early phase of the transition by exploiting differences in the speed of adjustments between K and N. Of course, in the long run, the increased tax rate in the case with commitment reduces the ratio of K to N, leading to a decrease in w and an increase in r. However, during their adjustment, the ratio of K to N increases in the early phase

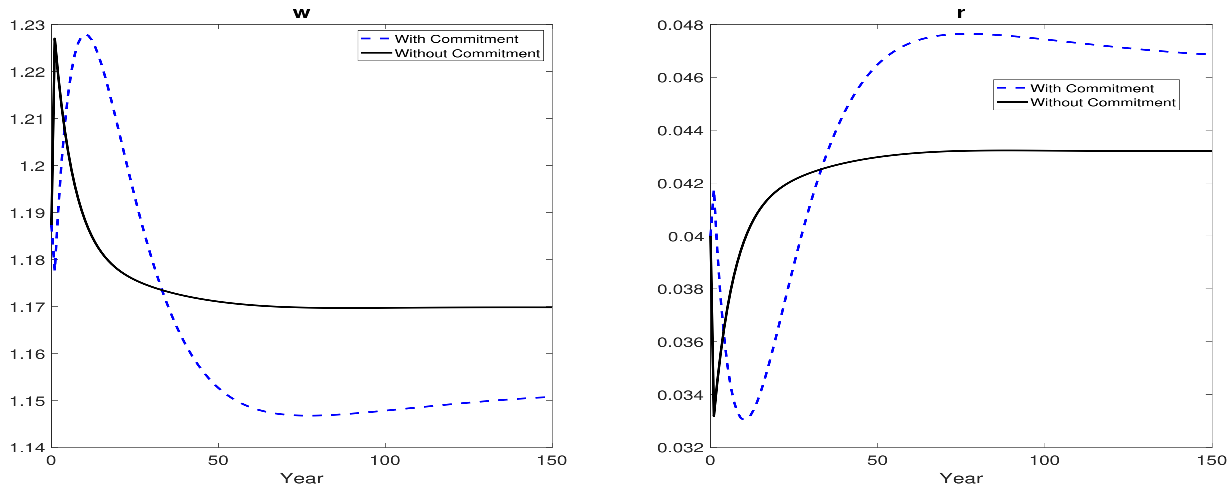

Figure 4: Dynamics of Wage and Interest Rate According to Government Commitment

of the transition because the adjustment of K is slower than that of N, leading to a rise in w and a reduction in r. These price changes help improve the welfare of the low-income group, who are more represented by the government, through pecuniary externalities during the early transition.

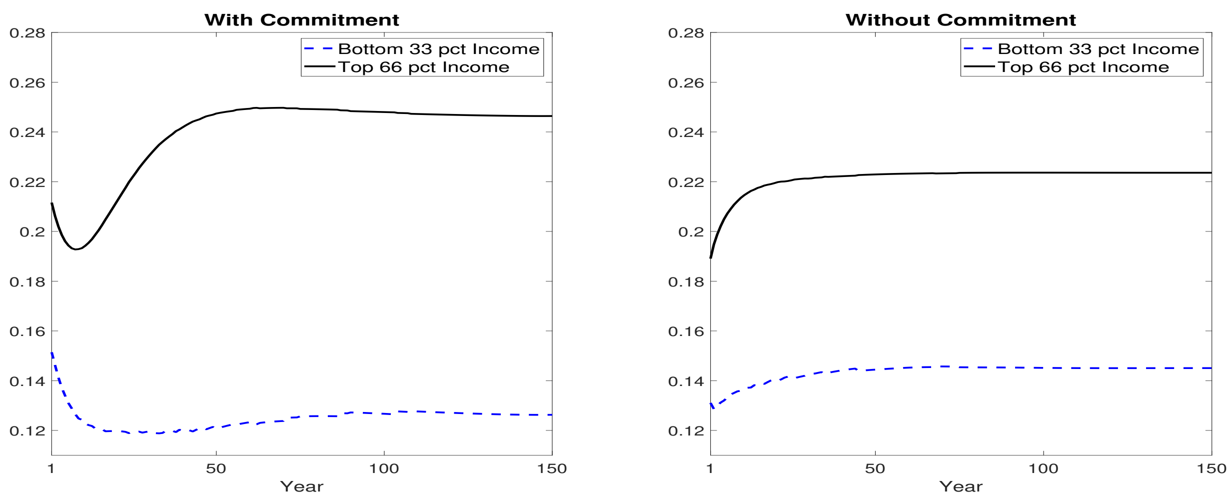

Figure 5: Proportion of Capital Income According to Government Commitment

Figure 5 illustrates the proportion of capital income out of total disposable income across income groups, according to the government's commitment technologies. The left panel of Figure 5 indicates that in the economy with commitment, low-income individuals reduce their precautionary savings, which is not observed for the middle- and high-income groups. Initially, all income groups experience a decrease in their capital income proportion as the market interest rate declines during the early transition—*retrospective distributional effects*. However, while middle- and high-income individuals increase their capital income proportion as the interest rate rises afterward, low-income individuals maintain their reduced savings in the long run. This decrease in precautionary savings facilitates a reduction in after-tax income inequality among low-income individuals because inequality in capital income is more severe than inequality in other types of income, such as labor income and transfers. Given the government's inclination toward the interests of low-income individuals, the reduction in their income inequality in the early transition leads to front-loaded welfare gains resulting from income redistribution.

The availability of government commitment technologies plays a crucial role in driving differences in savings across income groups. The right panel of Figure 5 shows that without commitment, the economy does not have such permanent decline in savings for low-income individuals. Low-income individuals in the economy without commitment have larger precautionary savings because the government does not provide as substantial transfers as the government with commitment in the long run. These findings imply that the front-loaded welfare gains from both types of economic forces are driven by the government's ability to commit to future policies in the long run.

Table 3: Difference in Welfare Changes between the Short and Long Runs

	With Commitment	Without Commitment
Welfare (CEV)	+4.36 pp	+1.34 pp

In exchange for these upfront welfare gains, a government with commitment incurs welfare losses in the long run. Table 3 shows that the difference in welfare changes between the short and long run is substantial (+4.36 pp). In the long run, w is lower and r is higher than their initial values, creating negative pecuniary externalities for the low-income group, as shown in Figure 4. Additionally, as depicted in Figure 3, there is no further reduction in after-tax income inequality in the long run, resulting in minimal welfare gains from income redistribution.

Recall that, as Proposition 3 suggests, a government with commitment coordinates the entire tax-transfer scheme by balancing pecuniary externalities, income redistribution, and reductions in lump-sum transfers over different periods. Therefore, these quantitative findings imply that the government with commitment takes front-loads welfare gains through income redistribution and positive pecuniary externalities while enduring welfare losses from diminished income redistribution and negative pecuniary externalities in the long run.

Note that the above time-lagged strategy is not credible without commitment. Suppose that the government without commitment is in the long-run equilibrium of the economy with commitment. As Proposition 2 shows, a lack of commitment causes the government to measure the costs and benefits of changing a tax rate considering its impacts only in periods since the implementation.

Since any front-loaded welfare gain is disregarded, the marginal cost of raising the tax rate is greater than that of the initial economy. Proposition 2 indicates that the government, in this case, will decide a one-time reduction in its tax rate and transfers, which is all it can do due to the lack of commitment, in the next period. Therefore, the time-lagged strategy is not credible in the case without commitment.

This time-consistent policy (without commitment) leads to more severe economic inequalities, thereby hidering welfare improvements. Individuals will rationally understand the government's motive to reduce its tax rate and transfers in the next period. Changes in their expectations regarding the policy will cause individuals to increase precautionary savings and labor supply. This increase in precautionary motives prevents short-run welfare gains at the expense of long-run welfare losses, which is pronounced in the case with commitment. As Table 3 shows, the difference in welfare changes between the short and long runs is much smaller in the case without commitment (+1.34 pp) than the case with commitment (+4.36 pp). As shown in Table 2, this time-consistent policy is quantitatively worse than the time-inconsistent policy in terms of managing welfare outcomes, resulting in smaller improvements in welfare in the short run.

6 Conclusion

This paper examines how government commitment affects the optimal design of transfers along the transition. To this end, I arrange a dynamic game with successive governments in the standard incomplete-markets model and characterize and solve for its equilibria, both with and without government commitment. I find that when making policy decisions, commitment makes a difference in how the government exploits pecuniary externalities, causing a time inconsistency problem.

I assess the magnitude of this qualitative difference using a quantitative method in a calibrated economy. The quantitative analysis shows that commitment has significant impacts on the government's policy decisions along the equilibrium path. The key mechanism is that commitment enables the government to provide substantial transfers in the long run, resulting in upfront welfare gains while delaying welfare losses to the long run. Without commitment, the government lacks the ability to provide such long-term public insurance and cannot obtain as large welfare gains as the government with commitment.

Note that the solution method itself could provide many opportunities for studying unexplored research topics. Given the fundamental feature of Reiter (2010), this solution method can be compatible with aggregate uncertainty. This research direction makes it possible to extend the fiscal policy analyses with incomplete markets Bhandari et al. (2017a,b); Le Grand and Ragot (2025), investigating the implications of government commitment. Such analyses are left for future work.

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Online Appendix

Appendix A Proofs of Propositions and Lemmas

Lemma 1. *Under the CARA preference setting, for $t = 1$, individual decision rules for savings, $a_{t+1} = g_{a,t}(a_t, \epsilon_t)$ are linear in assets, a_t, and idiosyncratic shock, ϵ, with identical coefficients across individuals. That is, for $t = 1, 2$,*

$$a_{t+1} = g_{a,t}(a_t, \epsilon_t) = \kappa_{a,t}(\tau_t) \cdot a_t + \kappa_{\epsilon,t}(\tau_t) \cdot \epsilon_t + Q(\tau_t)$$

where, given τ_t, $\kappa_{a,t}(\tau_t)$, $\kappa_{\epsilon,t}(\tau_t)$, and $Q(\tau_t)$ are constants that are identical across all individuals. Hence,

$$K_{t+1} = \Gamma_t(K_t, \tau_t) = \kappa_{a,t}(\tau_t) \cdot K_t + \kappa_{\epsilon,t}(\tau_t) \cdot E[\epsilon_t] + Q_t(\tau_t).$$

Proof. : The CARA preference utility allows closed-form solutions for individuals' decision rules, as shown in in Wang (2003); Acharya et al. (2023). In period 2, given w_2, r_2, τ_2 and T_2, the Lagrangian problem for individuals associated with (a_2, ϵ_2) is given by

$$\mathcal{L} = -\frac{e^{-\sigma c_2}}{\sigma} + \chi e^{-\frac{(\epsilon_2 - n_2)}{\chi}} + \lambda_2((1-\tau_2)w_2 n_2 + r_2 a_2 + T_2 - c_2). \tag{64}$$

The FOCs are as follows:

$$[c_2]: \quad e^{-\sigma c_2} = \lambda_2 \tag{65}$$

$$[n_2]: \quad e^{-\frac{(\epsilon_2 - n_2)}{\chi}} = (1-\tau_2)w_2\lambda_2. \tag{66}$$

The above FOCs imply that

$$n_2(a_2, \epsilon_2) = \chi \ln(1-\tau_2)w_2 - \sigma\chi \cdot c_2(a_2, \epsilon_2) + \epsilon_2 \tag{67}$$

Substituting (67) into the utility function, we can represent the utility function only with c_2 as follows:

$$u(c_2, \epsilon_2 - n_2(c_2)) - \frac{e^{-\sigma c_2}}{\sigma} + \chi e^{-\frac{(\epsilon_2 - n_2)}{\chi}} = -e^{-\sigma c_2} \cdot \Big(\frac{1 + \sigma\chi(1-\tau_2)w_2}{\sigma}\Big). \tag{68}$$

Likewise, by substituting (67) into the individual budget, I have

$$(1 + (1 - \tau_2)w_2\sigma\chi)c_2 = r_2 \cdot a_2 + (1 - \tau_2)w_2 \cdot \epsilon_2 + \Xi_2(\tau_2, w_2, T_2) \tag{69}$$

where $\Xi_2(\tau_2, w_2, T_2) = T_2 + \chi(1 - \tau_2)w_2 \ln[(1 - \tau_2)w_2]$, which is identical across individuals. Note that $\chi(1 - \tau_2)w_2 \ln[(1 - \tau_2)w_2]$ represents the substitution effect and $(1 + (1 - \tau_2))w_2\sigma\chi c_2)$ represents the income effect.

Then, the decision rule for consumption , $c_2(a_2, \epsilon_2)$, is given by:

$$c_2(a_2, \epsilon_2) = \frac{1}{1 + (1 - \tau_2)w_2\sigma\chi} \cdot \Big(r_2 \cdot a_2 + (1 - \tau_2)w_2 \cdot \epsilon_2 + \Xi_2(\tau_2, w_2, T_2)\Big) \tag{70}$$

Substituting (70) and (67), the households value in period 2, $v_2(a_2, \epsilon_2)$, is as follows:

$$\begin{aligned} v_2(a_2, \epsilon_2) &= -\Big(\frac{1 + \sigma\chi(1 - \tau_2)w_2}{\sigma}\Big)e^{-\sigma c_2} \\ &= -\Big(\frac{1 + \sigma\chi(1 - \tau_2)w_2}{\sigma}\Big)e^{-\left(\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\cdot\left(r_2\cdot a_2+(1-\tau_2)w_2\cdot\epsilon_2+\Xi_2(\tau_2,w_2,T_2)\right)\right)}. \end{aligned} \tag{71}$$

In period 1, n_1 can be arranged in terms of c_1 by using the FOCs as before:

$$n_1(a_1, \epsilon_1) = \chi \ln(1 - \tau_1)w_1 - \sigma\chi \cdot c_1(a_1, \epsilon_1) + \epsilon_1 \tag{72}$$

Substituting (72) into the utility function and the individual budget, the Lagrangian problem is given by:

$$\begin{aligned} \mathcal{L} = -\Big(\frac{1 + \sigma\chi(1 - \tau_1)w_1}{\sigma}\Big)e^{-\sigma c_1} + \beta E_{\epsilon_2}[v_2(a_2, \epsilon_2)] \\ +\lambda_1\big((1 - \tau_1)w_1\epsilon_1 + r_1a_1 + \chi(1 - \tau_1)w_1 \ln[(1 - \tau_1)w_1] + T_1 - (1 + (1 - \tau_1)w_1\sigma\chi)c_1 - a_2). \end{aligned}$$

Then, the FOC is as follows:

$$e^{-\sigma c_1} = \beta E_{\epsilon_2}\Big[\frac{\partial v_2(a_2, \epsilon_2)}{\partial a_2}\Big]. \tag{73}$$

$\beta E_{\epsilon_2}\left[\frac{\partial v_2(a_2,\epsilon_2)}{\partial a_2}\right]$ can be arranged as follows:

$$\begin{aligned}\beta E_{\epsilon_2}\Big[\frac{\partial v_2(a_2,\epsilon_2)}{\partial a_2}\Big] &= \beta E_{\epsilon_2}\Big[\Big(\frac{1+\sigma\chi(1-\tau_2)w_2}{\sigma}\Big)e^{-\sigma c_2}\cdot(\sigma\cdot\frac{\partial c_2}{\partial a_2})\Big] = \beta r_2 E_{\epsilon_2}\Big[e^{-\sigma c_2}\Big] \\ &= \beta r_2 E_{\epsilon_2}\Big[e^{-\left(\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\right)\cdot\left(r_2\cdot a_2+(1-\tau_2)w_2\cdot\epsilon_2+\Xi_2(\tau_2,w_2,T_2)\right)}\Big] \\ &= e^{-\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\left(r_2a_2+\Xi_2(\tau_2,w_2,T_2)-\left(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\right)\ln(\beta r_2)\right)}\cdot E_{\epsilon_2}\Big[e^{-\left(\frac{(1-\tau_2)w_2\sigma}{1+(1-\tau_2)w_2\sigma\chi}\right)\epsilon_2}\Big]\end{aligned} \tag{74}$$

Because $\log(\epsilon)\sim N(0,\sigma_\epsilon^2)$, $E_{\epsilon_2}\Big[e^{-\left(\frac{(1-\tau_2)w_2\sigma}{1+(1-\tau_2)w_2\sigma\chi}\right)\epsilon_2}\Big] = e^{-\frac{1}{2}\left(\frac{(1-\tau_2)w_2\sigma}{1+(1-\tau_2)w_2\sigma\chi}\right)^2\sigma_\epsilon^2}$. Thus,

$$\beta E_{\epsilon_2}\Big[\frac{\partial v_2(a_2,\epsilon_2)}{\partial a_2}\Big] = e^{-\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\left(r_2a_2+\Phi_2(\tau_2,w_2,r_2,T_2)\right)} \tag{75}$$

where $\Phi_2(\tau_2,w_2,r_2,T_2) = \Xi_2(\tau_2,w_2,T_2) - \left(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\right)\ln(\beta r_2) - \frac{1}{2}\cdot\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\cdot((1-\tau_2)w_2)^2\sigma_\epsilon^2$. Plugging (75) into the FOC (73), and taking the log, I obtain,

$$\begin{aligned}c_1 &= \frac{1}{1+(1-\tau_2)w_2\sigma\chi}\Big(r_2a_2+\Phi_2(\tau_2,w_2,r_2,T_2)\Big) \qquad (76)\\ &= \frac{1}{1+(1-\tau_2)w_2\sigma\chi}\Big[r_2\Big((1-\tau_1)w_1\epsilon_1+r_1a_1+\Xi_1(\tau_1,w_1,T_1)-(1+(1-\tau_1)w_1\sigma\chi)c_1\Big) \\ &\quad+\Phi_2(\tau_2,w_2,r_2,T_2)\Big]\end{aligned}$$

where $\Xi_1(\tau_1,w_1,T_1) = T_1+\chi(1-\tau_1)w_1\ln[(1-\tau_1)w_1]$. Then, the decision rule for consumption in period 1, $c_1(a_1,\epsilon_1)$ is given by

$$c_1(a_1,\epsilon_1) = \frac{r_2\,r_1\,a_1+r_2\,(1-\tau_1)w_1\,\epsilon_1+r_2\Xi_1(\tau_1,w_1,T_1)+\Phi_2(\tau_2,w_2,r_2,T_2)}{1+(1-\tau_2)w_2\sigma\chi+r_2(1+(1-\tau_1)w_1\sigma\chi)}. \tag{77}$$

Substitute (77) into the budget constraint appearing in the budget constraint. Then, the decision rule for savings $a_2(a_1,\epsilon_1)$ is given by

$$\begin{aligned}a_2(a_1,\epsilon_1) =& \frac{1+(1-\tau_2)w_2\sigma\chi}{1+(1-\tau_2)w_2\sigma\chi+r_2(1+(1-\tau_1)w_1\sigma\chi)}\cdot\Big(r_1\,a_1+(1-\tau_1)w_1\,\epsilon_1+\Xi_1(\tau_1,w_1,T_1)\Big) \\ &-\left(\frac{1+(1-\tau_1)w_1\sigma\chi}{1+(1-\tau_2)w_2\sigma\chi+r_2(1+(1-\tau_1)w_1\sigma\chi)}\right)\cdot\Phi_2(\tau_2,w_2,r_2,T_2).\end{aligned} \tag{78}$$

Note that $a_2(a_1, \epsilon_1)$ is linear in a_1 and ϵ_1 and identical for all individuals.

Hence, for $t = 1$,

$$K_{t+1} = \Gamma(K_t, \tau_t) = \kappa_{a,t}(\tau_t) \cdot K_t + \kappa_{\epsilon,t}(\tau_t) \cdot e^{\frac{\sigma_\epsilon^2}{2}} + Q_t(\tau_t) \tag{79}$$

where

$$\begin{aligned}
\kappa_{a,1}(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)r_1}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)} \\
\kappa_{\epsilon,1}(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)(1-\tau_1)w_1}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)} \\
Q_1(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)\Xi_1(\tau_1, w_1, T_1)}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)} \\
&\quad - \left(\frac{1+(1-\tau_1)w_1\sigma\chi}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)}\right) \cdot \Phi_2(\tau_2, w_2, r_2, T_2).
\end{aligned}$$

□

Lemma 2. $\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = \frac{\partial r_2}{\partial K_2} + \frac{\partial r_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = 0$ *under the Cobb-Douglas production function.*

Proof. Note that

$$\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = F_{NK,2} + F_{NN,2}\frac{\partial N_2}{\partial K_2}.$$

Because $F_{NN}N + F_{NK}K = 0$,

$$\frac{\partial w_2}{\partial K_2} + \frac{\partial w_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} = F_{NK,2} + F_{NN,2}\frac{\partial N_2}{\partial K_2} = F_{NK,2}\Big(1 - \frac{K_2}{N_2}\frac{\partial N_2}{\partial K_2}\Big) = F_{NK,2}\Big(1 - \sigma_{K,N}\Big) = 0$$

because the elasticity of substitution between capital and labor, $\sigma_{K,N}$, is one for the Cobb-Douglas production function. Likewise, $F_{KK}K + F_{KN}N = 0$ and

$$\begin{aligned}
\frac{\partial r_2}{\partial K_2} + \frac{\partial r_2}{\partial N_2} \cdot \frac{\partial N_2}{\partial K_2} &= F_{KK,2} + F_{KN,2}\frac{\partial N_2}{\partial K_2} = \frac{N_2}{K_2}F_{NK,2}\Big(\frac{K_2}{N_2}\frac{\partial N_2}{\partial K_2} - 1\Big) \\
&= \frac{N_2}{K_2}F_{NK,2}\Big(\sigma_{K,N} - 1\Big) = 0
\end{aligned}$$

□

Lemma 3. $\frac{\partial \Gamma_1(K_1, \tau_1)}{\partial \tau_1} < 0$ when savings driven by precautionary motives and patience are sufficiently large. Specifically, β and σ_ϵ^2 must satisfy

$$\Delta(\beta,\sigma_\epsilon^2) > r_2\Big(r_1K_1 + T_1 + \chi(1-\tau_1)\ln[(1-\tau_1)w_1]\Big)$$

where

$$\Delta(\beta,\sigma_\epsilon^2) = \Big(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\Big)\ln(\beta r_2) + \frac{1}{2}\cdot\frac{\sigma((1-\tau_2)w_2)^2\sigma_\epsilon^2}{1+(1-\tau_2)w_2\sigma\chi} + r_2(1-\tau_1)w_1e^{\sigma_\epsilon^2}.$$

Proof. From the proof in Lemma 1,

$$K_2 = \Gamma(K_1,\tau_1) = \kappa_{a,1}(\tau_1)\cdot K_1 + \kappa_{\epsilon,1}(\tau_1)\cdot e^{\frac{\sigma_\epsilon^2}{2}} + Q_1(\tau_1) \tag{80}$$

where

$$\begin{aligned}
\kappa_{a,1}(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)r_1}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)}\\
\kappa_{\epsilon,1}(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)(1-\tau_1)w_1}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)}\\
Q_1(\tau_1) &= \frac{(1+(1-\tau_2)w_2\sigma\chi)\Xi_1(\tau_1,w_1,T_1)}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)}\\
&\quad - \left(\frac{1+(1-\tau_1)w_1\sigma\chi}{1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi)}\right)\cdot\Phi_2(\tau_2,w_2,r_2,T_2).
\end{aligned}$$

Then, $\frac{\partial K_2}{\partial\tau_1}$ is given by:

$$\frac{\partial K_2}{\partial\tau_1} = \underbrace{\frac{Aw_1\sigma\chi}{B^2}\left[r_2\left(r_1K_1 + (1-\tau_1)w_1e^{\sigma_\epsilon^2} + \Xi_1\right) + \Phi_2\right]}_{\text{X}} + \underbrace{\frac{A}{B}\left(-w_2e^{\sigma_\epsilon^2} + \frac{\partial\Xi_1}{\partial\tau_1}\right)}_{\text{Y}} \tag{81}$$

where

$$\begin{gathered}
A = 1+(1-\tau_2)w_2\sigma\chi > 0\\
B = 1+(1-\tau_2)w_2\sigma\chi + r_2(1+(1-\tau_1)w_1\sigma\chi) > 0\\
\Xi_1 = T_1 + \chi(1-\tau_1)w_1\ln[(1-\tau_1)w_1]\\
\Phi_2 = \Xi_2 - \Big(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\Big)\ln(\beta r_2) - \frac{1}{2}\cdot\frac{\sigma}{1+(1-\tau_2)w_2\sigma\chi}\cdot((1-\tau_2)w_2)^2\sigma_\epsilon^2.
\end{gathered}$$

Then, $\frac{\partial K_2}{\partial\tau_1}$ is negative if both X and Y are negative. X is negative when

$$\begin{gathered}
\Big(\frac{1+(1-\tau_2)w_2\sigma\chi}{\sigma}\Big)\ln(\beta r_2) + \frac{1}{2}\cdot\frac{\sigma((1-\tau_2)w_2)^2\sigma_\epsilon^2}{1+(1-\tau_2)w_2\sigma\chi} + r_2(1-\tau_1)w_1e^{\sigma_\epsilon^2}\\
> r_2\Big(r_1K_1 + T_1 + \chi(1-\tau_1)\ln[(1-\tau_1)w_1]\Big)
\end{gathered} \tag{82}$$

Y is negative when

$$\frac{\partial \Xi_1}{\partial \tau_1} = -\chi((\ln[(1-\tau_1)w_1]+1) < w_1 e^{\sigma_\epsilon^1} \tag{83}$$

This relation holds true in all economically meaningful cases except when the after-tax rate $(1-\tau_1)w_1$ becomes extremely small. □

Proposition 3. When deciding τ_2, the government with commitment satisfies

$$\int E_{i,1}\left[\frac{\partial \tau_1}{\partial \tau_2}\cdot \mathcal{G}_{i,1} + \beta\Big(1+\frac{\partial \tau_2}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2}\Big)\cdot \mathcal{G}_{i,2}\right]\mu_1(d(i)) = -\int E_{i,1}\left[\beta\,\frac{\Delta K_2}{\Delta \tau_2}\cdot \mathcal{L}_{i,2}\right]\mu_1(d(i))$$

where $\frac{\partial \tau_1}{\partial \tau_2}$ and $\frac{\Delta K_2}{\Delta \tau_2}$ are non-zero.

Proof. Let's assume that the tax path is on the optimal Ramsey solution. Then, the partial derivative of value for individual i in period 1, $\frac{\partial v_{i,1}(\tau_1,\tau_2)}{\partial \tau_2}$ is given by:

$$\frac{\partial v_{i,1}(\tau_1,\tau_2)}{\partial \tau_2} = E_{i,1}\left[u_{c,i,1}\cdot\frac{dc_{i,1}}{d\tau_2} - u_{l,i,1}\frac{dn_{i,1}}{d\tau_2} + \beta\cdot\Big(u_{c,i,2}\cdot\frac{dc_{i,2}}{d\tau_2} - u_{l,i,1}\frac{dn_{i,2}}{d\tau_2}\Big)\right] \tag{84}$$

where

$$\frac{dc_{i,1}}{d\tau_2} = \frac{\partial c_{i,1}}{\partial \tau_1}\cdot\frac{\partial \tau_1}{\partial \tau_2} \tag{85}$$

$$\frac{dn_{i,1}}{d\tau_2} = \frac{\partial n_{i,1}}{\partial \tau_1}\cdot\frac{\partial \tau_1}{\partial \tau_2} \tag{86}$$

$$\frac{dc_{i,2}}{d\tau_2} = \frac{\partial c_{i,2}}{\partial \tau_2}\cdot\Big(1+\frac{\partial \tau_2}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2}\Big) + \frac{\partial c_{i,2}}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2} \tag{87}$$

$$\frac{dn_{i,2}}{d\tau_2} = \frac{\partial n_{i,2}}{\partial \tau_2}\cdot\Big(1+\frac{\partial \tau_2}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2}\Big) + \frac{\partial n_{i,2}}{\partial K_2}\cdot\frac{\Delta K_2}{\Delta \tau_2} \tag{88}$$

Note $\frac{\Delta K_2}{\Delta \tau_2} = \frac{\partial K_2}{\partial \tau_1}\cdot\frac{\partial \tau_1}{\partial \tau_2}$. Recall that the derivation of Proposition 2 implies

$$u_{c,i,t}\cdot\frac{\partial c_{i,t}}{\partial \tau_t} - u_{l,i,t}\frac{\partial n_{i,t}}{\partial \tau_t} = \mathcal{G}_{i,t} \tag{89}$$

$$u_{c,i,t}\cdot\frac{\partial c_{i,t}}{\partial K_t} - u_{l,i,t}\frac{\partial n_{i,t}}{\partial K_t} = \mathcal{L}_{i,t} \tag{90}$$

where

$$\mathcal{G}_{i,t} = u_{c,i,t}\cdot(w_t + \tau_t F_{NN,t}\cdot\frac{\partial N_t}{\partial \tau_t})(N_t - n_{i,t}) + u_{c,i,t}\cdot\Big(\frac{n_{i,t}}{N_t} - \frac{a_{i,t}}{K_t}\Big)\Big(\frac{K_t}{N_t}\Big)^{\theta}(-\theta)(1-\theta)\frac{\partial N_t}{\partial \tau_t} + u_{c,i,t}\cdot\tau_t w_t\frac{\partial N_t}{\partial \tau_t} \tag{91}$$

$$\mathcal{L}_{i,t} = u_{c,i,t}\cdot\tau_t w_t\frac{\partial N_t}{\partial K_t} \tag{92}$$

Because $\{\tau_1, \tau_2\}$ is the Ramsey solution, $\int \frac{\partial v_{i,1}(\tau_1,\tau_2)}{\partial \tau_2}\mu_1(d(i)) = 0$. This condition implies

$$\int E_{i,1}\left[\frac{\partial \tau_1}{\partial \tau_2} \cdot \mathcal{G}_{i,1} + \beta\Big(1 + \frac{\partial \tau_2}{\partial K_2} \cdot \frac{\Delta K_2}{\Delta \tau_2}\Big) \cdot \mathcal{G}_{i,2}\right]\mu_1(d(i)) = -\int E_{i,1}\left[\beta\, \frac{\Delta K_2}{\Delta \tau_2} \cdot \mathcal{L}_{i,2}\right]\mu_1(d(i)).$$

To prove $\frac{\partial \tau_1}{\partial \tau_2} \neq 0$ & $\frac{\Delta K_2}{\Delta \tau_2} \neq 0$, we need to use $v_1(a_1, \epsilon_1)$. Using the decision rules in the proof of Lemma 1, $v_1(a_1, \epsilon_1)$ is given by

$$v_1(a_1, \epsilon_1) = -\frac{e^{-\sigma c_1(a_1,\epsilon_1)}}{\sigma} \cdot \Big(\frac{1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + \sigma\chi(1-\tau_1)w_1)}{r_2}\Big) \tag{93}$$

where

$$c_1(a_1, \epsilon_1) = \frac{r_2\, r_1\, a_1 + r_2\,(1-\tau_1)w_1\,\epsilon_1 + r_2\Xi_1(\tau_1, w_1, T_1) + \Phi_2(\tau_2, w_2, r_2, T_2)}{1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + (1-\tau_1)w_1\sigma\chi)}.$$

Using the implicit function theorem,

$$\frac{\partial \tau_1}{\partial \tau_2} = -\frac{\frac{\partial \int v_{i,1}\mu_1(d(i))}{\partial \tau_2}}{\frac{\partial \int v_{i,1}\mu_1(d(i))}{\partial \tau_1}} = -\frac{\int \frac{\partial v_{i,1}}{\partial \tau_2}\mu_1(d(i))}{\int \frac{\partial v_{i,1}}{\partial \tau_1}\mu_1(d(i))} \neq 0 \tag{94}$$

where

$$\frac{\partial v_{i,1}}{\partial \tau_1} = e^{-\sigma c_{i,1}}\frac{\partial c_{i,1}}{\partial \tau_1} = e^{-\sigma c_{i1}}\Bigg(\frac{-r_2(w_1\epsilon_1 - \chi\ln(1-\tau_1)w_1 + 1)}{1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + (1-\tau_1)w_1\sigma\chi)}$$
$$+\frac{r_2 w_1\sigma\chi(r_2\, r_1\, a_1 + r_2\,(1-\tau_1)w_1\,\epsilon_1 + r_2\Xi_1(\tau_1, w_1, T_1) + \Phi_2(\tau_2, w_2, r_2, T_2)))}{[1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + (1-\tau_1)w_1\sigma\chi)]^2}\Bigg) \neq 0 \tag{95}$$

$$\frac{\partial v_{i,1}}{\partial \tau_2} = e^{-\sigma c_{i,1}}\frac{\partial c_{i,1}}{\partial \tau_2} = e^{-\sigma c_{i,1}} \cdot \Bigg(\frac{\sigma w_2^2\sigma_\epsilon^2(1-\tau_2)[1 + (1-\tau_2)w_2\sigma\chi) + \sigma\chi(1-\tau_2)w_2]}{[1 + (1-\tau_2)w_2\sigma\chi)^2(1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + (1-\tau_1)w_1\sigma\chi)]}$$
$$+\frac{w_2\sigma\chi(r_2\, r_1\, a_1 + r_2\,(1-\tau_1)w_1\,\epsilon_1 + r_2\Xi_1(\tau_1, w_1, T_1) + \Phi_2(\tau_2, w_2, r_2, T_2)}{[1 + (1-\tau_2)w_2\sigma\chi + r_2(1 + (1-\tau_1)w_1\sigma\chi)]^2}\Bigg) > 0. \tag{96}$$

Because Lemma 4 indicates $\frac{\partial K_2}{\partial \tau_1} < 0$,

$$\frac{\Delta K_2}{\Delta \tau_2} = \frac{\partial K_2}{\partial \tau_1} \cdot \frac{\partial \tau_1}{\partial \tau_2} \neq 0. \tag{97}$$

□

Appendix B Numerical Solution Algorithm

Solving the Markov-Perfect Equilibria (MPE) of consecutive governments entails heavy computational burdens with heterogeneous agents. As in standard macroeconomic heterogeneous agent models, individual decisions should be consistent with the aggregate law of motion for the distribution of agents. On top of that, the aggregate tax policy function must be compatible with individual decisions and the aggregate law of motion for the distribution of agents. In other words, these three equilibrium objects—individual decisions, the law of motion for the distribution, and the tax policy function—have to be consistent with each other in the Markov-perfect equilibrium.

I address this computational issue by taking ideas from the Backward Induction method of Reiter (2010). This method discretizes the aggregate state into finite grid points. For each aggregate grid point, the Backward Induction algorithm allows updating the aggregate law of motion while solving the decision rules thanks to the existence of the proxy distribution. This means that for each aggregate grid point, the backward induction algorithm would make it possible to approximate not only the aggregate law of motion for the distribution; but also the tax policy function consistent with the choice of government lacking commitment. With the value function, this endogenous tax policy outcome can be directly obtained when the proxy distribution is explicitly available.

Unfortunately, the original method in Reiter (2010) cannot directly be applied to the MPE models because the existence of off-the-equilibrium paths makes it challenging to arrange the proxy distribution. In the model of Krusell and Smith (1998), for which the method of Reiter (2010) is originally designed, the distribution of TFP shocks Z is stationary, thus all the aggregate states Z are not measure zero. With a positive probability, all the states Z are realized on the equilibrium path. However, the MPE economy does not have this property. Let us think about a case in a stationary distribution. In this equilibrium, this optimal policy is obtained by comparing among one-time deviation policies. Some tax paths would not be reached at all on the equilibrium path.

I have three variations from the original backward induction method. First, I have to approximate not only the aggregate law of motion for distributions but also the tax policy function that is endogenous. I find these mappings in a non-parametric way, as in Reiter (2010) distribution of the simulations as the previous proxy distribution for each aggregate state. Finally, I modify the construction of the reference distributions in Reiter (2002, 2010), reflecting the features of economies in the MPE wherein how many times a policy off-equilibrium takes place is unknown before simulations.

Here, I show how to apply the algorithm to the case with proportional income taxes. Note that I solve all the value functions in the following steps with the Endogenous Grid Method of Carroll (2006).

B.1 Notation and Sketch of the Solution Method

The aggregate law of motion Γ and the tax policy function Ψ are evolved with the distribution μ that is an infinite dimensional equilibrium object, and thus it not not feasible in computations. To handle this issue, the Backward Induction method replaces μ with m, a set of moments from the distribution and discretize it. Here, I take the mean of the distribution and discretize it, $M = \{m_1, \cdots, m_{N_m}\}$. This setting allows me to define the aggregate law motion and the tax policy function on each grid m_{i_m} such that $m' = G(m_{i_m}, \tau)$ where $\tau = P(m_{i_m})$. Note that G and P do not rely on a parametric law.

Across a grid of aggregate states m_{i_m}, each point selecting a proxy distribution, the Backward Induction method simultaneously solves for households' decision rules and an intratemporally consistent end-of-period distribution. This implies a future approximate aggregate state consistent with households' expectation ($m' = G(m_{i_m}, \tau)$) for any τ. Likewise, the backward induction can find the tax policy function that is consistent with the choice of the government, by using the household's value functions and the proxy distribution ($\tau = P(m_{i_m})$). These mappings imply that G interacts with P. Given P, first, I find G during the iteration of value functions and then update P with the value function and proxy distribution. I repeat this until P is convergent.

Given a distribution over individual states at each aggregate grid point m_{i_m}, my goal is to obtain the law of motion for households distribution G and the tax policy function P that are intratemporally consistent with the end-of-period distribution and the choice of the government. Explicitly,

$$m' = G(m_{i_m}, \tau) \tag{98}$$

$$\tau = P(m_{i_m}) \tag{99}$$

$$w = W(m_{i_m}) \tag{100}$$

$$T = TR(m_{i_m}) \tag{101}$$

(98) is to approximate Γ, (99) is to do Ψ, (100) is the mapping for the market wage, and (101) is the mapping for transfers.

The backward induction method explicitly computes G, P, τ, W, and T, given a set of proxy distributions before the simulation step. An issue is that computing $G(m_{i_m}, \tau)$ in solving the value is costly because it depends on τ not only on the equilibrium path but also off-the-equilibrium path. To address this issue, I reduce $G(m_{i_m}, \tau)$ into $\tilde{G}(m_{i_m}) = G(m_{i_m}, P(m_{i_m}))$ while solving the value function; retrieve $G(m_{i_m}, \tau)$ with the converged value function and the proxy distribution. Note that $G(m_{i_m}, \tau)$ must also satisfy an intratemporal consistency.

B.2 Computing the Aggregate Mappings given a Set of Proxy Distributions

(1) Given $v^n(a, \epsilon; m)$ and $\tau = P^q(m,)$, where $n = 1, 2, \cdots$ and $q = 1, 2, \cdots$ denote the rounds of iteration, at grid m_{i_m}, where $i_m = 1, \cdots, N_m$ is the grid index for m, solve for intratemporally consistent m'.

a) Guess m' . Using v^n and P^q, solve for $a' = g_a^{n+1}(a, \epsilon; m_{i_m})$ and $n = g_n^{n+1}(a, \epsilon; m_{i_m})$ using

$$v^{n+1}(a, \epsilon; m_{i_m}) = \max_{c,a',n} u(c, 1-n) + \beta \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} v^n(a', \epsilon', m') \tag{102}$$

such that

$$c + a' = (1-\tau)w(m_{i_m})\epsilon n + (1 + r(m_{i_m}) - \delta)a + T(m_{i_m})$$

$$\tau = P^q(m_{i_m})$$

b) Using the proxy distribution, $\mu(a, \epsilon; m_{i_m})$, compute the distribution consistent with capital stock at the end of period $\tilde{m}'$, wage $\tilde{w}$, and transfers $\tilde{T}$.

$$\tilde{m}' = \int g_a^{n+1}(a, \epsilon; m_{i_m})\mu(\mathbf{d}(a \times \epsilon); m_{i_m}) \tag{103}$$

$$\tilde{w}(m_{i_m}) = (1-\theta)\left(\frac{m_{i_m}}{\tilde{N}}\right)^{\theta} \tag{104}$$

$$\tilde{T}(m_{i_m}) = P^q(m_{i_m}) \cdot w(m_{i_m})\tilde{N} - G \tag{105}$$

where

$$\tilde{N} = \int g_n^{n+1}(a, \epsilon; m_{i_m})\epsilon\, \mu(\mathbf{d}(a \times \epsilon); m_{i_m})$$

c) If $\max\left\{|\tilde{m}' - m'|, |\tilde{w} - w|, |\tilde{T} - T|\right\}$ >precision, update m', w, and T; set $r = \theta\left(\frac{w}{1-\theta}\right)^{\frac{\theta-1}{\theta}} - \delta$; and return to a).

(2) Having found $m' = \tilde{G}^q(m_{i_m})$, $w = W^q(m_{i_m})$, and $T = TR^q(m_{i_m})$, use (102) to define $v^{n+1}(a, \epsilon; m)$ consistent with $v^n(a', \epsilon'; G^q(m_{i_m}), P^q(m_{i_m}))$. If $||v^{n+1} - v^n|| >$ precision, $n = n + 1$ and return to (1).

(3) For each grid (m_{i_m}, τ_{i_τ}), where $i_\tau = 1, \cdots, N_\tau$ is the grid index for τ, retrieve $G^q(m_{i_m}, \tau_{i_\tau})$ by solving for intratempollay consistent $\hat{m}'$.

a) For each (m_{i_m}, τ_{i_τ}), guess $\hat{m}'$. With v^∞, solve for $a' = \hat{g}_a(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ and $n = \hat{g}_n(a, \epsilon; m_{i_m}, \tau_{i_\tau}, \tau'_{i_\tau})$ using

$$\hat{v}(a, \epsilon; m_{i_m}, \tau_{i_\tau}) = \max_{c,a',n} u(c, 1-n) + \beta \sum_{\epsilon'} \pi_{\epsilon,\epsilon'} v^\infty(a', \epsilon', m')$$

such that

$$c + a' = (1 - \tau_{i_\tau})\hat{w}(m_{i_m}, \tau_{i_\tau})\epsilon n + (1 + \hat{r}(m_{i_m}, \tau_{i_\tau}) - \delta)a + \hat{T}$$

b) For each (m_{i_m}, τ_{i_τ}), using the proxy distribution, $\mu(a, \epsilon; m_{i_m})$, compute the distribution consistent with the end of period aggregate capital stock.

$$\hat{m}' = \int \hat{g}_a(a, \epsilon; m_{i_m}, \tau_{i_\tau})\mu(\mathbf{d}(a \times \epsilon); m_{i_m})$$

$$\hat{w}(m_{i_m}, \tau_{i_\tau}) = (1-\theta)\left(\frac{m_{i_m}}{\hat{N}}\right)^\theta$$

$$\tilde{T}(m_{i_m}, \tau_{i_\tau}) = \tau_{i_\tau}\hat{w}\hat{N} - G$$

where

$$\hat{N} = \int \hat{g}_n(a, \epsilon; m_{i_m}, \tau_{i_\tau})\epsilon\, \mu(\mathbf{d}(a \times \epsilon); m_{i_m})$$

c) If $\max\left\{|\tilde{m}' - \hat{m}'|, |\tilde{w} - \hat{w}|, |\tilde{T} - \hat{T}|\right\} >$ precision, update $\hat{m}', \hat{w}$, and $\hat{T}$; set $\hat{r} = \theta\left(\frac{\hat{w}}{1-\theta}\right)^{\frac{\theta-1}{\theta}} - \delta$; and return to a).

(4) Having found $m' = G^q(m_{i_m}, \tau_{i_\tau})$, keep $G^q(m_{i_m}, \tau_{i_\tau})$. Note that here is no update of the value.

(5) For each aggregate grid m_{i_m}, find $\tau^{m,q}(m_{i_m})$.

a) Given $(a, \epsilon; m_{i_m})$, using $\hat{v}(a, \epsilon; m_{i_m}, \tau_{i_\tau})$ in (3) - a), solve $\psi^q(m_{i_m})$ as follows:

$$\psi^q(m_{i_m}) = \underset{\tilde{\tau}}{\text{argmax}}\, \hat{V}(m_{i_m}, \tilde{\tau}) = \underset{\tilde{\tau}}{\text{argmax}} \int \hat{v}(a, \epsilon; m_{i_m}, \tilde{\tau})\mu(\mathbf{d}(a \times \epsilon); m_{i_m}) \quad (106)$$

The golden section search is used to find $\Psi^q(m_{i_m})$ with a cubic spline for $\hat{V}$ over $\tilde{\tau}$.

b) For each aggregate grid m_{i_m}, if $P^q(m_{i_m}) = \psi^q(m_{i_m})$, G^q and P^q are the solutions, given the proxy distribution. Then, go to the next step. Otherwise, they are not the solutions. Take $P^{q+1} = \omega \cdot P^q + (1-\omega) \cdot \psi^q$, and go back to (1).

B.3 Constructing the Reference Distributions

Until now, I have solved G and P for a given set of proxy distributions. In the following step, I will simulate the economy and update the distribution selection function, as in Reiter (2002, 2010); but, the simulation step in this paper is substantially different from that in his method. He addresses Krusell and Smith (1998) model where aggregate uncertainty exists. Thus, what matters in his papers is to obtain the Ergodic set that is not affected by the initial distribution.

However, in economies without government commitment, it is important to obtain not only distributions on the equilibrium path but also those off-the-equilibrium path. For example, let us think of an economy without commitment in the stationary equilibrium. Then, there will be a unique value of $\tau^* = P(m^*)$ and $m^* = G(m^*, \tau^*)$. In this case, I may not know the value of other alternatives because this economy has nothing but the unique equilibrium path. This difficulty might lead the previous studies to employ local solution methods in solving this type of the MPE. By constrast, my approach is a global solution method, which means I need proxy distributions over all types of off-the-equilibrium paths.

To reserve distributions off-the-equilibrium path, I use the proxy distributions in the previous step as the initial distribution for the simulation. For each (m_{i_m}, τ_{i_τ}), I run a simulations for T periods from the proxy distribution $\mu_0 = \mu(a, \epsilon; m_{i_m})$, implying the number of simulations is N_m and that of simulation outcomes is $T \times N_m$. Note that any type of m_{i_m} will be observed at least once in the simulations. For each m_{i_m}, using $\mu_0 = \mu(a, \epsilon; m_{i_m})$ and v^∞ from the previous step, I simulate the economy in a forward manner. I compute the market cleared w_t and r_t and transfers T_t satisfying the government budget condition for each simulation period $t = 1, \cdots, T$. In addition, I solve the government's problem τ_t for each simulation period $t = 1, \cdots, T$ with the $m' = G(m_{i_m}, \tau_{i_\tau})$ obtained in the previous step.

I gather all the simulated distributions and rearrange the index as $\tilde{t} = 1, \cdots, T \times N_m$. In creating the reference distributions from the simulation, I need a measure of distance for the moments of a distribution. For m, define an inverse norm

$$d(m_0, m_1) = (m_0 - m_1)^{-4} \tag{107}$$

In contrast to an economy with uncertainty, the initial simulation results should be preserved, having to be used to construct the reference distributions off-the-equilibrium path (non-Ergodic set). For each t, when m_t with $m_t \in [m_k, m_{k+1})$,

$$d_1(m_k) = d_1(m_k) + (m_t - m_k)^{-4}$$
$$d_1(m_{k+1}) = d_1(m_{k+1}) + (m_t - m_{k+1})^{-4}$$

Above m_k is the k-th grid point for m. Note that distances between a given node and non-adjacent moments are not taken into account, which is different from the corresponding step in Reiter (2002, 2010).

I construct the reference distributions for each m_{i_m} using the above, when $m_{\tilde{t}} \in [m_{i_m}, m_{i_m+1})$,

$$\mu^r(a, \epsilon; m_{i_m}) = \sum_{\tilde{t}=1}^{T\times N_m} \frac{d(m_{i_m}, m_{\tilde{t}})}{d_1(m_{i_m})} \mu_{\tilde{t}}(a, \epsilon). \tag{108}$$

Each reference distribution is a weighted sum of distributions over the simulation only when simulated moments are adjacent to a given pair of grid points m_{i_m}. Since the simulation moments are not on an Ergodic set, this should be considered.

I arrange the finite grid, which is the distribution support, as explicit. The distribution over (a, ϵ) used below size $(N_a \times N_\epsilon)$ with $\epsilon \in E = \{\epsilon_1, \cdots, \epsilon_{N_\epsilon}\}$ and $a \in A = \{a_1, \cdots, a_{N_a}\}$. I represent $\mu^r(a, \epsilon; m_{i_m})$ using $\mu^r_{i_a,i_\epsilon}(i_m)$, indexing $(a_{i_a}, \epsilon_{i_\epsilon})$ over $A \times E$ for m_{i_m}. The moment of a reference distribution, $\sum_{i_\epsilon}^{N_\epsilon} \mu^r_{i_a,i_\epsilon}(i_m) a_{i_a}$, will not be consistent with m_{i_m}. However, the updated proxy distribution at i_m will have this property.

B.4 Updating the Proxy Distributions

Following Reiter (2002, 2010), for each aggregate grid i_m, I solve for μ_{i_a,i_ϵ}, the proxy distribution, as the solution to a problem that minimizes the distance to the reference distribution while imposing that each type of sums to its reference value and moment consistency.

$$\min_{\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon}} \sum_{i_a=1}^{N_a} \sum_{i_\epsilon=1}^{N_\epsilon} \left(\mu_{i_a,i_\epsilon} - \mu^r_{i_a,i_\epsilon}(i_m)\right)^2 \tag{109}$$

$$\sum_{i_a=1}^{N_a} \mu_{i_a,1_\epsilon} = \sum_{i_a=1}^{N_a} \mu^r_{i_a,i_\epsilon}(i_m) \text{ for } i = 1, \cdots, N_\epsilon \tag{110}$$

$$\sum_{i_\epsilon=1}^{N_\epsilon} \sum_{i_a=1}^{N_a} \mu_{i_a,i_\epsilon} \cdot a_{i_a} = m_{i_m} \tag{111}$$

$$\mu_{i_a,i_\epsilon} \geq 0 \tag{112}$$

The first-order condition for μ_{i_a,i_ϵ} with λ_i as the LaGrange multiplier for (110) and ω the multiplier (111) is

$$2(\mu_{i_a,i_\epsilon} - \mu^r_{i_a,i_\epsilon}(i_m)) - \lambda_i - \omega \cdot a_{i_a} = 0 \tag{113}$$

If I ignore the non-negative constraints for probabilities in (112), I have N_ϵ constraint in (110). 1

constraint in (111) and $N_a \times N_\epsilon$ first-order conditions in (112). These are a system of $N_a \times N_\epsilon + N_\epsilon + 1$ linear equations in $\left(\{\mu_{i_a,i_\epsilon}\}_{i_a=1,i_\epsilon=1}^{N_a,N_\epsilon}, \{\lambda_{i_\epsilon}\}_{i_\epsilon}^{N_\epsilon}, \omega\right)$.

I construct a column vector **x**. The first block of **x** are the stack of the elements from the proxy distribution, such that $\mathbf{x}(j) = \mu_{i_a,i_\epsilon}$ where $j = (i_\epsilon - 1) \times N_a + i_a$. Next are the N_ϵ multipliers λ_i, followed by one multiplier ω. I solve for **x** using a system of linear equations, $\mathbf{Ax} = \mathbf{b}$ in Figure 6. The non-zero element of **A** and **b** are described here. The coefficients for μ_{i_a,i_ϵ} are entered into **A** as

$$\mathbf{A}((i_\epsilon - 1) \times N_a + i_a, (i_\epsilon - 1) \times N_a + i_a) = 2 \tag{114}$$

$$\mathbf{A}(N_\epsilon \times N_a + i_\epsilon, (i_\epsilon - 1) \times N_a + i_a) = 1 \text{ for } i_\epsilon = 1, \cdots, N_\epsilon \tag{115}$$

$$\mathbf{A}(N_\epsilon \times N_a + N_\epsilon + 1, (i_\epsilon - 1) \times N_a + i_a)) = a_{i_a}. \tag{116}$$

The coefficient for λ_i are entered in **A**, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$, as

$$\mathbf{A}((i_\epsilon - 1) \times N_a + i_a, N_\epsilon \times N_a + i_\epsilon) = -1 \tag{117}$$

The coefficients for ω sets the following elements of **A**, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$,

$$\mathbf{A}((i_\epsilon - 1) \times N_a + i_a, N_\epsilon \times N_a + N_\epsilon + 1) = -a_{i_a}. \tag{118}$$

The elements of **b** are, for $i_\epsilon = 1, \cdots N_\epsilon$ and $i_a = 1, \cdots, N_a$,

$$\mathbf{b}((i_\epsilon - 1) \times N_a + i_a) = 2\mu^r_{i_a,i_\epsilon}(i_m, i_\tau) \tag{119}$$

$$\mathbf{b}(N_\epsilon \times N_a + i_\epsilon) = \sum_{i_a=1}^{N_a} \mu^r_{i_a,i_\epsilon}(i_m, i_\tau) \tag{120}$$

$$\mathbf{b}(N_\epsilon \times N_a + N_\epsilon + 1) = m_{i_m}. \tag{121}$$

I solve $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$ iteratively using an active set method corresponding to probabilities that are not set to 0.

To solve the linear system, I use the active set approach to non-negative constraints in Reiter (2002, 2010). If any of the first $N_\epsilon \times N_a$ elements of **x** are negative, the constraint $\mu_{i_a,1_\epsilon} \geq 0$ has been violated for some $(i_\epsilon - 1)N_a + i_a = j \in J_0$ where

$$J_0 = \{j | 1 \leq j \leq N_a \times N_\epsilon \text{ and } \mathbf{x}(j) < 0\}. \tag{122}$$

For some $O > 0$, set the most negative O elements indexed in J_0 to 0, $\mu_{i_a,i_\epsilon} = 0$. Remove the $j - th$ row and column of A along with the $j - th$ element of **b**. Solve the reduced system with O

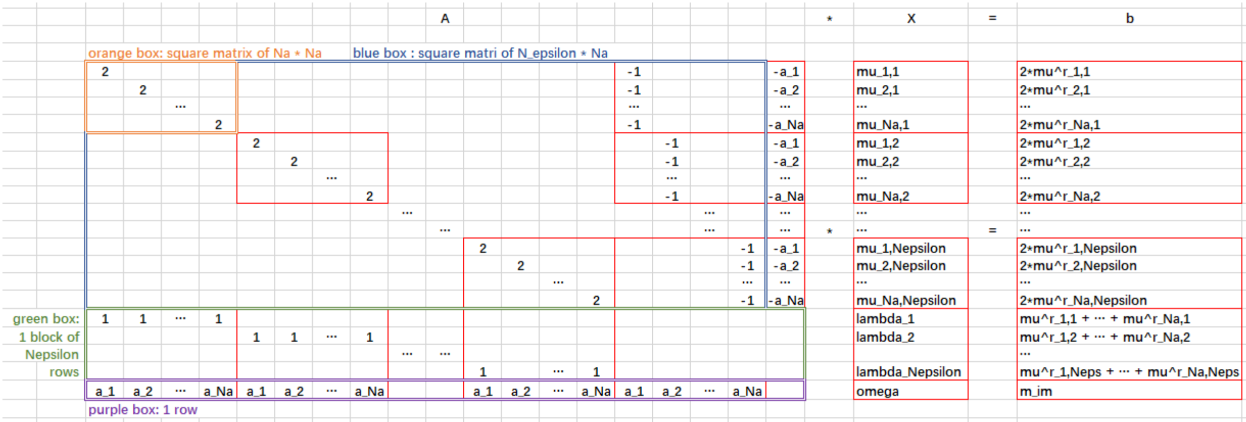

Figure 6: $\mathbf{A} \times \mathbf{x} = \mathbf{b}$

less rows. If any of the $N_\epsilon \times (N_a - O)$ elements are negative, again discard the most negative O. I repeat this procedure until the most negative elements of $\mathbf{x}$ is larger than a precision level. This iteratively implements the non-negativity of probabilities (122).

Table 4: Setting for Computation

	num. of nodes	Description
N_a	400(400)	asset (distribution)
N_ϵ	10	persistence wage process
N_m	10	aggregate capital (aggregate)
N_τ	10	income tax (aggregate)

Table 4 shows the setting of the grids in this paper. With this setting, I continue to repeat the whole steps above until no improvement in accuracy statistic proposed by Den Haan (2010). I find that the mean errors on the equilibrium path are sufficiently small (considerably less than 0.02% for all cases) and the maximum of the errors are also reasonably small (not exceeding 0.03% for all cases). Furthermore, the method is substantially efficient in a usual personal computer.

Table 5: Accuracy and Efficiency of the Solution Method

	Proportional Income Tax
Run time	20.49 min
AVG(DH) of m	0.219%
AVG(DH) of w	0.069%
AVG(DH) of τ	0.233%
MAX(DH) of m	0.856%
MAX(DH) of w	0.287%
MAX(DH) of τ	0.289%

$AVG(\cdot)$ and $MAX(\cdot)$ are computed on the equilibrium path.
Processor: AMD Ryzen Threadripper 3960X @ 3.8GHz, RAM: 256GB

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